

# MHD Free Convective Flow Past a Vertical Porous Plate in a Porous Medium in the Presence of Soret and Dofour Effects

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## Abstract:

**A** steady MHD free convective heat and mass transfer flow past a semi-infinite vertical porous plate in the presence of Soret and Dofour effects has been studied in this paper. The governing equations are solved numerically by Runge – Kutta fourth order method along with shooting technique. The effect of various governing parameters for velocity, temperature, concentration; skin-friction, Nusselt number and Sherwood number are also obtained. The effects of various parameters have been shown numerically and discussed graphically.

**Keywords:** MHD

## I. INTRODUCTION

In recent years, the flows of fluid through porous media are of principal interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholars due to their application in many branches of science and technology, viz., in the field of agriculture engineering to study the underground water resources, seepage of water in river-beds, in petroleum technology to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification processes. The convection problem in porous medium has also important applications in geothermal reservoirs and geothermal energy extractions. Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. Considering these applications, steady MHD free convective flow past a heated vertical flat plate has been studied by many researchers such as Gupta [1], Lykoudis [2] and Nanda and Mohanty [3]. Some of them are Raptis and Kafoussias [5] investigated heat and mass transfer effects on steady MHD over a porous medium bounded by an infinite vertical porous plate with constant heat flux. Hiremath and Patil [6] studied the effect on free convection currents on the oscillatory flow through a porous medium, which is bounded by vertical plane surface of constant temperature. Vajravelu and Hadjinicolaou [7] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. A comprehensive review of the studies of convective heat transfer mechanism through porous media has been made by Nield and Bejan [8]. Anghel et al [9] studied the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. El-Amin [10] studied the MHD free convection and mass transfer flow in a micropolar fluid over a stationary vertical plate with constant suction. Hossain et al [12] studied problem of the natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Kim [13] found that the effects of heat and mass transfer on MHD micropolar flow over a vertical moving porous plate in a porous medium. Postelnicu [14] analyzed the influence of magnetic field on heat and mass transfer from vertical surfaces in porous media considering Soret and Dufour effects.

Chaudhary and Sharma [18] considered combined heat and mass transfer by laminar mixed convection flow from a vertical surface with induced magnetic field. The interest in these recent problems generates from their importance in liquid metals, electrolytes and ionized gases. Many researchers have studied MHD free convective heat and mass transfer flow in a porous medium. Alamet al. [19] investigated the Dufour and Soret effects on steady MHD mixed convective and mass transfer flow past a semi-infinite vertical plate. Fluctuating heat and mass transfer on three-dimensional flow through porous medium with variable permeability has been discussed by Sharma et al [20]. Chamkha and Ben-Nakhi [21] analyzed MHD mixed convection-radiation interaction along a permeable surface immersed in a porous medium in the presence of Soret and Dufour's effects. A comprehensive account of the available information in this field is provided in books by Pop and Ingham [11], Ingham and Pop [15], Vafai [17], Vadasz [25], etc.

Makinde and Ogulu [23] studied the effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Kesavaiah et al [28] reported that the effects of the chemical reaction and radiation absorption on unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Mohammed Ibrahim and Bhaskar Reddy [30] investigated heat and mass transfer effects on steady MHD free convective flow along a stretching surface with dissipation, heat generation and radiation. Singh and Shweta Agarwal [31] studied the Heat transfer in a second-grade fluid over an exponentially stretching sheet through porous medium with thermal radiation and

elastic deformation under the effect of magnetic field. Mohammed Ibrahim et al [32] found the radiation effects on MHD free convection flow of a micropolar fluid past a stretching surface over a non-Darcian porous medium.

At high operating temperatures, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are example of such engineering areas. The effect of radiation on MHD flow and heat transfer problem has become more important industrially.

The study of heat generation in moving fluids is important as it changes the temperature distribution and the particle deposition rate particularly in nuclear reactor cores, fire and combustion modeling, electronic chips and semiconductor wafers. Heat generation is also important in the context of exothermic or endothermic chemical reaction.

But in the above-mentioned studies, Dufour and Soret terms have been neglected from the energy and concentration equations respectively. It has been found that energy flux can be generated not only by temperature gradient but also by concentration gradient as well. The energy flux caused by concentration gradient is called Dufour effect and the same by temperature gradient is called the Soret effect. These effects are very significant when the temperature and concentration gradient are very high. Many researchers have studied Dufour and Soret effects on free convective heat and mass transfer flow in a porous medium.; some of them are M.S. Alam and M.M Rahman [16], Mansour et al [24], El-Aziz [22], Afify [26], Kishore et al [27], Alam and Ahammad [29], SreedharSarma et al [32], Raju et al [34], Vedavathi et al [35].

The aim of this paper is to discuss the Dufour and Soret effects on MHD free convection flow past a vertical porous plate placed in porous medium in the presence of chemical reaction, thermal radiation and heat source. The set of governing equations and boundary equation of the problem are transformed into a set of nonlinear ordinary differential equation with assisting of similarity transformations are solved using the shooting method along with fourth order Runge-Kutta integration scheme. The effects of different physical parameters on the velocity, temperature and concentration profiles as well as the local skin-friction coefficient, local Nusselt number and local Sherwood number are presented graphically and in tabular form. To verify the obtained results, we have compared the present numerical results with previous work by Alam and Rahman [16]. The comparison results show a good agreement and we confident that our present numerical results are accurate.

## II. MATHEMATICAL FORMULATION

A steady two-dimensional flow of an incompressible and electrical conducting viscous fluid, along an infinite vertical porous plate embedded in a porous medium is considered. The  $x$ - axis is taken on the infinite plate, and parallel to the free-stream velocity which is vertical and the  $y$ - axis is taken normal to the plate. A magnetic field  $B_0$  of uniform strength is applied transversely to the direction of the flow. Initially the plate and the fluid are at same temperature  $T_\infty$  in a stationary condition with concentration level  $C_\infty$  at all points. The plate starts moving impulsively in its own plane with velocity  $U_0$ , its temperature is raised to  $T_w$  and the concentration level at the plate is raised to  $C_w$ . The fluid is assumed to be slightly conducting, and hence the magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Roseland approximation is used to describe the radiative heat flux in the energy equation. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussineq's approximation). Then, under the above assumptions, the governing boundary layer equations are

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u - \frac{b}{K} u^2 \tag{2}$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) \tag{3}$$

Concentration equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

where  $u, v$  are the Darcian velocities components in the  $x$  and  $y$  directions respectively,  $\nu$  is the kinematic viscosity,  $g$  is the acceleration due to gravity,  $\rho$  is the density,  $\beta$  is the coefficient of volume expansion with temperature,  $\beta^*$  is the volumetric coefficient of expansion with concentration,  $b$  is the empirical constant is  $T, T_w$  and  $T_\infty$  are the temperature

of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively,  $C, C_w$  and  $C_\infty$  are the corresponding concentrations,  $K$  is the Darcy permeability,  $\sigma$  is the electric conductivity,  $\alpha$  is the thermal diffusivity,  $c_p$  is the specific heat at constant pressure,  $k_T$  is the thermal diffusion ratio,  $c_s$  is the concentration susceptibility, the term  $Q_0(T - T_\infty)$  is assumed to be amount of heat generated or absorbed per unit volume and  $Q_0$  is a constant, which may take on either positive or negative values,  $q_r$  is the radiative heat flux in the  $y$ -direction,  $D_m$  is the coefficient of mass diffusivity.

The boundary conditions for velocity, temperature and concentration fields are given by

$$u = U_0, v = v_0(x), T = T_w, C = C_w \text{ at } y = 0$$

$$u = 0, v = 0, T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty \tag{5}$$

where  $U_0$  is the uniform velocity and  $v_0(x)$  is the velocity of suction at the plate.

