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A Study on Radio Labeling of Diameter N-2 and Caterpillar Graphs

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Abstract –

Radio labeling of graphs is a specific type of graph labeling. The basic type of graph labeling is vertex coloring; this is where the vertices of a graph G are assigned different colors so that adjacent vertices are not given the same color. A k-coloring of a graph G is a coloring that uses k colors. The chromatic number of a graph G is the minimum value for k such that a k-coloring exists for G [2].

Keywords-* *Radio labeling, vertex coloring, k-coloring,caterpillar, N-2

I. INTRODUCTION AND PRELIMINARY RESULTS

Radio labeling of simple connected graphs is a specific type of graph labeling. The basic type of graph labeling is vertex coloring; this is where the vertices of a graph G are assigned different colors so that adjacent vertices are not given the same color. A k-coloring of a graph G is a coloring that uses k colors. The chromatic number of a graph G is the minimum value for k such that a k-coloring exists for G.

Radio labeling is another type of graph labeling that evolved as a way to use graph theory to try to solve a problem. Radio labeling addresses the channel assignment problem: how to assign radio channels or frequencies to different radio transmitters in an optimal way. This means we want to assign radio channels so that two radio transmitters that are geographically close to one another do not have channels with frequencies that interfere with one another. This problem of channel assignment was first put into a graph theoretic context by Hale [6]. In terms of graph theory, the vertices of a simple connected graph represent the locations of the radio transmitters, or radio stations, with the labels of the vertices corresponding to channels or frequencies assigned to the stations.

II. MAIN RESULTS

In this section, we present the main results about the new graph classes constructed using the old ones and study their on radio labeling N-2 diameter and caterpillar graph.

A. *k-radio Labeling*

There have been various restrictions used on labeling, or coloring, graphs in an e_ort to model the channel assignment problem. Chartrand and Zhang discussed the use of k-radio coloring of graphs and distance 2 labeling [3]. The k-radio coloring condition of graphs is when, given a graph G with diameter D and $1 \leq k \leq D$ with $f : V(G) \rightarrow Z+$ a coloring, the inequality

$d(u; v) + |f(u) - f(v)| \geq 1 + k$

is satisfied for all vertices u; v in G. The largest number used as a label under the labeling f is called the span of f. When k-radio labeling a graph, one tries to minimize the span of that particular graph.

When k = 1, k-radio coloring can be used to determine the chromatic number of a graph G. If f is a 1-radio labeling for G, then for adjacent vertices x and y, the condition that needs to be satis_ed becomes 1 + jf(x) □ f(y)j _ 1 + 1 which implies that jf(x)□f(y)j _ 1.

Definition. Given an ordering x1,……, xn of the vertices of a graph G and the associated radio labeling f, we say that f requires jumps if $\sum_{i=1}^{n=1} J_f(x_i, X_{i+1}) \geq 1$

Proposition 1.

Let G be a simple connected graph with n vertices and let x……….. xn be any ordering of the vertices of G with f the associated radio labeling. Then,

f(xn) = (n - 1)(D + 1) + f(x1) -$\sum^{n=1}$i=1 d(xi, xi+1) +$\sum^{n□1}$i=1 Jf (xi, xi+1).

Proof. The result is obtained by adding up the equations

d(x1, x2) + f(x2) - f(x1) = D + 1 + Jf (x1, x2);
d(x2, x3) + f(x3) - f(x2) = D + 1 + Jf (x2, x3);
.
.
d(xn-1; xn) + f(xn) - f(xn□1) = D + 1 + Jf (xn-1; xn):

Lemma 1. Let G be a graph with vertices v1…… vn and edges e1…. em. Let p be a bijection from the vertices of G to the set fx1…… xng. Let Pj be a fixed shortest path from xj to xj+1. Let n(ei) be the number of paths Pj that contain the edge ei. Then the following hold:

1. Each edge can appear in any path Pj at most once.
2. Let { e^k_{i1}............e^k_{ir} } be the set of all the edges incident to x_k. Then $n(e^k_{i1})$+………$n(e^k_{ir})$ is even unless k = 1 or k = n in which case the sum is odd.
3. Suppose e_i is an edge so that removing it from the graph gives a disconnected two component graph where the two components are denoted A and B. Furthermore assume that if x_j and x_{j+1} are both contained in the same component, then so is Pj . Then $n(e_i) \leq 2\min\{ |V(A)|, |V(B)| \}$

.4. Let fei1 ; :::; eirg be a set of edges so that no two of them are ever contained in the same Pj . Then n(ei1) +….. + n(eir) ≤ n - 1.

Proof. The first conclusion follows from the fact that Pj is a shortest path so it cannot contain any cycles.

The second conclusion follows from the fact that if xk is not the endpoint of a path P_j but the vertex is included in this path, two of its incident edges belong to the path. If x_k is the endpoint of a path, then exactly one of its incident edges is part of the path. For 1 < k < n, x_k is the endpoint of exactly two paths while each of x1 and x_n is an endpoint of exactly one of the paths.

A path Pj contains the edge e_i if and only if its endpoints are in different components of the graph obtained by deleting e_j. This observation verifies the third conclusion.The final conclusion follows from the fact that there are n - 1 paths and any edge can appear in a path at most once.Sometimes we will need a generalization of the third condition of Lemma 1, i.e., we will need to simultaneously remove multiple edges to disconnect a graph.

The following lemma describes the corresponding result in this case. In this thesis, this generalization will only be needed when we consider graphs with n vertices and diameter n - 2 that are not tree graphs in section 3.3.

