

$T_{\tilde{g}}(1,2)^*$ -SPACES

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Abstract-

In this paper we discussed about A bitopological space X is called an $gT_{\tilde{g}}(1,2)^*$ -space if every $(1,2)^*$ - g -closed set in it is $\tilde{g}(1,2)^*$ -closed. And A bitopological space X is called a $T_{\tilde{g}}(1,2)^*$ -space if every $\tilde{g}(1,2)^*$ -closed subset of X is $\tau_{1,2}$ -closed in X . and we are also going to prove that Every $(1,2)^*$ - T_b -space is $T_{\tilde{g}}(1,2)^*$ -space but not conversely

Keywords- Bitopological spaces, $\tilde{g}(1,2)^*$ -closed set, $\tilde{g}(1,2)^*$ -open set, bicontinuous transformation, homeomorphisms

I. INTRODUCTION

Levine [11] introduced the notion of $T_{1/2}$ -spaces which properly lies between T_1 -spaces and T_0 -spaces. Many authors studied properties of $T_{1/2}$ -spaces: Dunham [9], Arenas et al. [3] etc. In this chapter, we introduce the notions called $T_{\tilde{g}}(1,2)^*$ -spaces, $gT_{\tilde{g}}(1,2)^*$ -spaces and ${}_aT_{\tilde{g}}(1,2)^*$ -spaces and obtain their properties and characterizations

II. PRELIMINARIES

Definition 2.1

A subset A of a bitopological space X is called a $(1,2)^*$ -pre open set [12] if $A = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$. The complement of $(1,2)^*$ -pre open set is called $(1,2)^*$ -pre closed. The $(1,2)^*$ -pre closure [12] of a subset A of X , denoted by $(1,2)^*\text{-pcl}(A)$ is defined to be the intersection of all $(1,2)^*$ -preclosed sets of X containing A . It is known that $(1,2)^*\text{-pcl}(A)$ is a preclosed set. For any subset A of an arbitrarily chosen bitopological space, the $(1,2)^*$ -semi-interior [59] (resp. $(1,2)^*\text{-}\alpha$ -interior [12], $(1,2)^*\text{-pre interior}$ [12] of A , denoted by $(1,2)^*\text{-sint}(A)$ (resp. $(1,2)^*\text{-}\alpha\text{int}(A)$, $(1,2)^*\text{-pint}(A)$), is defined to be the union of all $(1,2)^*$ -semi-open (resp. $(1,2)^*\text{-}\alpha$ -open, $(1,2)^*\text{-preopen}$) sets of X contained in A

Remark 2.2

The collection of all $(1,2)^*\text{-}\tilde{g}$ -open (resp. $(1,2)^*\text{-}\omega$ -open, $(1,2)^*\text{-}\alpha$ - g -open, $(1,2)^*\text{-gsp}$ -open, $(1,2)^*\text{-gs}$ -open, $(1,2)^*\text{-}\alpha$ -open, $(1,2)^*\text{-g}^*\text{p}$ -open) sets is denoted by $(1,2)^*\text{-}\tilde{g}O(X)$ (resp. $(1,2)^*\text{-}\omega O(X)$, $(1,2)^*\text{-}\alpha G O(X)$, $(1,2)^*\text{-GSPO}(X)$, $(1,2)^*\text{-GS}O(X)$, $(1,2)^*\text{-}\underline{\alpha}O(X)$, $(1,2)^*\text{-G}^*\text{PO}(X)$). We denote the power set of X by $P(X)$.

Definition 2.3 [15]

A bitopological space X is called

- (i) $(1,2)^*\text{-}T_{1/2}$ -space if every $(1,2)^*\text{-}g$ -closed subset of X is $\tau_{1,2}$ -closed in X .
- (ii) $(1,2)^*\text{-}T_b$ -space if every $(1,2)^*\text{-}gs$ -closed subset of X is $\tau_{1,2}$ -closed in X .

Definition 2.4 [1]

Let X be a bitopological space and $A \subseteq X$. We define the $(1,2)^*\text{-}\hat{g}$ -closure of A (briefly $(1,2)^*\text{-}\hat{g}\text{-cl}(A)$) to be the intersection of all $(1,2)^*\text{-}\hat{g}$ -closed sets containing A .