Using the Rosseland approximation for radiation, radiative heat flux is given by Sparrow and Cess[4]

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{6}$$

where  $\sigma^*$  and  $k^*$  are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow are such that the term  $T^4$  may be expressed as a linear function of temperature. Hence, expanding  $T^4$  in a Taylor series about  $T_\infty$  and neglecting higher order terms we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Using equations (6) and (7) equation (3) becomes

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^*} \frac{1}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) \tag{8}$$

The equations (2), (4) and (8) are coupled, parabolic and nonlinear partial differential equations and hence analytical solution is not possible. Therefore, numerical technique is employed to obtain the required solution. Numerical computations are greatly facilitated by non-dimensionalization of the equations. Proceeding with the analysis, we introduce the following similarity transformations and dimensionless variables which will convert the partial differential equations from two independent variables  $(x, y)$  to a system of coupled, non-linear ordinary differential equations in a single variable  $(\eta)$  i.e., coordinate normal to the plate.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\eta = y \sqrt{\frac{U_0}{2\nu x}}, \quad \psi = \sqrt{\nu x U_0} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{9}$$

where  $f(\eta)$  is the dimensionless stream function and  $\psi$  is the dimensional stream function defined in the usual way

$$u = \frac{\partial \psi}{\partial x} \text{ and } v = -\frac{\partial \psi}{\partial y}$$

Clearly the continuity equation (1) is identically satisfied, Then introducing the relation (9) into equation (1) we obtain

$$u = U_0 f'(\eta) \text{ and } v = \sqrt{\frac{\nu U_0}{2x}} (\eta f' - f) \tag{10}$$

Further introducing equations (9) and (10) into momentum equation (2), Energy equation (8) and Concentration equation (4) we obtain the following local similarity equations

$$f''' + ff'' + Gr\theta + Gc\phi - Mf' - \frac{1}{Da Re} f' - \frac{Fs}{Da} f'^2 = 0 \tag{11}$$

$$\left(1 + \frac{16}{3R}\right) \theta'' + Pr f \theta' + Pr Du \phi'' + Pr Q \theta = 0 \tag{12}$$

$$\phi'' + Scf \phi' + ScSr \theta'' = 0 \tag{13}$$

where

$$Gr = \frac{g\beta(T_w - T_\infty)2x}{U_0^2} \text{ is the Grashof number, } Gm = \frac{g\beta^*(C_w - C_\infty)2x^2}{\nu U_0} \text{ is modified Grashof number,}$$

$$M = \frac{\sigma B_0^2 2x}{\rho U_0} \text{ is the magnetic field parameter, } Da = \frac{K}{2x^2} \text{ is the Darcy number, } Re = \frac{U_0 x}{\nu} \text{ is the Reynolds number,}$$

$$Fs = \frac{b}{x} \text{ is the Forchheimer number, } Pr = \frac{\nu}{\alpha} \text{ is the Prandtl number, } R = \frac{k^* \alpha \rho c_p}{\sigma T_\infty^3} \text{ is the Radiation parameter,}$$

$$Du = \frac{D_m K_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)} \text{ is the Dufour number, } Sr = \frac{D_m K_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)} \text{ is the Soret number, } Q = \frac{Q_0 \nu}{\rho c_p U_0^2} \text{ is the}$$

heat generation parameter,  $Sc = \frac{\nu}{D_m}$  is the Schmidt number.

The corresponding boundary conditions are

$$f = f_w, f' = 1, \theta = 1, \phi = 1 \text{ at } \eta = 0,$$

$$f' = 0, \theta = 0, \phi = 0 \text{ as } \eta \rightarrow \infty \tag{14}$$

Where  $f_w = -v_0 \sqrt{\frac{2x}{\nu U_0}}$  is the dimensionless suction velocity and primes denote partial differentiation with respect to the variable  $\eta$ .

The parameters of engineering interest for the present problem are the skin-friction coefficient, the Nusselt number and the Sherwood number, which are given respectively by the following expressions. Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$\frac{1}{2} Re^{\frac{1}{2}} C_f = f''(0) \tag{15}$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form, in terms of Nusselt number, is given by

$$Nu Re^{\frac{1}{2}} = -\theta'(0) \tag{16}$$

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form, in terms of Sherwood number, is given by

$$Sh Re^{\frac{1}{2}} = -\phi'(0) \tag{17}$$

Where  $Re = \frac{U_0 x}{\nu}$  is the Reynold's number.

### III. SOLUTION OF THE PROBLEM

The numerical solutions of the non-linear differential equations (11) – (13) under the boundary conditions (14) have been performed by applying a shooting method along with the fourth order Runge-Kutta method. First of all, higher order non-linear differential equations (11) – (13) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. From this process of numerical computation, the skin-friction coefficient, the Nusselt number and Sherwood number which are respectively proportional to  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  are also sorted out and their numerical values are presented in a tabular form.

### IV. RESULTS AND DISCUSSION

From the numerical computations, dimensionless velocity, temperature and concentration profiles as well as the skin-friction coefficient, Nusselt number and Sherwood number are found for different values of the various physical parameters occurring in the problem. The value of Prandtl number  $Pr$  is taken to be 0.71 which corresponds to air and the value of Schmidt number  $Sc$  is chosen 0.22, which represents hydrogen at 25<sup>0</sup> C and 1 atm. Due to free convection problem positive large values of  $Gr = 12$  and  $Gm = 6$  are chosen. The value of  $Re$  is kept 100 and  $F_s$  equal to 1.0. The values of Dufour number and Soret number are chosen in such a way that their product is constant provided that the mean temperature  $T_m$  is constant as well. However, the values of Darcy number  $Da = 1.0$ , magnetic field parameter  $M = 1.0$ , suction parameter  $f_w = 0.5$ , radiation parameter  $R = 1.0$ , heat generation parameter  $Q = 1.0$  are chosen arbitrarily. The numerical results for velocity, temperature and concentration profiles are displayed in Figures. 2 to 13.

The effect of Grashof number  $Gr$  on the velocity field is presented in Figure.1. The Grashof number  $Gr$  signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As Grashof number  $Gr$  increases the velocity of the fluid increases. Figure.2. present velocity profiles in the boundary layer for various values of modified Grashof number  $Gm$ . The modified Grashof number  $Gm$  defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As modified Grashof number  $Gm$  increases the fluid velocity increases. The effect of Darcy number  $Da$  on the temperature field is shown in Figure.3. From this figure, we observe that velocity increases with the increase of Darcy number  $Da$ . For large Darcy number porosity of the medium increases, hence fluid flows quickly. The effect of Reynolds number  $Re$  on the velocity fields are shown in Figure.4. It is noted that negligible effect of Reynolds number on velocity profiles.

Figures. 5(a) – 5(c) depicts the effect of Forchheimer number  $Fs$  on the velocity, temperature and concentration profiles. It is observed from Figure. 5(a) that the velocity of the fluid decreases with the increase of Forchheimer number  $Fs$ . Since Forchheimer number  $Fs$  represents the inertial drag, thus an increase in the Forchheimer number  $Fs$  increases the resistance to the flow and so a decrease in the fluid velocity ensues. It is noticed from Figure. 5(b) that temperature of the fluid increases with increase of Forchheimer number  $Fs$ , since as the fluid is decelerated; energy is dissipated as heat and serves to increase temperature. From Figure. 5(c), it is observed that the concentration of the fluid increases with increase of the Forchheimer number  $Fs$ .

Figure.6 (a), 6(b) and 6(c) display the velocity, temperature and concentration profiles for different values of magnetic field parameter  $M$  when the other parameters are fixed. An applied of a magnetic field within boundary layer has produced resistive-type force which known as Lorentz force. This force act to retard the fluid motion along surface and simultaneously increase its temperature and concentration values. Therefore, one can see that the velocity boundary layer thickness decreases with the increase of magnetic field parameter  $M$  as shown in Figure.6 (a). However, the temperature and concentration increase with the increasing of the magnetic field parameter  $M$  shown in Figure.6 (b) and Figure.6(c).

Figure.7 (a). Illustrates the velocity profiles for different values of the Prandtl number  $Pr$ . The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Figure.7 (b), it is observed that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of  $Pr$  are equivalent to increasing the thermal conductivities, and therefore heat can diffuse away from the heated plate more rapidly than for higher values of  $Pr$ . Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

The effect of the radiation parameter  $R$  on the velocity and temperature profiles is shown in Figures. 8(a) and 8(b) respectively. Figure.8 (a) shows that velocity profiles decreases with an increase in the radiation parameter  $R$ . From Figure.8 (b), it is seen that the temperature decreases as the radiation parameter  $R$  increase. This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

Figures. 9(a) and 9(b) depict the velocity and temperature profiles for different values of the heat generation parameter  $Q$ . It is noticed that an increase in the heat generation parameter  $Q$  results in an increase in velocity and temperature within the boundary layer. Figure.10 (a), 10(b) and 10(c) show the combination effects of the Dufour and Soret numbers on the fluid velocity, temperature and concentration respectively. The Dufour number  $Du$  and Soret number  $Sr$  represent the thermal- diffusion and diffusion-thermal effects in this problem. Figure.10 (a), shows the influences of the Dufour and Soret number on the variations of the fluid velocity. For the case of increasing Dufour number and decreasing Soret number, it is seen that the velocity profiles decreases.