III. GRAPHS WITH N VERTICES AND DIAMETER N -2

Much of this chapter will be devoted to studying a family of graphs which we call spire graphs, which are paths with an extra leg vertex. More formally, we have the following:

A. Definition.

Let n, s € Z where n ≥ 4 and 2 ≤ s ≤ n - 2. The spire graph $S_{n,s}$ is
the graph with vertices v1…….. vn and edges { $|(v_i , v_{i+1})|$ i = 1; 2....... n-2} together with the edge (v_s , v_n). The vertex v_n is called the spire. Without loss of generality we will always assume that s ≤ n/2

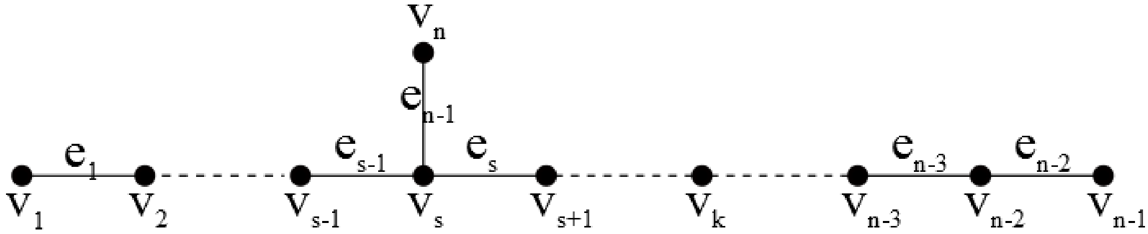

Fig. 1 SPIRE GRAPH

Theorem 3 (Upper bound for $S_{n,s}$). Let $S_{n,s}$ be a spire graph, where 2 ≤ s ≤ [n/2]
Then,

$$R\,n(s_{n,s}) = \begin{cases} 2k^2 - 4k + 2s + 3 \text{ if } n = 2k \text{ and } 2 \leq s \leq k - 2; \\ 2k^2 - 2k \text{ if } n = 2k \text{ and } s = k -1; \\ 2k^2 - 2k + 1 \text{ if } n = 2k \text{ and } s = k; \\ 2k^2 - 2k + 2s \text{ if } n = 2k + 1: \end{cases}$$

B. Proof:

To establish this bound we de_ne a labeling with the appropriate span. The cases for n even and n odd are discussed separately.

Case I: First consider the case when n = 2k for some k € Z. The upper bounds for cases when k < 7 that are not included in this proof are shown explicitly in Appendix A.

Subcase A: 2 ≤ s ≤ k - 2 and k ≥ 7. Order the vertices of $S_{n,s}$ into three
groups as follows:

Group I: v_k; v_{2k}; v_{k+4}; v_5; v_{k+3}; v_3; v_{k+2}; v_4;

Group II: v_{k+5}; v_6; v_{k+6}; v_7……… v_{k+m}; v_{m+1}………$v_{k+(k-3)}$; $v_{k-2;}$

Group III: v2k-2; v2; vk+1; v1; v2k-1; vk-1.

In this ordering Group I always contains the same 8 vertices and Group III always contains the same 6 vertices. Group II follows the indicated pattern and contains n - 14 vertices.

Now, rename the vertices of Sn;s in the above ordering by x1; x2……….. xn where

x1 = vk, x2 = v2k, etc. In Table 3.1 we define a labeling f of Sn;s. We will let f(x1) = 1. The first column in the table gives the order in which the vertices are labeled, i.e., the inequality f(xi) > f(xi-1) always holds.

The second column reminds the reader which vertex we are labeling. In the third column we have computed the distance between xi and xi+1. Finally in the last column we give the difference between the labels f(xi) and f(xi+1). Given that f(x1) = 1, one can use the last column to compute f(xi) by summing the first i - 1 entries of the column and then adding one to this sum.

Claim: The function f defined in Table 3.1 is a radio labeling on Sn;s.

Table 3.1 Radio Labeling on Sn;s.

xi	**Vertex Names**	**d(xi; xi+1)**	**f(xi+1) - f(xi)**
X1	V_K	K-S+1	K+S-2
X2	V_{2K}	K-S+5	K+S-6
X3	$V_{K+!}$	K-1	K
X4	V_5	K-2	K+1
X5	V_{K+3}	K	K-1
X6	V_3	K-1	K
X7	V_{K+2}	K-2	K+1
X8	V_4	K+1	K-2
x9	vk+5	k – 1	K
x10	v6	k	K -1
...	.	.	.
...	.	.	.
...	.	.	.
...	.		
x2m-1	.	k – 1	K
x2m □ 1		k	k
...	vk+m	.	.
...	vm+1	.	.
...	.	.	.
...	.		
Xn-7	.	k - 1	K
Xn-6	.	K	k □ 1
	vk+(k-3)		
	vk-2		
Xn-5	v2k-2	2k – 4	4
Xn-4	v2	k – 1	K
Xn-3	vk+1	K	K-1
Xn-2	v1	2k – 2	2
Xn-1	v2k-1	K	K-1
xn	vk-1	n/a	n/a

IV. CONCLUSION

It is an improved lower bound for a particular type of caterpillar with one center edge such that) is even, but the exact radio number has not yet been determined. However, some more specific conclusions can be made about edge-balanced, vertex-balanced, and almost vertex-balanced caterpillars .Corollaries 2 and 3 establish a way to determine when the bound for the radio number given by Proposition 2 is increased for an edge-balanced, vertex-balanced, or almost vertex-balanced caterpillar G based on the structure of G

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