III. PROPERTIES OF $T_{\tilde{g}}(1,2)^*$ -SPACES

Definition 3.1

- (i) A bitopological space X is called $(1,2)^*\text{-}\hat{g}\text{-}R_0$ if and only if for each $(1,2)^*\text{-}\hat{g}$ -open set G and $x \in G$ implies $(1,2)^*\text{-}\hat{g}\text{-cl}(\{x\}) \subseteq G$.
- (ii) A subset A of a bitopological space X is called $(1,2)^*\text{-}g^*\text{-preclosed}$ (briefly $(1,2)^*\text{-}g^*\text{p}$ -closed) set if $(1,2)^*\text{-pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*\text{-}g$ -open in X . The complement of $(1,2)^*\text{-}g^*\text{p}$ -closed set is called $(1,2)^*\text{-}g^*\text{preopen}$ set

Definition 3.2

A bitopological space X is called

- (i) $(1,2)^*-\hat{g}$ -T0 if and only if to each pair of distinct points x, y of X , there exists a $(1,2)^*-\hat{g}$ -open set containing one but not the other.
- (ii) $(1,2)^*-\hat{g}$ -T1 if and only if to each pair of distinct points x, y of X , there exist a pair of $(1,2)^*-\hat{g}$ -open sets, one containing x but not y , and the other containing y but not x .

Definition 3.3

A bitopological space X is called

- (i) $(1,2)^*-\alpha$ T_b-space if every $(1,2)^*-\alpha$ g-closed subset of X is $\tau_{1,2}$ -closed in X .
- (ii) $(1,2)^*-\tau_\omega$ -space if every $(1,2)^*-\omega$ -closed subset of X is $\tau_{1,2}$ -closed in X .
- (iii) $(1,2)^*-\tau_p^*$ -space if every $(1,2)^*-\text{g}^*\text{p}$ -closed subset of X is $\tau_{1,2}$ -closed in X .
- (iv) $(1,2)^*-\tau_p^*$ -space if every $(1,2)^*-\text{gsp}$ -closed subset of X is $(1,2)^*-\text{g}^*\text{p}$ -closed in X .
- (v) $(1,2)^*-\alpha$ T_a-space if every $(1,2)^*-\alpha$ g-closed subset of X is $(1,2)^*-\text{g}$ closed in X .
- (vi) $(1,2)^*-\alpha$ -space if every $(1,2)^*-\alpha$ -closed subset of X is $\tau_{1,2}$ -closed in X .
- (vii) T $(1,2)^*-\ddot{g}$ -space if every $(1,2)^*-\ddot{g}$ -closed subset of X is $\tau_{1,2}$ -closed in X .

Theorem 3.4

For a bitopological space X , each of the following statement is equivalent:

- (i) X is a $(1,2)^*-\hat{g}$ -T₁.
- (ii) Each one point set is $(1,2)^*-\hat{g}$ -closed set in X .

Definition 3.5

A bitopological space X is called a T \tilde{g} $(1,2)^*$ -space if every \tilde{g} $(1,2)^*$ -closed subset of X is $\tau_{1,2}$ -closed in X .

Example 3.6

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}, \{a, b\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ are called $\tau_{1,2}$ -closed. Then $(1,2)^*-\tilde{G} C(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X\}$. Thus X is an T \tilde{g} $(1,2)^*$ -space.

Example 3.7

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a, b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{c\}\}$ are called $\tau_{1,2}$ -closed. Then $(1,2)^*-\tilde{G} C(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$. Thus X is not an T \tilde{g} $(1,2)^*$ -space.

Proposition 3.8

Every $(1,2)^*-\alpha$ T_b-space is T \tilde{g} $(1,2)^*$ -space but not conversely.

Proof

Follows from Proposition 3.16.
 The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}, \{a, b\}\}$. Thus X is not an $(1,2)^*-\alpha$ T_b-space but it is T \tilde{g} $(1,2)^*$ -space.

Proposition 3.10

Every T \tilde{g} $(1,2)^*$ -space is $(1, 2)^*-\underline{\alpha}$ -space but not conversely.

Proof

Follows from Proposition 3.26. The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b, c\}, \{a, b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a\}, \{c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -closed. Then $(1,2)^*-\square C(X) = \{\emptyset, X, \{a\}, \{c\}, \{b, c\}\}$ and $(1,2)^*-\tilde{G} C(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. Thus X is an $(1,2)^*-\underline{\alpha}$ -space but not an T \tilde{g} $(1,2)^*$ -space.