Figure.10(b), illustrate the effects of the Dufour and Soret number on the variations of the fluid temperature. From Figure.10 (b), we observe that an increasing Dufour number and decreasing Soret number, it is seen that the temperature profiles increases. The Dufour term describes the effect of concentration gradients as noted in Equation (12), plays a vital role in assisting the flow and able to increase thermal energy in the boundary layer. This is the evident for the increasing values in the fluid temperature as the Dufour number  $Du$  increase and the Soret number  $Sr$  decrease. In

Figure.10(c), as increasing Dufour number  $Du$  and simultaneously decreasing Soret number  $Sr$  has implies significant effects on the concentration profiles. The Soret term exemplifies the temperature gradient effects on the variation of concentration as noted in Equation (13). It is observed as the Dufour number increase and Soret number is decrease, the concentration values are found to be decreases.

The influence of Schmidt number  $Sc$  on the velocity and concentration profiles is plotted in Figures.11 (a) and 11 (b) respectively. As the Schmidt number  $Sc$  increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figures. 11 (a) and 11 (b).

The effects of suction parameter  $f_w$  on the velocity profiles are shown in Figure.12 (a). It is found from Figure.12 (a) that the velocity profiles decrease monotonically with the increase of suction parameter indicating the usual fact that suction stabilizes the boundary layer growth. The effects of suction parameter on the temperature and concentration field are displayed in Figure.12 (b) and Figure.12(c) respectively. From Figure.12 (b), it is noticed that the temperature decreases with an increase of suction parameter  $f_w$ . From Figure.12 (c), it is observed that the concentration decreases with an increase of suction parameter  $f_w$ .

The variation of skin-friction coefficient, heat and mass transfer coefficient with radiation parameter  $R$  and magnetic field parameter  $M$  are shown in Figures.13 (a), 13 (b) and 13 (c) respectively. We observe that the effect of increasing  $M$  is the decrease in the heat and mass transfer and skin friction coefficient. On the other hand, the magnitude of the heat and mass transfer increases while that of skin friction coefficient decreases as radiation parameter  $R$  increases.

Table 1 shows the comparison of  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  with those reported by M.S. Alam and M.M Rahman[26], which show a good agreement and we confident that our present numerical results are correct.

Table 1 Comparison values of  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  with different values of Soret and Dufour numbers.

$Sr$	$Du$	$f''(0)$		$-\theta'(0)$		$-\phi'(0)$	
		Alam and Rahman [26]	Present	Alam and Rahman [26]	Present	Alam and Rahman [26]	Present
2.0	0.03	3.4231141	3.42938	1.0283189	1.02699	0.1296854	0.12749
1.0	0.06	3.3457474	3.35237	1.0155338	1.01386	0.2992750	0.297643
0.5	0.12	3.3162482	3.32295	1.0019868	1.00022	0.3844602	0.382995
0.4	0.15	3.3141130	3.32085	0.9965224	0.994735	0.4017999	0.400363
0.2	0.30	3.3287043	3.33558	0.9718535	0.969957	0.4381199	0.436742
0.1	0.60	3.3828661	3.38997	0.9248360	0.922756	0.4602605	0.458919

Table 2, and 3 shows the effects of Grashof number  $Gr$ , modified Grashof number  $Gm$ , Darcy number  $Da$ , magnetic parameter  $M$ , suction parameter  $f_w$ , Prandtl number  $Pr$ , radiation parameter  $R$ , heat generation parameter  $Q$ , and Schmidt number  $Sc$  on the physical parameters skin-friction coefficient  $f''(0)$ , Nusselt number  $-\theta'(0)$  and Sherwood number  $-\phi'(0)$  respectively. All  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  increases as Grashof number  $Gr$ , modified Grashof number  $Gm$ , Darcy number  $Da$ , and suction parameter  $f_w$  increases.  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  decreases as magnetic field parameter  $M$  increases.  $-\theta'(0)$  increase as Prandtl number  $Pr$  or radiation parameter  $R$  increases, while it is decreases as heat generation parameter  $Q$ .  $-\phi'(0)$  increase as Schmidt number  $Sc$  increases.

Table 2: Numerical values of skin-friction coefficient ( $Cf$ ), Nusselt number ( $Nu$ ) and Sherwood number ( $Sh$ ) for,  $Pr = 0.71, Fs = 1.0, Re = 100, R = 1.0, Du = 0.12, Sr = 0.5, Sc = 0.22, Q = 1.0$ .

$Gr$	$Gm$	$Da$	$M$	$f_w$	$Cf$	$Nu$	$Sh$
12	6.0	1.0	1.0	0.5	6.34454	0.277773	0.921802
5	6.0	1.0	1.0	0.5	3.42049	0.287881	0.879047
10	6.0	1.0	1.0	0.5	5.54205	0.297256	0.910302
12	2.0	1.0	1.0	0.5	5.05881	0.288464	0.907464
12	4.0	1.0	1.0	0.5	5.70716	0.297754	0.914682
12	6.0	2.0	1.0	0.5	7.43037	0.246511	0.941016
12	6.0	3.0	1.0	0.5	7.90166	0.289481	0.949455
12	6.0	1.0	2.0	0.5	5.50345	0.295472	0.907285
12	6.0	1.0	3.0	0.5	4.76842	0.229296	0.894937
12	6.0	1.0	1.0	1.0	6.65476	0.307874	0.981983
12	6.0	1.0	1.0	2.0	7.12034	0.375616	1.10796

Table 3: Numerical values of skin-friction coefficient ( $Cf$ ), Nusselt number ( $Nu$ ) and Sherwood number ( $Sh$ ) for  $Gr = 12.0, Gm = 6.0, Da = 1.0, M = 1.0, f_w = 0.5, Du = 0.12, Sr = 0.5$

$Pr$	$R$	$Q$	$Sc$	$Cf$	$Nu$	$Sh$
0.71	1.0	1.0	0.22	6.34454	0.277773	0.921803
1.0	1.0	1.0	0.22	6.1504	0.355013	0.910383
1.5	1.0	1.0	0.22	5.8776	0.470762	0.893565
0.71	2.0	1.0	0.22	6.06827	0.34458	0.909912
0.71	3.0	1.0	0.22	5.90103	0.389547	0.902282
0.71	1.0	0.1	0.22	6.27499	0.366377	0.911571
0.71	1.0	0.5	0.22	6.3056	0.327556	0.916059
0.71	1.0	1.0	0.6	6.09895	0.277768	1.26436
0.71	1.0	1.0	0.78	6.01588	0.277751	1.41114

Finally, the effects of Soret number  $Sr$  and Dufour number  $Du$  on the skin-friction coefficient, Nusselt number and Sherwood number are shown in Table 4. The behavior of these parameters is self-evident from the Table 4 and hence they will not discuss any further due to brevity.

Table 4: Numerical values of skin-friction coefficient ( $Cf$ ), Nusselt number ( $Nu$ ) and Sherwood number ( $Sh$ ) for,  $Gr = 12.0$ ,  $Gm = 6.0$ ,  $Da = 1.0$ ,  $M = 1.0$ ,  $f_w = 0.5$ ,  $Du = 0.12$ ,  $Sr = 0.5$ ,  $Pr = 0.71$ ,  $Fs = 1.0$ ,  $Re = 100$ ,  $R = 1.0$ ,  $Sc = 0.22$ ,  $Q = 1.0$ .

