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On Nano Weakly Generalized Continuous Functions

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Abstract:

The aim of this paper is to introduce and study the concept of new class of function called Nano weakly generalized continuous functions. Some of its basic properties are analyzed. The relationship between the Nano weakly generalized continuous functions and existing function are established. Also the Nano weakly generalized irresolute functions are introduced and its properties have been discussed.

Keywords: Nano topological space, Nano closed sets, Nano continuous functions , Nano weakly generalized closed set and Nano irresolute function.

I. INTRODUCTION

In 1991, Balachandran [1] *et.al,* introduced and studied the notions of generalized continuous functions. Different types of generalizations of continuous functions were studied by various authors in the recent development of Topology.

The notion of Nano topology was introduced by Lellis Thivagar [6] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure. He has also defined Nano continuous functions, Nano open mapping, Nano closed mapping and Nano Homeomorphism.

In Bhuvaneswari [2] *et.al*, introduced and studied some properties of Nano generalized closed sets in Nano topological spaces.

Nagaveni [16] ,[11] introduced a class of sets called weakly generalized closed sets in general topology in 1999 and in Nano topology in 2015 respectively.

In this paper we have introduced a new class of continuous functions called Nano weakly generalized continuous functions and discuss some of its properties.

Throughout this paper $(U,\tau_R(X))$ is a Nano Topological space with respect to X Where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R and $(V,\tau_{R'}(Y))$ is a Nano Topological space with respect to Y Where $Y \subseteq V$, R' is an equivalence relation on V, V/R' denotes the family of equivalence classes of V by R'.

II. PRELIMINARIES

.**Definition:2.1 [6]** Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation.Elements belonging to the same equivalence class are said to be indiscernible with one another.The pair (U,R) is said to be the approximation space. Let $X \subseteq U$,

1. The Lower approximation of X with respect to R is the set of all objects,which can be for certain classified as X with respect to R and is defined by $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$.Where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is defined by $u_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and is defined by $B_R(X) = U_R(X) - L_R(X)$.

Proposition: 2.2 [6] If (U, R) is an approximation space and $X, Y \subseteq U$ .Then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$
(iii) $U_R(X \cup Y) = L_R(X) \cup L_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition: 2.3[6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.

1. U *and* $\phi \in \tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ forms a topology on U called as the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets.

Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$

Definition:2.4 [6] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), U_R(X)\}$ is the basis for $\tau_R(X)$.

Definition:2.5 [6] If $(U, \tau_R(X))$ is a Nano Topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by Nint(A).Nano interior is the largest Nano open subset of A.

The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A).It is the smallest Nano closed set containing A.

Definition:2.6 [6] Let $(U, \tau_R(X))$ be a Nano Topological space with respect to X and $A \subseteq U$. Then A is said to be

(i) Nano semi- open if $A \subseteq Ncl(N\operatorname{int}(A))$
(ii) Nano pre- open if $A \subseteq N\operatorname{int}(Ncl(A))$
(iii) Nano α - open if $A \subseteq N\operatorname{int}(Ncl(N\operatorname{int}(A)))$
(iv) Nano Regular open if $A = N\operatorname{int}(Ncl(A))$

NSO(U,X), NPO(U,X), NRO(U,X) and NαO(U,X) respectively denote the famies of all Nano semi open , Nano pre open, Nano regular open and Nano α open subsets of U.

Definition:2.7 [2] Let $(U, \tau_R(X))$ be a Nano Topological space. A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set if $Ncl(A) \subseteq V$ where $A \subseteq V$ and V is Nano open.

Definition:2.8 [18] A subset A of a Nano Topological space $(U, \tau_R(X))$ is called Nano α generalized closed set if $N\alpha cl(A) \subseteq V$ where $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition:2.9 [11] Let $(U, \tau_R(X))$ be a Nano Topological space. A subset A of $(U, \tau_R(X))$ is called Nano weakly generalized closed (briefly Nwg-closed) set if $Ncl(N\operatorname{int}(A)) \subseteq V$ where $A \subseteq V$ and V is Nano open.

