

Recent Development in Power System Dynamic State Estimation

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Abstract—

Due to the expansion of power systems there is a challenge of real time monitoring and control by placing important parameters at various points for efficient functioning of the power system. There can be drastic changes during hourly load fluctuations, component outages, or network switching. In such conditions, the inclusion of predicted values could degrade the power system state estimation hence the need of state estimation is must. Synchronized Phasor Measurement Units (PMU) are being placed along with the GPS (global positioning system) to enhance the monitoring of the power system. It depends on the measurement numbers, types and locations. This paper aims to present the main categories for PMUs placement strategies which considers main factors that required for enhancing the performance of a state estimation process such as reliability, accuracy and the special requirements of the modern distribution grid.

Keywords— Phasor Measurement Unit, Global Positioning Systems, State Estimations, Weighted Least Square Method, Observability

I. INTRODUCTION

The enlargement of power system has made more and more intricacy so real time checking and control became very significant for the consistent operation of power system and which the Energy Management System (EMS) cures. In this system, the State estimation forms the spine of the energy management system by providing a database of the real time state of the system for using in other EMS functions [1]. Therefore, efficient and accurate state estimation is a prerequisite for an efficient and consistent operation of the power system.

State estimation is a technique where the voltage, magnitude and angles at all the buses of a power system are recorded from the existing measurements. These measurements are obtained through a suitable communication standard and are processed through state estimation procedures, to obtain the voltage magnitude and angle at all the buses of the network under consideration. As we know that the power system is a quasi-static system, hence changes slowly but progressively which are driven by dynamic load so the generations also have to be adjusted accordingly, which in turn changes the flows and injections across the system, which makes the entire system dynamic in nature. Therefore, in order to have a continuous monitoring of the power system, we must perform the state estimation at regular interval of time. However, by the expansion of power system, along with addition of generations and loads, this system becomes more complex for such monitoring, which requires heavy computing resources. Hence, static state estimators may not capably capture this dynamic behavior of the power system. This lead to the necessity of another set of procedures called the "Dynamic State Estimation" (DSE) techniques, where we use the actual physical modeling of the time varying nature of the power system. These procedures have dual advantages of being more accurate and possessing the ability to predict the state of the system at next step. That is, from the knowledge of the state vector at an instant of time "t", and the mathematical model of the system, the DSE techniques forecast the state vector of the power system at the next instant of time i.e. "t+1". This predicting ability has remarkable advantages, as security analysis can now be performed one-time stamp ahead and hence allows more time for the operator to take control actions, especially in cases of any emergency [2]. Hence, DSE algorithms for power systems form an important branch of power system state estimation techniques, with a potential to impact the very nature of operation of the real time monitoring and control of power systems.

Currently, there is a need for secure operation of power system because of the advanced dependency on electricity over different aspects of our life activities including the activities which are related to national security and human safety which is employed by computer-aided programs like the state estimation operation that achieve the requirements of steady state monitoring inside the control hubs [3]. These hubs receive their data from different power system elements by different meters that connected with Remote Terminal Units (RTU). The RTUs transfer the measured quantities and other required data via the Supervisory Control and Data Acquisition (SCADA) system. This system delivers the orthodox measurements, which contain voltage magnitudes in each bus bar, real and reactive power in the buses, real, and reactive power flows in lines to the control centers for using them by the Energy Management System (EMS) programs. In practice, SCADA system cannot capture the measurements of transient periods and other emergency conditions, which are necessary for robust monitoring and dynamic state estimation operation [4]. However, a drastic change occurred in this field by the development of synchronized Phasor Measurement Units (PMU) in 1980s. This event provides the state estimator with a device that can deliver efficient and fast measurement data with high sampling rates via

Global Positioning System (GPS) [5]. PMU can be considered as a real-time measurement station because of its capability to measure both the voltage magnitude and voltage phase angle in addition to all currents of incidents branches on that bus with a speed of microseconds [4], [6]. Then, these measurements are delivered instantaneously to join the EMS. Therefore, prime focus is given to the PMU placements methods rather than the conventional placements method. The significant features of PMUs like the high precision time references are provided by the Global Positioning Satellite (GPS).

The employment of PMUs is involved within the using of so-called Smart Meters, which include the Intelligent Electronic Devices (IED). These devices facilitate Smart Grid initiative [7]; therefore, the recent orientation for making power grid to be 'smart', adds new attention for PMUs placement strategies. Generally, modern distribution grids provide another challenge for PMUs placement methods because of the increasing of necessity on renewable energy sources, which are inserted continuously in distribution grid. These distributed generators (DG) convert the distribution grid to be an active system and change its power flow paths [8].

We cannot place these efficient and expensive devices because they make several sources of cost such as the GPS communications cost, installation cost and maintenance cost, which represent a burden on the decision of using these devices widely. Therefore, to benefit from PMUs advantages, efficient placement strategies should be implemented to justify the cost of modernizing the measurement system. This is the main motivation of PMUs placement studies. Earlier papers that deal with PMUs placement literature review like that [7]- [9] have classified the PMUs placement problem, according to the methods and algorithms that are used for examining the problem of choosing the best number and location of PMUs installation. In fact, these methods, such as: genetic algorithm, particle swarm optimization and integer programming have different limitations and requirements for each objective of PMUs installation and it can be used for different goal within different cases. Therefore, this paper is approached to address the problem of PMUs placement according to main objectives and reasons for their installations in a particular power system. We also put some light on the reduction of PMUs number, reliability, accuracy and especial requirements for modern distribution grids. These selective objectives are discussed in section III, while the next section introduces a brief review of the main formulas of state estimation and the observability concept. However, the essential constraint, which imposes the minimum number of measurement set in power systems, is the ability of state estimation operation to be solvable, and the power system to be an observable [10]. This paper grants a review of the advances in dynamic state estimation techniques based on our survey of research papers available in literature.