$Sr$	$Du$	$Cf$	$Nu$	$Sh$
0.5	0.12	6.34454	0.277773	0.921803
1.0	0.12	6.35474	0.278037	0.920257
2.0	0.12	6.37527	0.278545	0.917239
0.5	0.03	6.3325	0.284933	0.920915
0.5	0.06	6.33651	0.282548	0.921211

### V. CONCLUSIONS

In this paper, a mathematical model has been presented for the influence of radiation and heat generation on MHD free convective flow past a vertical porous plate in a porous medium under the influence of Dufour and Soret effects. Using the similarity transformation, a set of ordinary differential equations has been derived for the conservation of mass, momentum and species diffusion in the boundary layer. These nonlinear, coupled differential equations solved under valid boundary conditions using fourth order Runge-Kutta method with shooting technique. The conclusions of the study are as follows:

- The velocity increases with increase of Grashof number and modified Grashof number.
- The velocity decreases with an increase in the magnetic field parameter and Permeability parameter.
- The temperature and velocity of the fluid increases with increase of Radiation parameter.
- The temperature and velocity of the fluid increases with increase of heat source parameter.
- As increasing Radiation parameter, the skin-friction coefficient and Nusselt number decrease.
- As increases Dufour number and increasing Soret number, it is seen that the temperature profiles increase.
- As the Schmidt number  $Sc$  increases the concentration decreases.