Definition:2.10 The map $f : (U, \tau_R(X)) \to (V, \tau_R{}'(Y))$ from a Nano Topological space $(U, \tau_R(X))$ to a Nano Topological space $(V, \tau_R{}'(Y))$ is

(i) Nano continuous [7] if f^{-1} (A) is Nano- open in U for every Nano-open set A in V.
(ii) Nano g-continuous [3] if f^{-1} (A) is Ng- open in U for every Nano-open set A in V.
(iii) Nano pre- continuous [10] if f^{-1} (A) is Nano- pre-open in U for every Nano-open set A in V.
(vi) Nano semi- continuous [10] if f^{-1}(A) is Nano-semi- open in U for every Nano-open set A in V.
(v) Nano α - continuous [10] if f^{-1}(A) is Nano - α - open in U for every Nano-open set A in V.
(vii) Nano α -generalized continuous [19] if f^{-1}(A) is Nano- α -generalized open in U for every Nano-open set A in V.

Theorem 2.11[11] If A is a Nano closed subset of a Nano topological space $(U, \tau_R(X))$, then A is Nano weakly generalized closed set.

Theorem 2.12 [11] Let A be a Nano generalized closed set of a Nano topological space $(U, \tau_R(X))$, then A is Nano weakly generalized closed set.

Theorem 2.13 [11] If A is Nano α - closed set in a nano topological space $(U, \tau_R(X))$ then A is Nano weakly generalized closed set.

Theorem 2.14 [11] Every Nano α - generalized closed set is Nano weakly generalized closed set.

III. NANO WEAKLY GENERALIZED CONTINUOUS FUNCTION

Definition 3.1 The map $f:(U, \tau_R(X)) \to (V, \tau_R{}'(Y))$ is Nwg-continuous on U if the inverse image of every Nano closed set in V is Nano weakly generalized closed in U.

Theorem:3.2 Let $f:(U, \tau_R(X)) \to (V, \tau_R{}'(Y))$ be a mapping from a Nano topological space $(U, \tau_R(X)$ to Nano topological space $(V, \tau_R{}'(Y)$ then f is Nwg-continuous on U if the inverse image of every Nano open set in V is Nano weakly generalized open in U.

Proof : Let $f:(U, \tau_R(X)) \to (V, \tau_R{}'(Y))$ be a Nwg-continuous function and A be Nano open in V. Then V-A is is Nano closed. Since f is Nwg-continuous $f^{-1}(V-A)$ is Nwg-closed. $f^{-1}(V-A) = U - f^{-1}(A)$ is Nwg-closesd in U.

Therefore $f^{-1}(A)$ is Nwg-open.

Conversely let the inverse image of every Nano open set in V is Nano weakly generalized open in U.

Let B be nano closed in V. Then V-B nano open set in V.Then $f^{-1}(V-B)$ is Nano open in U. $f^{-1}(V-B) = U - f^{-1}(B)$ is Nano open in U. Therefore $f^{-1}(B)$ is Nwg-closed in U. Thus the inverse image of every nano closed set in V is Nwg-closesd in U.therefore f is nano continuous on U.

Example : 3.3 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then the Nano Topology is $\tau_R(X) = \{U, \phi, \{a\}, \{b,d\}, \{a,b,d\}\}$.

Let $V = \{p,q,r,s\}$ with $V/R' = \{\{p\},\{s\},\{q,r\}\}$ and $Y = \{p,r\}$. Then the Nano Topology is $\tau_R{}'(Y) = \{V, \phi, \{p\}, \{q,r\}, \{p,q,r\}\}$.