Proposition 3.12

Every $(1,2)^*$ -Tb-space is $T \tilde{g} (1,2)^*$ -space but not conversely.

Proof

Follows from Proposition 3.26.

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}\}$. Then $(1, 2)^*$ -GSC(X) = P(X). Thus X is $T \tilde{g} (1,2)^*$ -space but not $(1, 2)^*$ -Tb-space.

Proposition 3.14

Every $(1,2)^*$ -*sTp-space and $(1,2)^*$ -Tp*-space is $T(1,2)^*$ - \tilde{g} -space but not conversely.

Proof

Follows from Proposition 3.30 and Definition 3.3 (iii) and (vi).The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the set in $\{\emptyset, X, \{b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a, c\}\}$ are called $\tau_{1,2}$ -closed. We have $(1, 2)^*$ -GSPC(X) = $\{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. And $(1,2)^*$ -G*PC(X) = $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. Thus is neither $(1, 2)^*$ -*sTb-space nor $(1, 2)^*$ -Tp*-space.

Remark .3.16

$T \tilde{g} (1,2)^*$ -spaces and $(1, 2)^*$ - τ_1 spaces are independent.

Example 3.17

- (i) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}, \{a, c\}\}$. Then X is $T \tilde{g} (1,2)^*$ -space but not $(1, 2)^*$ -T ω -space.
- (ii) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then X is $(1, 2)^*$ -T ω -space but not $T \tilde{g} (1,2)^*$ -space.

Remark.3.18

We conclude from the next two examples that $T \tilde{g} (1,2)^*$ -spaces and $(1,2)^*$ - α -spaces are independent.

Example 3.19

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$ Thus X is an $T \tilde{g} (1,2)^*$ -space but not an $(1,2)^*$ - α -space.

Example 3.20

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, c\}\}$ Thus X is an $(1, 2)^*$ - α -space but not an $T \tilde{g} (1,2)^*$ -space.

Theorem 3.21

For a bitopological space X the following properties are equivalent:

- (i) X is a $T \tilde{g} (1,2)^*$ -space.
- (ii) Every singleton subset of X is either $(1,2)^*$ - \hat{g} -closed or $\tau_{1,2}$ -open.

Proof

(i) \Rightarrow (ii). Assume that for some $x \in X$, the set $\{x\}$ is not a $(1,2)^*$ - \hat{g} -closed in X. Then the only $(1,2)^*$ - \hat{g} -open set containing $\{x\}$ is X and so $\{x\}$ is $\tilde{g} (1,2)^*$ -closed in X. By assumption $\{x\}$ is $\tau_{1,2}$ -closed in X or equivalently $\{x\}$ is $\tau_{1,2}$ -open.

(ii) \Rightarrow (i). Let A be a $\tilde{g} (1,2)^*$ -closed subset of X and let $x \in \tau_{1,2}$ -cl(A). By assumption $\{x\}$ is either $(1,2)^*$ - \hat{g} -closed or $\tau_{1,2}$ -open

Case (a) Suppose that $\{x\}$ is $(1,2)^*$ - \hat{g} -closed. If $x \in A$, then $(1,2)^*$ -cl(A) = A contains a nonempty $(1,2)^*$ - \hat{g} -closed set $\{x\}$, which is a contradiction to Theorem 4.7. Therefore $x \notin A$.

Case (b) Suppose that $\{x\}$ is $\tau_{1,2}$ -open. Since $x \in \tau_{1,2}$ -cl(A), $\{x\} \cap A \neq \emptyset$ and so $x \in A$. Thus in both case, $x \in A$ and therefore $\tau_{1,2}$ -cl(A) = A or equivalently A is a $\tau_{1,2}$ -closed set of X.

Theorem 3.22

For a bitopological space X the following properties hold:

- (i) If X is $(1,2)^*$ - \hat{g} -T1, then it is $T \tilde{g} (1,2)^*$.
- (ii) If X is $T \tilde{g} (1,2)^*$, then it is $(1,2)^*$ - \hat{g} -T0.

Proof

(i) The proof is obvious from Theorem 3.4.