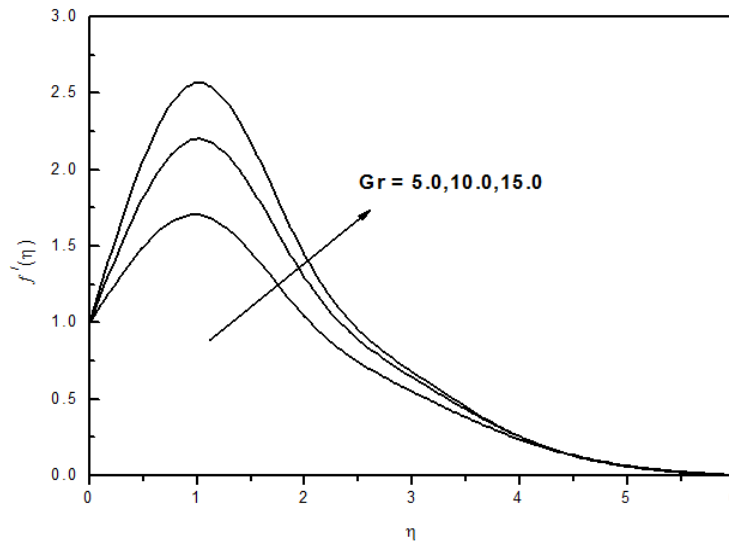


Figure.1. Velocity profiles for different values of  $Gr$

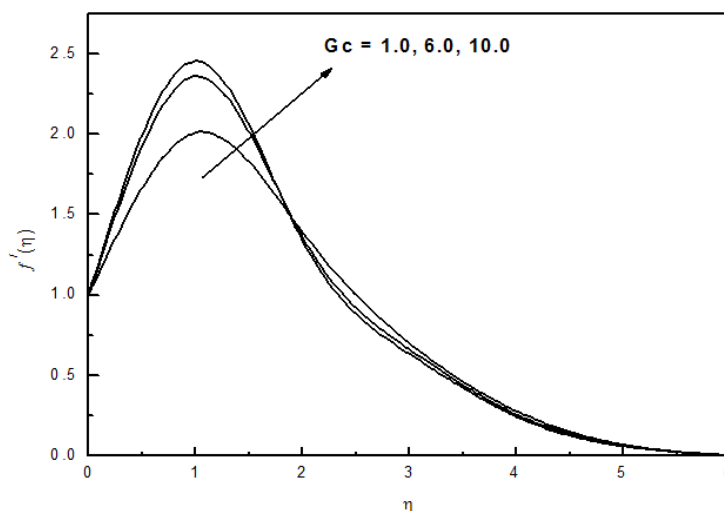


Figure.2. Velocity profiles for different values of  $Gm$

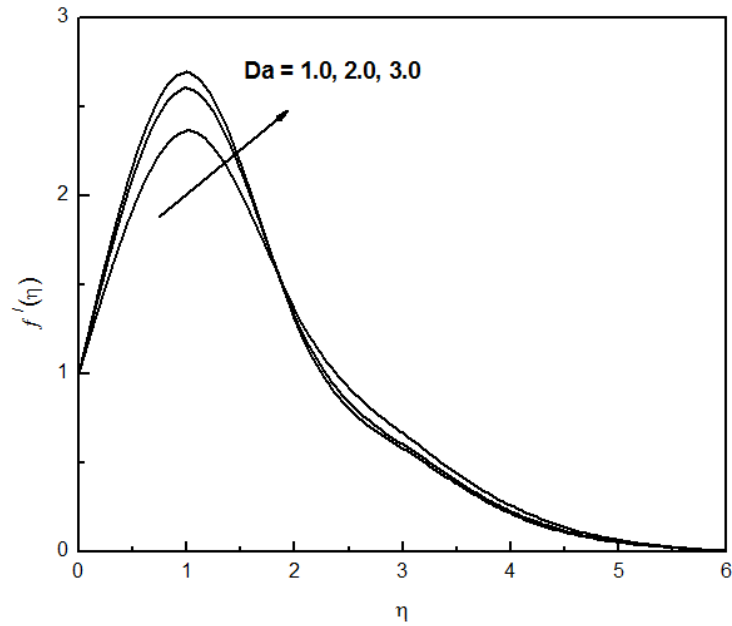


Figure.3. Velocity profiles for different values of  $Da$

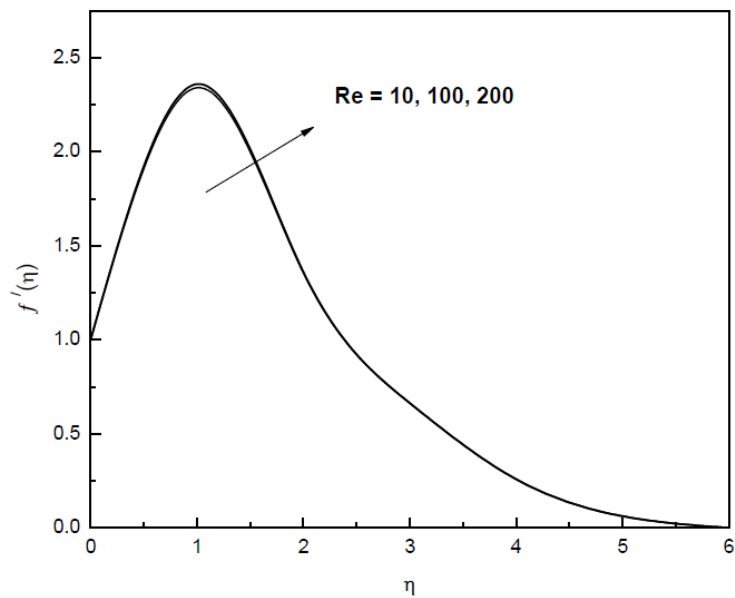


Figure.4. Velocity profiles for different values of  $Re$

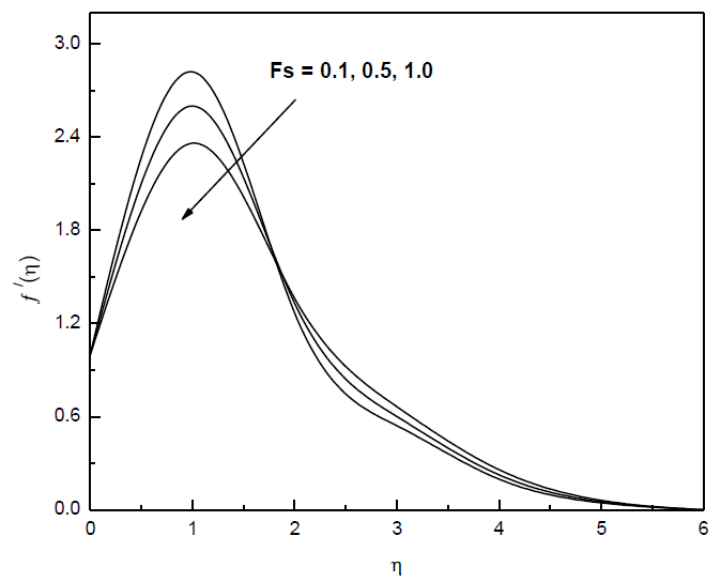


Figure.5(a). Velocity profiles for different values of  $F_s$



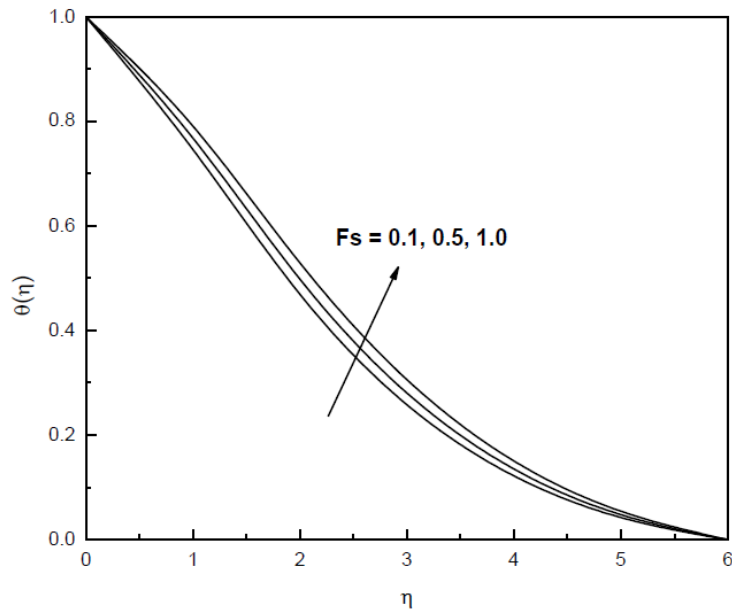


Figure.5(b). Temperature profiles for different values of  $F_s$

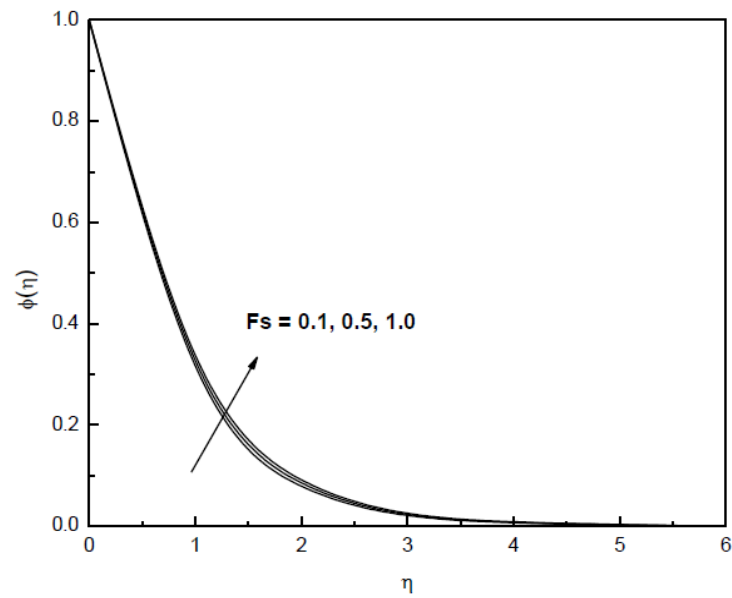


Figure.5(c). Concentration profiles for different values of  $F_s$

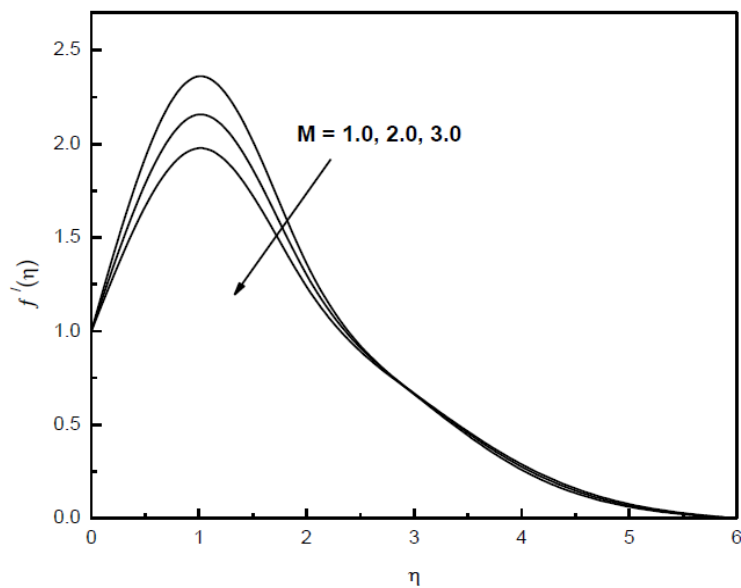


Figure.6 (a). Velocity profiles for different values of  $M$