Let $f:(U, \tau_R(X)) \to (V, \tau_R{}'(Y))$ by $f(a) = q, f(b) = r, \ f(c) = s, f(d) = p$ Then f is Nwg- continuous function. Since $f^{-1'}(\{p,q,r\}) = \{a,b,d\}$, $f^{-1}(\{q,r\}) = \{a,b\}$, $f^{-1}(\{p\}) = \{d\}$ $f^{-1}(\phi) = \phi$ and $f^{-1}(V) = U$ .Where $\{a,b,d\}, \{a,b\}, \{d\}, \phi$ and U are Nwg-open sets in the Nano topological space $(U, \tau_R(X))$ .Hence f is Nwg-continuous function.

Remark 3.4 : composition of two Nwg- continuous function need not be a Nwg- continuous function.

Example 3.5 Let $U = V = W = \{a,b,c,d\}$ with $\tau_R(X) = \{U, \phi, \{a\}, \{b,d\}, \{a,b,d\}\}$

$\tau_{R'}(Y) = \{U, \phi, \{b\}, \{a,c\}, \{a,b,c\}\}$ $\tau_{R''}(Z) = \{U, \phi, \{c\}, \{a,b\}, \{a,b,c\}\}$

Let $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = b, f(b) = c, \ f(c) = d, f(d) = a$

Let $g:(V, \tau_{R'}(Y) \to (W, \tau_{R''}(Z))$ be the identity map. Functions f and g are Nwg- continuous function but their composition $g \circ f:(U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is not Nwg- continuous function since the inverse image of the Nano open sets $\{a,b\}$, $\{a,b,c\}$ are $\{a,d\}, \{a,b,d\}$ respectively. But they are not Nwg-closed sets.

Theorem 3.6 : Every Nano continuous functions is Nwg-continuous function.

Proof: Let $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a Nano continuous function.Let A be a Nano closed set in the Nano Topological space $(V, \tau_{R'}(Y))$.Then the inverse image of A under the map f is Nano closed in the Nano topological space $(U, \tau_R(X))$.Since every Nano closed set is Nwg-closed set $f^{-1}(A)$ is Nwg-closed set. Hence f is Nwg-continuous.

Theorem 3.7 : Every Nano generalized continuous functions is Nwg-continuous function.

Proof: Let $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a Nano continuous function.Let A be a Nano closed set in the Nano Topological space $(V, \tau_{R'}(Y))$.Then the inverse image of A under the map f is Nano generalized closed in the Nano topological space $(U, \tau_R(X))$.Since every Nano generalized closed set is Nwg-closed set by theorem 2.12, $f^{-1}(A)$ is Nwg-closed set. Hence f is Nwg-continuous.

Theorem 3.8 : .Every Nano Pre continuous functions is Nwg-continuous function.

Proof: The theorem is true since every Nano pre closed set is Nwg-closed set.

Theorem 3.9 Every Nano alpha continuous functions is Nwg-continuous function.

Proof: The proof of the theorem follows from the fact that every Nano alpha closed set is Nwg-closed set.

Theorem 3.10.Every Nano alpha generalized continuous functions is Nwg-continuous function.

Proof: The statement of the theorem is true from the theorem 2.14.

Remark: 3.11 The converse of the above need not be true as shown in the following example.

Example : 3.12 Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{b,c\},\{a\},\{d\},\{e\}\}$ and $X = \{b,d\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{d\},\{b,c\},\{b,c,d\}\}$.

Let $V = U = \{a,b,c,d,e\}$ with $V/R' = \{\{a,b\},\{c,d\},\{e\}\}$ and $Y = \{a,c,d\}$. Then the Nano topology is $\tau_R{}'(Y) = \{V,\phi,\{c,d\},\{a,b\},\{a,b,c,d\}\}$.

Let $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ be the identity map. Then f is Nwg- continuous function. But not Nano continuous function.Since $f^{-1'}(\{a,b,e\}) = \{a,b,e\}$ which is not Nano closed in the Nano topological space $(U,\tau_R(X))$.