(ii) Let x and y be two distinct elements of X. Since the space X is $T \tilde{g} (1,2)^*$, we have that $\{x\}$ is $(1,2)^*$ - \hat{g} -closed or $\tau_{1,2}$ -open. Suppose that $\{x\}$ is $\tau_{1,2}$ -open. Then the singleton $\{x\}$ is a $(1,2)^*$ - \hat{g} -open set such that $x \in \{x\}$ and $y \notin \{x\}$. Also, if $\{x\}$ is $(1,2)^*$ - \hat{g} -closed, then $X \setminus \{x\}$ is $(1,2)^*$ - \hat{g} -open such that $y \in X \setminus \{x\}$ and $x \notin X \setminus \{x\}$. Thus, in the above two cases, there exists a $(1,2)^*$ - \hat{g} -open set U of X such that $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$. Thus, the space X is $(1,2)^*$ - \hat{g} -T0.

Theorem 3.23

For a $(1,2)^*$ - \hat{g} -R bitopological space X the following properties are equivalent:

- (i) X is $(1,2)^*$ - \hat{g} -T0.
- (ii) X is $T \tilde{g} (1,2)^*$.
- (iii) X is $(1,2)^*$ - \hat{g} -T1.

Proof

It suffices to prove only (i) \Leftrightarrow (iii). Let $x \neq y$ and since X is $(1,2)^*$ - \hat{g} -T0, we may assume that $x \in U \setminus \{y\}$ for some $(1,2)^*$ - \hat{g} -open set U. Then $x \in X \setminus (1,2)^*$ - \hat{g} -cl($\{y\}$) and $X \setminus (1,2)^*$ - \hat{g} -cl($\{y\}$) is $(1,2)^*$ - \hat{g} -open. Since X is $(1,2)^*$ - \hat{g} -R0, we have $(1,2)^*$ - \hat{g} -cl($\{x\}$) \cap $(1,2)^*$ - \hat{g} -cl($\{y\}$) \cap $(1,2)^*$ - \hat{g} -cl($\{y\}$) \cap $(1,2)^*$ - \hat{g} -cl($\{x\}$) = \emptyset . There exists $(1,2)^*$ - \hat{g} -open set V such that $y \in V \setminus \{x\}$ and X is $(1,2)^*$ - \hat{g} -T1.

IV. $gT \tilde{g} (1,2)^*$ -SPACES

Definition .4.1

A bitopological space X is called an $gT \tilde{g} (1,2)^*$ -space if every $(1,2)^*$ -g-closed set in it is $\tilde{g} (1,2)^*$ -closed.

Example 4.2

- (i) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then X is an $gT \tilde{g} (1,2)^*$ -space.
- (ii) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Then X is not $gT \tilde{g} (1,2)^*$ -space.

Proposition 4.3

- (i) Every $(1,2)^*$ -T1/2-space is $gT \tilde{g} (1,2)^*$ -space but not conversely.
- (ii) Every $(1,2)^*$ -T1/2-space is $T \tilde{g} (1,2)^*$ -space but not conversely.

Proof

Follows from Definitions. The converse of Proposition 4.3 need not be true as seen from the following example.

Example 4.4

- (i) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, c\}\}$. Then X is $gT \tilde{g} (1,2)^*$ -space but not an $(1,2)^*$ -T1/2-space.
- (ii) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Then X is $T \tilde{g} (1,2)^*$ -space but not an $(1,2)^*$ -T1/2-space.

Remark 4.5

$T \tilde{g} (1,2)^*$ -spaces and $gT \tilde{g} (1,2)^*$ -spaces are independent.

Example 4.6

- (i) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, c\}\}$. Then X is $T \tilde{g} (1,2)^*$ -space but it is not $gT \tilde{g} (1,2)^*$ -space.
- (ii) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b, c\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then X is $gT \tilde{g} (1,2)^*$ - space but it is not $gT \tilde{g} (1,2)^*$ -space.

Theorem 4.7

If X is a $gT \tilde{g} (1,2)^*$ -space, then every singleton subset of X is either $(1,2)^*$ -g-closed or $\tilde{g} (1,2)^*$ -open.

Proof

Assume that for some $x \in X$, the set $\{x\}$ is not a $(1,2)^*$ -g-closed in X . Then $\{x\}$ is not a $\tau_{1,2}$ -closed set, since every $\tau_{1,2}$ -closed set is a $(1,2)^*$ -g-closed set. So $\{x\}$ is not $\tau_{1,2}$ -open and the only $\tau_{1,2}$ -open set containing $\{x\}$ is X itself. Therefore $\{x\}$ is trivially a $(1,2)^*$ -g-closed set and by assumption, $\{x\}$ is an $\tilde{g} (1,2)^*$ -closed set or equivalently $\{x\}$ is $\tilde{g} (1,2)^*$ -open. The converse of Theorem 4.7 need not be true as seen from the following example.