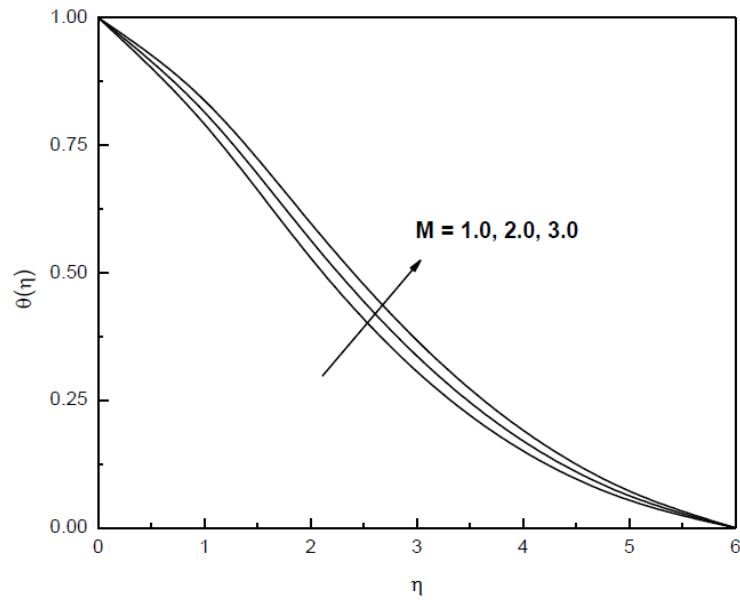


Figure.6 (b). Temperature profiles for different values of  $M$

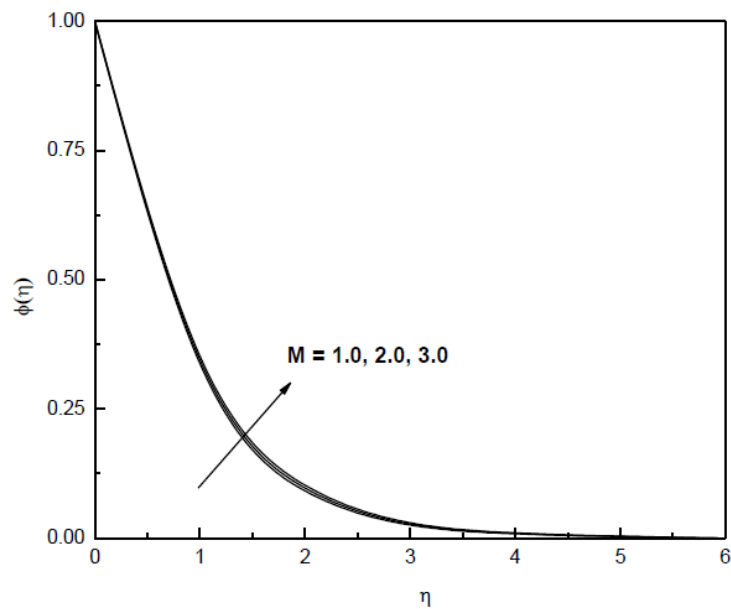


Figure.6(c). Concentration profiles for different values of  $M$

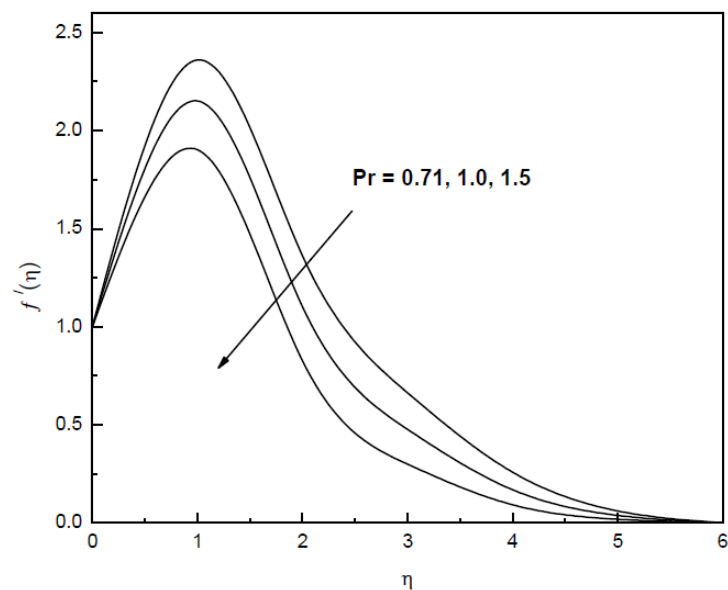


Figure.7(a). Velocity profiles for different values of  $Pr$

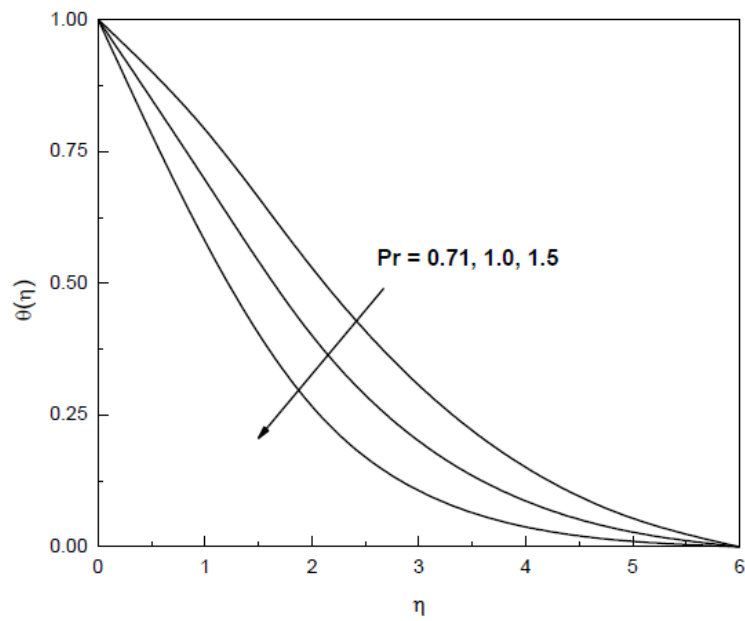


Figure.7(b). Temperature profiles for different values of  $Pr$

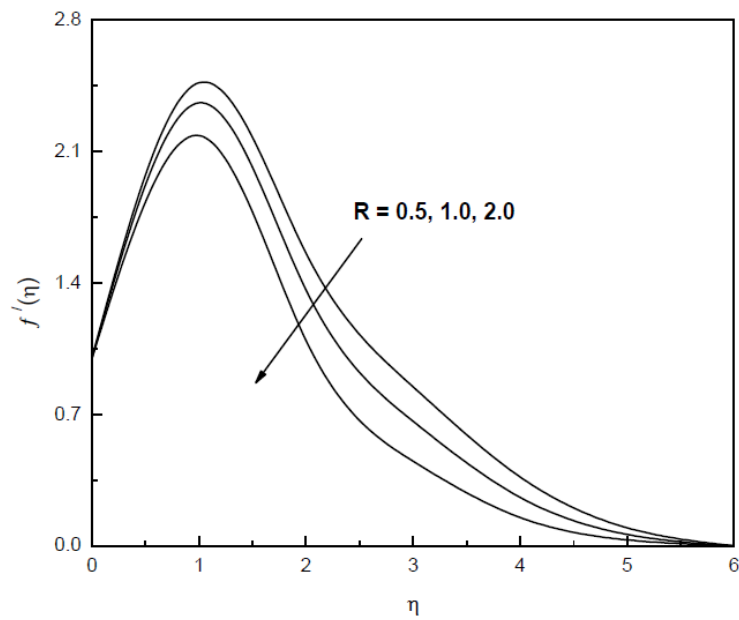


Figure.8(a). Velocity profiles for different values of  $R$

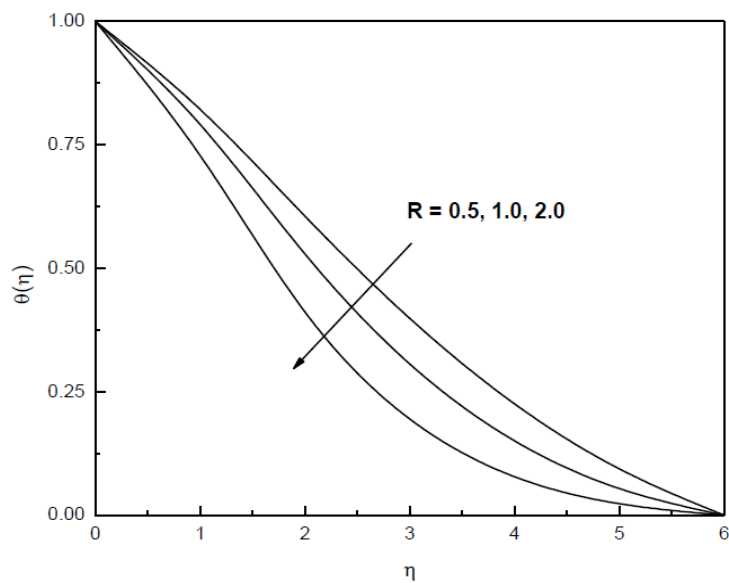


Figure.8(b). Temperature profiles for different values of  $R$

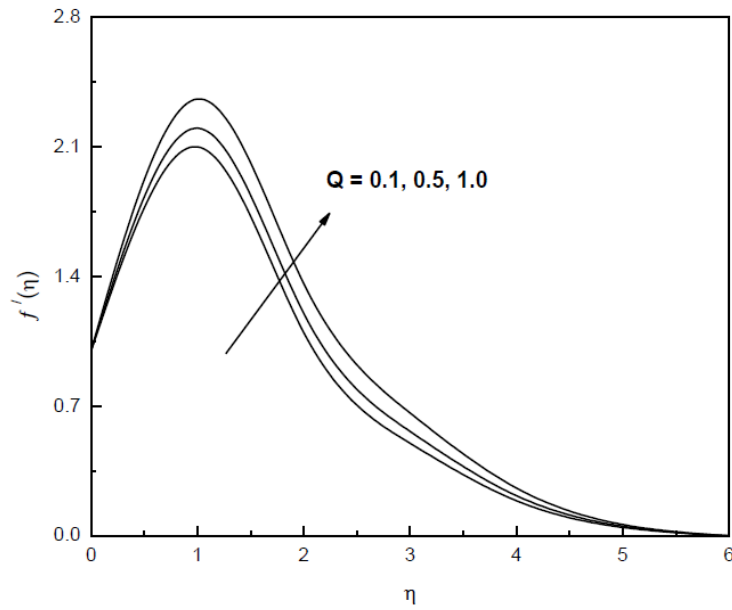


Fig 9(a) Velocity profiles for different values of  $Q$ .

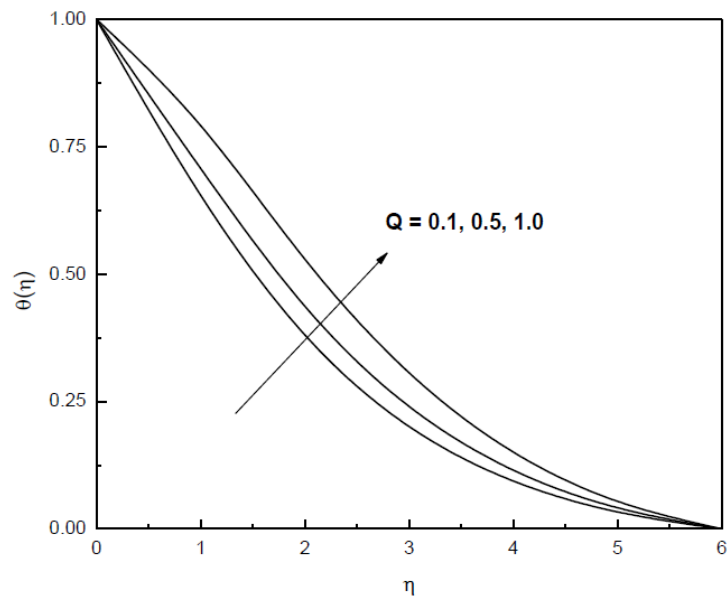


Fig 9(b) Temperature profiles for different values of  $Q$ .