Example : 3.13 Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{b,c\},\{a\},\{d\},\{e\}\}$ and $X = \{b,d\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{d\},\{b,c\},\{b,c,d\}\}$.

Let $V = U = \{a,b,c,d,e\}$ with $V/R' = \{\{a,b\},\{c,d\},\{e\}\}$ and $Y = \{a,c,d\}$. Then the Nano topology is $\tau_R{}'(Y) = \{V,\phi,\{c,d\},\{a,b\},\{a,b,c,d\}\}$.

Let $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ be defined as $f(a)=a, f(b)=b, f(c)=d, f(d)=e, f(e)=c$

Then f is Nwg- continuous function. But not Nano generalized continuous function. Since $f^{-1'}(\{e\}) = \{d\}$ which is not Nano generalized closed in the Nano topological space $(U,\tau_R(X))$.

Example : 3.14 Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{b,c\},\{a\},\{d\},\{e\}\}$ and $X = \{b,d\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{d\},\{b,c\},\{b,c,d\}\}$.

Let $V = U = \{a,b,c,d,e\}$ with $V/R' = \{\{a,b\},\{c,d\},\{e\}\}$ and $Y = \{a,c,d\}$. Then the Nano topology is $\tau_R{}'(Y) = \{V,\phi,\{c,d\},\{a,b\},\{a,b,c,d\}\}$.

Let $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ be the identity map.

Then f is Nwg- continuous function. But not Nano-pre-continuous function. Since $f^{-1'}(\{c,d,e\}) = \{c,d,e\}$ which is not Nano pre closed in the Nano topological space $(U,\tau_R(X))$.

Example : 3.15 Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{b,c\},\{a\},\{d\},\{e\}\}$ and $X = \{b,d\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{d\},\{b,c\},\{b,c,d\}\}$.

Let $V = U = \{a,b,c,d,e\}$ with $V/R' = \{\{a,b\},\{c,d\},\{e\}\}$ and $Y = \{a,c,d\}$. Then the Nano topology is $\tau_R{}'(Y) = \{V,\phi,\{c,d\},\{a,b\},\{a,b,c,d\}\}$.

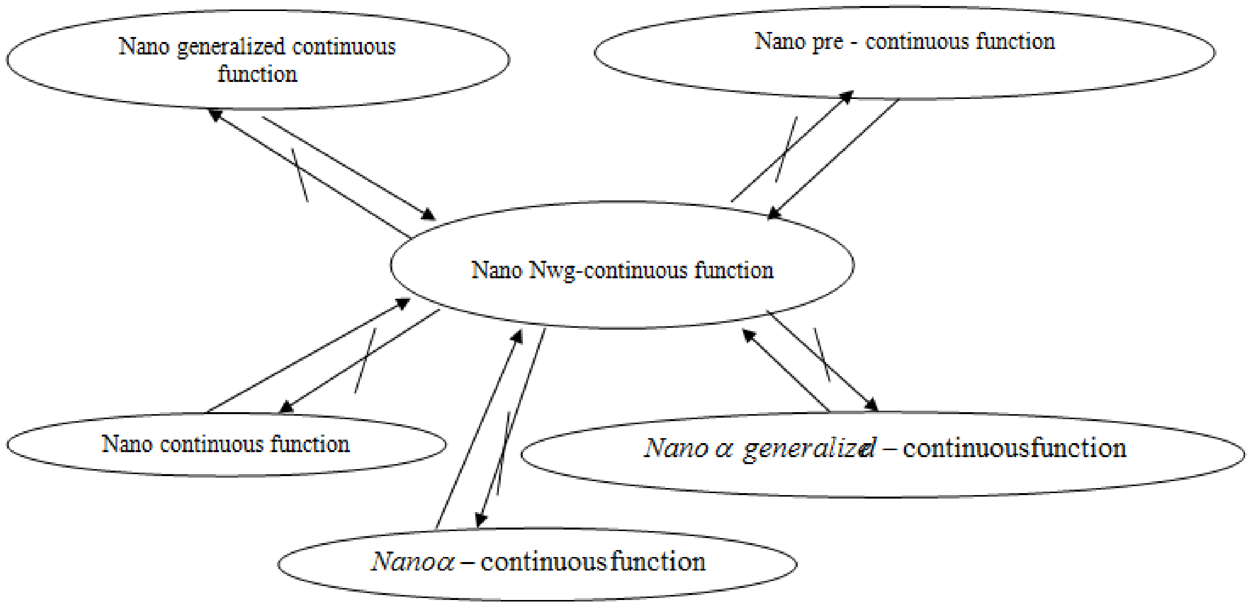
Let $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$ be the identity map.Then f is Nwg- continuous function. But not Nano alpha continuous function. Since $f^{-1'}(\{c,d,e\}) = \{c,d,e\}$ which is not Nano –alpha closed in the Nano topological space $(U,\tau_R(X))$.