Example 4.8

In the Example 3.6, the sets $\{a\}$ and $\{c\}$ are $(1,2)^*$ -g-closed in X and the set $\{b\}$ is $\tilde{g} (1,2)^*$ -open. But the space X is not an $gT \tilde{g} (1,2)^*$ -space.

Theorem 4.9

A space X is $(1,2)^*$ -T1/2 if and only if it is both $T \tilde{g} (1,2)^*$ and $gT \tilde{g} (1,2)^*$.

Proof

Necessity. Follows from Proposition 4.3.

Sufficiency. Assume that X is both $T \tilde{g} (1,2)^*$ and $gT \tilde{g} (1,2)^*$. Let A be a $(1,2)^*$ -g-closed set of X . Then A is $\tilde{g} (1,2)^*$ -closed, since X is a $gT \tilde{g} (1,2)^*$. Again since X is a $T \tilde{g} (1,2)^*$, A is a $\tau_{1,2}$ -closed set in X and so X is a $(1,2)^*$ -T1/2.

V. $\underline{g}T \tilde{g} (1,2)^*$ - SPACES

Definition .5.1

A bitopological space X is called a $\underline{g}T \tilde{g} (1,2)^*$ -space if every $\tilde{g} (1,2)^*$ -closed subset of X is $(1,2)^*$ - \underline{g} -closed in X .

Example 5.2

- (i) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, c\}\}$. Then X is $\underline{g}T \tilde{g} (1,2)^*$ -space.
- (ii) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a, c\}\}$. Then X is not $T \tilde{g} (1,2)^*$ -space.

Proposition 5.3

Every $(1,2)^*$ - $\underline{g}T_b$ -space is $\underline{g}T \tilde{g} (1,2)^*$ -space but not conversely.

Proof

The converse of Proposition 5.3 need not be true as seen from the following example.

Example 5.4

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b\}\}$. Then X is an $\underline{g}T \tilde{g} (1,2)^*$ -space but not an $(1,2)^*$ - $\underline{g}T_b$ -space.

Proposition 5.5

Every $T \tilde{g} (1,2)^*$ -space is a $\underline{g}T \tilde{g} (1,2)^*$ -space but not conversely.

Proof

Let X be an $T \tilde{g} (1,2)^*$ -space and let A be an $\tilde{g} (1,2)^*$ -closed set of X . Then A is a $\tau_{1,2}$ -closed subset of X and is $(1,2)^*$ - \underline{g} -closed. Therefore X is an $\underline{g}T \tilde{g} (1,2)^*$ -space. The converse of Proposition 5.5 need not be true as seen from the following example.

Example 5.6

Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{a\}\}$. Then X is an $\underline{\alpha} T \tilde{g} (1,2)^*$ -space but not an $T \tilde{g} (1,2)^*$ -space.

Theorem 5.7

If (X, τ) is a $\underline{\alpha} T \tilde{g} (1,2)^*$ -space then every singleton subset of X is either $(1,2)^* \wedge g$ -closed or $(1,2)^* \underline{\alpha}$ -open.

Proof

Let $x \in X$. Suppose $\{x\}$ is not a $(1,2)^* \wedge g$ -closed set of (X, τ_1, τ_2) . Then $X - \{x\}$ is not a $(1, 2)^* \wedge g$ -open set. So X is the only $(1, 2)^* \wedge g$ -open set containing $X - \{x\}$. So $X - \{x\}$ is a $\tilde{g} (1,2)^*$ -closed set of (X, τ_1, τ_2) . $X - \{x\}$ is a $(1, 2)^* \underline{\alpha}$ -closed set of (X, τ_1, τ_2) or equivalently $\{x\}$ is a $(1, 2)^* \underline{\alpha}$ -open set of (X, τ_1, τ_2) . The converse of Theorem 5.7 need not be true as seen from the following example.