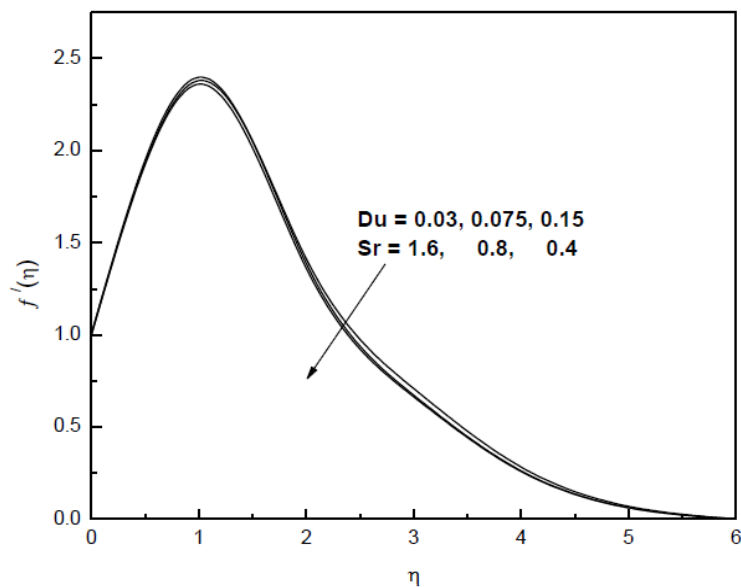


Figure.10 (a). Velocity profiles for different values of  $Sr$  and  $Du$

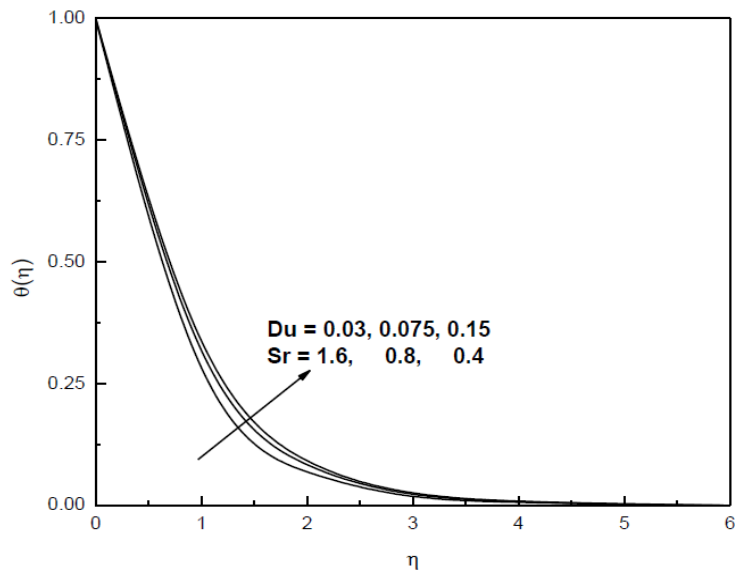


Figure.10 (b). Temperature profiles for different values of  $Sr$  and  $Du$

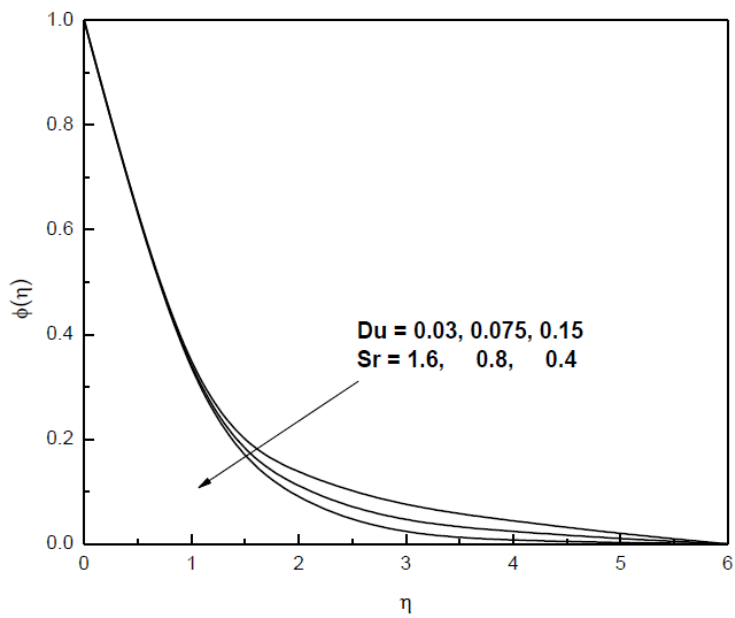


Figure.10(c). Concentration profiles for different values of  $Sr$  and  $Du$ .