Example : 3.16 Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{a,b\},\{c,d\},\{e\}\}$ and $X = \{a,c,d\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{c,d\},\{a,c\},\{a,b,c,d\}\}$.

Let $V = U = \{a,b,c,d,e\}$ with $V/R' = \{\{b,c\},\{a\},\{d\},\{e\}\}$ and $Y = \{b,d\}$. Then the Nano topology is $\tau_R{}'(Y) = \{V,\phi,\{b,c\},\{d\},\{b,c,d\}\}$.

Let be defined as $f(a)=d, f(b)=b, f(c)=c, f(d)=a, f(e)=e$ $f:(U,\tau_R(X)) \to (V,\tau_R{}'(Y))$. Then f is Nwg- continuous function. But not Nano alpha-generalized continuous function. Since $f^{-1'}(\{a,b,c,e\}) = \{b,c,d,e\}$ which is not Nano –alpha-generalized closed in the Nano topological space $(U,\tau_R(X))$.

Remark: 3.17 From the above discussion and result, the relationship between the Nano weakly generalized continuous function and existing continuous functions are as bellow in the diagrams.



IV. NANO WEAKLY GENERALIZED IRRESOLUTE FUNCTION

Definition 4.1 The map $f:(U,\tau_R(X))\to(V,\tau_R^{'}(Y))$ is Nwg-irresolute on U if the inverse image of every Nwg-closed set in V is Nwg-closed set in U.

Theorem 4.2: composition of two Nwg- irresolute function is again a Nwg- irresolute function.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and Let $g:(V,\tau_{R'}(Y)\to(W,\tau_{R''}(Z))$ are any two Nwg- irresolute functions. Let A be a Nwg-closed set in Z .Then $g^{-1}(A)$ is Nwg-closed set in V ,Since g is Nwg- irresolute function. $f^{-1}\left(g^{-1}(A)\right)$ is Nwg-closed set in U ,Since f is Nwg- irresolute function. Hence the composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is Nwg- irresolute function.

Theorem 4.3: If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nwg- irresolute function then f is Nwg- continuous function.

Proof: Let A be a Nano -closed set in V. then A is Nwg-closed set since every Nano closed set is Nwg-closed set. Then $f^{-1}(A)$ is Nwg-closed set in U, Since f is Nwg- irresolute function. Hence f is Nwg- continuous function.

Remark 4.4: The converse of the above need not be true as shown in the following example.

Example 4.5 In Example 3.3 f is is Nwg- continuous function but not Nwg- irresolute function since the set $\{a\}$ is not Nwg-closed set in U.

V. CONCLUSION

In this paper a new class of generalized function called "Nano weakly generalized continuous functions" is introduced and some of its properties were discussed. Further Nwg-irresolute function has been defined and its characterization is analyzed. We have established the relationship with other existing continuous functions in Nano Topological space with suitable examples. Also we observed that composition of two Nwg-irresolute functions is also Nwg-irresolute function.

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