Example 5.8

Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{c\}\}$ and $\tau_2 = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$. Then the sets $\{a\}$ and $\{c\}$ are $(1,2)^* \underline{\alpha}$ -open in X and the set $\{a\}$ is $(1,2)^* \wedge g$ -closed. But the space X is not an $\underline{\alpha} T \tilde{g} (1,2)^*$ -space.

VI. CONCLUSION

Topology as a branch of mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of transformation especially those properties that are invariant under a certain kind of equivalence and it is study of those properties of geometric configurations which remain invariant when these configurations are subjected to one-to-one bicontinuous transformation or homeomorphisms. Topology operates with more general concepts than analysis. Differential properties of a given transformation are non essential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of problems to which analysis cannot give the answer. In this paper I introduced $\tilde{g}(1,2)^*$ -closed maps, $\tilde{g}(1,2)^*$ -open maps, $\tilde{g}(1,2)^* \underline{\alpha}$ -closed maps and $\tilde{g}(1,2)^* \underline{\alpha}$ -open maps in bitopological spaces and obtain certain characterization of these classes of maps.

REFERENCE

- [1] Dharmalingam.K.M, Thamilisai.A, Ravi. O, $\tilde{g}(1,2)^*$ -Closed and $\tilde{g}(1,2)^*$ -Open sets in Bitopological spaces,
- [2] Thamilisai.A, $\tilde{g}(1,2)^*$ -Closed and $\tilde{g}(1,2)^*$ -Open maps in Bitopological spaces,
- [3] Arenas, F. G., Dontchev, J. and Ganster, M.: *On \square -sets and dual of generalized Continuity*, Questions Answers Gen. Topology, 15 (1997), 3-13.
- [4] Veera kumar, M. K. R. S.: *\tilde{g} -closed sets in topological spaces*, Bull. Allah. Math. Soc., 18 (2003), 99-112.
- [5] Zbigniew Duszynski, Rose Mary, S. and Lellis Thivagar, M.: *Remarks on α homeomorphisms*, Math. Maced, 7 (2009), 13-20.
- [6] Devi, R., Balachandran, K. and Maki, H.: *Semi-generalized closed maps and generalized semi-closed maps*, Mem. Fac. Kochi Univ. Ser. A. Math., 14 (1993), 41-54.
- [7] Antony Rex Rodrigo, J., Ravi, O., Pandi, A. and Santhana, C. M.: *On $(1,2)^* \wedge s$ -normal spaces and pre- $(1, 2)^* \wedge g$ -closed functions*, International Journal of Algorithms, Computing and Mathematics, 4(1) (2011), 29-42.
- [8] Ravi, O., Thivagar, M. L. and Nagarajan, A.: *$(1,2)^* \wedge \square g$ -closed sets and $(1,2)^* \wedge g \square$ -closed sets* (submitted).
- [9] Dunham, W.: *$T1/2$ -spaces*, Kyungpook Math. J., 17 (1977), 161-169.
- [10] Kayathri, K., Ravi, O., Thivagar, M. L. and Joseph Israel, M.: *Decompositions of $(1,2)^* \wedge rg$ -continuous maps in bitopological spaces*, Antarctica J. Math., 6(1) (2009), 13-23.
- [11] Lellis Thivagar, M., Ravi, O. and Abd El-Monsef, M. E.: *Remarks on bitopological $(1,2)^* \wedge$ -quotient mappings*, J. Egypt Math. Soc., 16(1) (2008), 17-25.
- [12] Ravi, O., Thivagar, M. L. and Hatir, E.: *Decomposition of $(1,2)^* \wedge$ -continuity and $(1,2)^* \wedge \alpha$ -continuity*, Miskolc Mathematical Notes., 10(2) (2009), 115-171.
- [13] Ravi, O. and Lellis Thivagar, M.: *A bitopological $(1,2)^* \wedge$ -semi-generalized continuous maps*, Bull. Malays. Math. Sci. Soc., (2), 29(1) (2006), 79-88.
- [14] Ravi, O., Ekici, E. and Lellis Thivagar, M.: *On $(1,2)^* \wedge$ -sets and decompositions of bitopological $(1,2)^* \wedge$ -continuous mappings*, Kochi J. Math., 3 (2008), 181-189.
- [15] Ravi, O., Kayathri, K., Thivagar, M. L. and Joseph Israel, M.: *Mildly $(1,2)^* \wedge$ -normal spaces and some bitopological functions*, Mathematical Bohemica, 135(1) (2010), 1-1