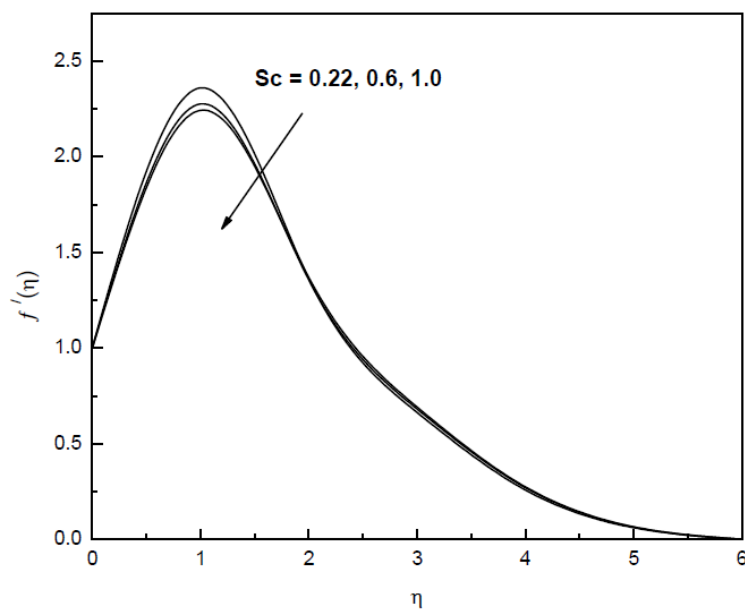


Figure.11 (a). Velocity profiles for different values of  $Sc$

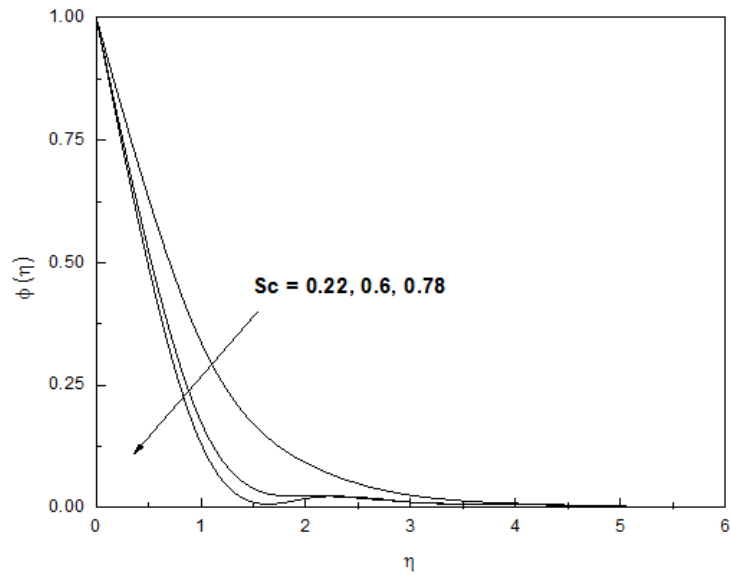


Figure.11(b). Concentration profiles for different values of  $Sc$

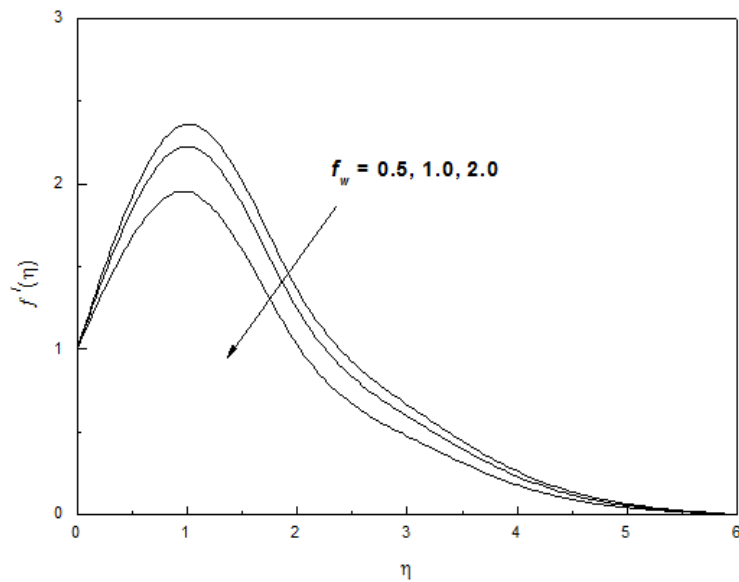


Figure.12(a). Velocity profiles for different values of  $f_w$

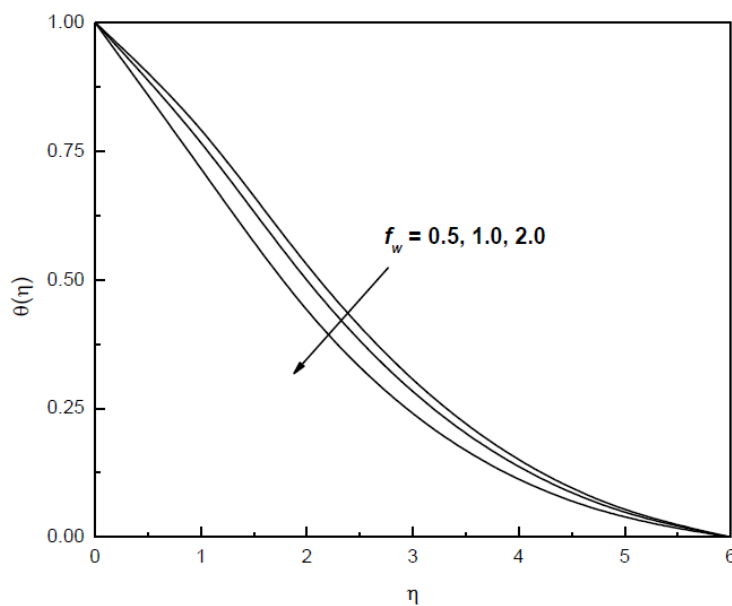


Figure.12 (b). Temperature profiles for different values of  $f_w$

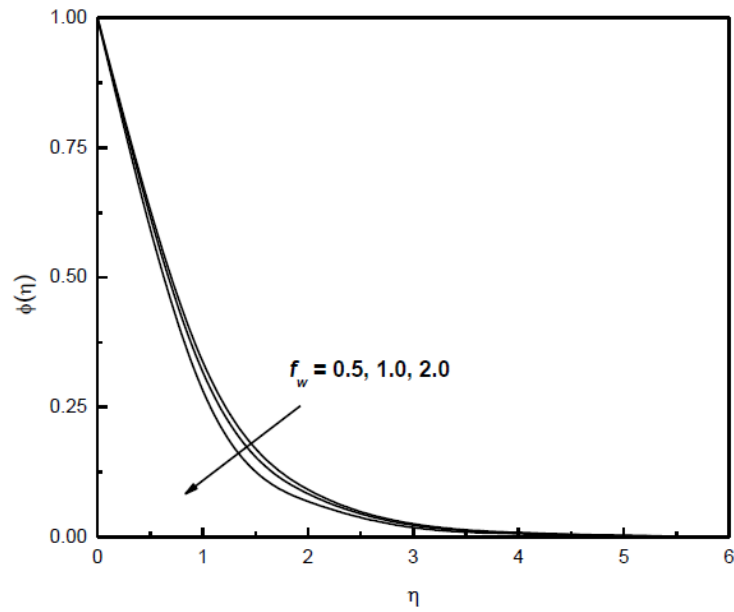


Figure.12(c). Concentration profiles for different values of  $f_w$

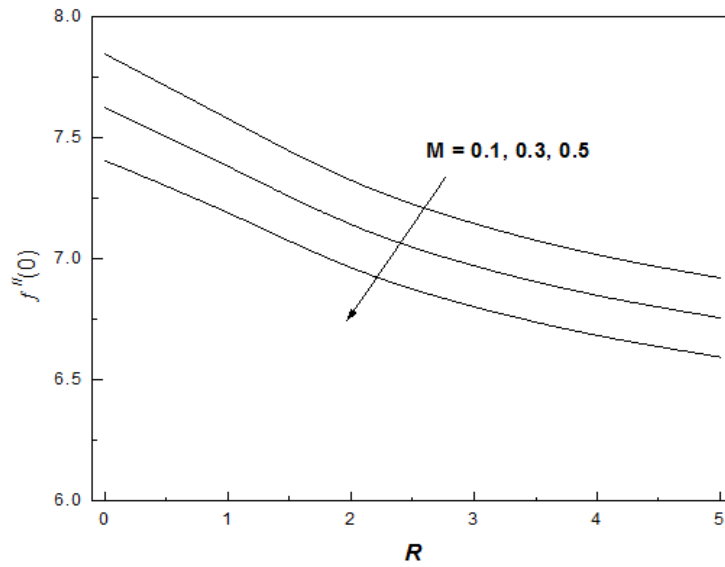


Figure.13 (a). Variation of  $f''(0)$  with  $R$  and  $M$ .

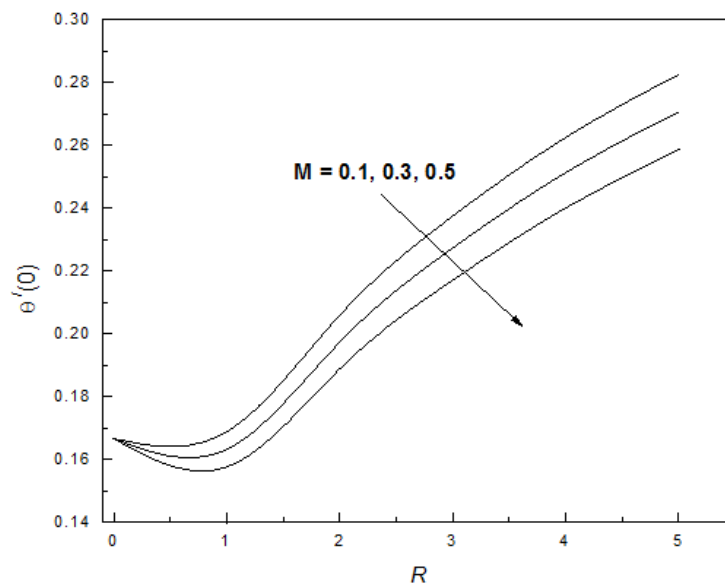


Figure.13 (b). Variation of the heat flux  $-\theta'(0)$  with  $R$  and  $M$ .

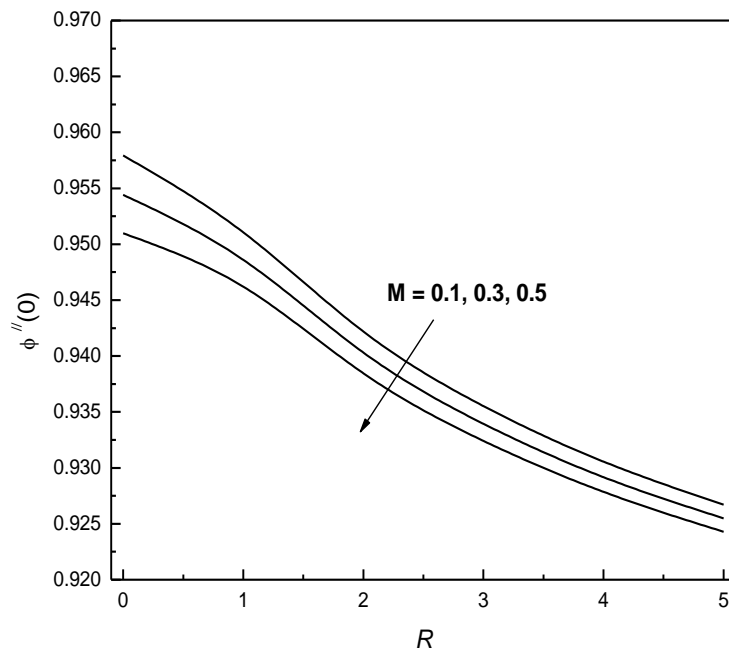


Figure.13(c). Variation of the mass flux  $-\phi''(0)$  with  $R$  and  $M$

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