We have also done a broad survey of the literature available on the dynamic estimation techniques and in this paper, an attempt has been made to give an overview of the various procedures, their applications and future trends, on these topics. Finally, the discussion and conclusion are presented in sections IV and V respectively.

II. DESIGN OF STATE ESTIMATION

Power system state estimation procedure aims to estimate the state vector x of the system which includes the voltage of bus and angles by using the available real-time monitoring. The measurement set consists of magnitude of bus voltage, real and reactive power currents, and real and reactive bus booster powers [4], [11]. It is then collected and information is processed and filtered by SE formulas to deliver the estimated state of the entire system for the monitors. However, the PMU can provide voltage magnitudes and their angles directly if it would be more affordable. The principle formula for state estimation solution is as follows [13]:

$$z = h(x) + \varepsilon \quad (1)$$

where: x is the state vector (including bus voltage magnitudes and phase angles), z is the measurement vector, and $h(x)$ is a non-linear vector function and ε as the error or noise vector.

The minimization of the additional function $J(x)$, if the observability condition is satisfied, results in the Weighted Least Square (WLS) state estimation.

$$J(x) = (z - Hx)^T R^{-1} (z - Hx) \quad (2)$$

Where: $H = \partial h(x) / \partial x$ is the measurement Jacobian matrix of $h(x)$ and R is the covariance matrix. Whenever a measurement set in a particular power system is enough to provide a unique solution for the estimated state vector, this power system can be declared as an observable system [11]. This minimum number of measurements could be equal to $(2n - 1)$, if n is the number of buses in the power system under test. Although, this condition of measurements availability could be changed according to their types, numbers and locations, the solution of WLS still associates with Jacobian matrix's characteristics [3], [11], [13]. The ability of predicting the state vector one step ahead is a very important advantage of DSE is which allows security analysis to be carried out in advance and also allows the operator to have more time during emergencies.

III. PMU'S PLACEMENT APPROACHES WITH SIMULATION RESULTS

From the PMU based static state estimation, we know that PMUs can be used along with the normal measurements in the state estimation procedure, so that the high quality measurements of the PMUs can drive the estimates of other low quality measurements towards their true values. At least 2 PMUs are necessary in order to calculate the voltage angular vector, as the voltage angle at a particular bus is the angular deviation of the voltage vector at that particular bus, with reference to the voltage vector at the reference bus.

In fact, the common goal of placement method is to achieve the observability of the power system by using minimal n PMUs numbers because these studies aim to turn the power system to be observable with the minimum cost. However, for robust and efficient state estimation, many requirements should be regarded in addition to PUMs numbers.

In fact, it could be a justified process due to the common requirements of observability conditions, which associated with numbers, and locations of meters in the power system [4]. Reference [3] is an evidence about the early orientation toward using the phasor measurement units (PMUs) and subsequently, solving the problem of efficient employment of these expensive devices. The authors in [3] have used a combination of dual search and graph theory for computing the minimal number of PMUs that are required for achieving the topological observability. In this paper, we applied the Graph theory along with the aid of depth-first search, for supplying the initial PMUs set which is used as an upper limit for the bisecting search method. This initial set of PMUs is represented by a minimum spanning sub graph, which contains all buses of the network. Then, this process is accelerated by guessing the candidate buses for PMU installation. These candidates are the buses, which have the largest number of branches. Although this method is relatively fast, more development is needed to reduce computing time as the researchers have recommended [3]. For testing this method, several power networks have been used such as: IEEE 14bus, IEEE 118-bus, WSCC, New England and Taiwan systems. Despite the studies like [3], until recent years, PMUs placement studies were taking additionally the conventional measurements placement into consideration. For example, Xu and Abur [4] have implemented a study about the mixed measurement set placement. By using integer-programming method, the authors have found the minimal number of PMUs that are required for obtaining the observability in two cases: with and without conventional measurements. They have used IEEE 14-, 57- and 118-bus networks for testing their method. In this field, a heuristic sequential elimination method has used by Rakpenthai et al. [13,] for finding the minimal number of PMUs. In fact, Madtharad et al. [14] used the same method previously with the conventional measurements. This method depends on the condition number of Jacobian matrix where the measurement location that has a minimum condition number is discarded. This process should be repeated for all locations; therefore, long computational time is required for using this method [13]. Graph theory [15] has been used again for maximizing power system observability by using PMU placement strategy. In addition, they used Greedy algorithm which produces close-to-optimal (optimality of 97%) performance. Furthermore, the authors [11] have proved that PMUs placement for achieving full observability is one of the hard and complicated problems, which are called nondeterministic polynomial (NP) problems. This method has been implemented on IEEE 30-, 57- and 118-bus systems [15]. Dongjie et al. [16] have used artificial intelligent techniques such as simulated annealing, Tabu search in addition to genetic algorithm. This combination has been used because Tabu search and simulating annealing have better performance in large-scale problems. This can be added to GA accuracy to optimize PMU location in a power grid.

In fact, recent studies tend to address more than one objective and to deal with different constraints of the PMUs placement problem. Therefore, papers that are more modern are discussed in the next sections. From the reliability point of view, the measurement placement problem should take into consideration the probability of meter failure, communication channels, and single or multiple-line outage. In this case the measurements set which is calculated according to item A solely, would not be sufficient for observability analysis in real operating conditions; consequently, measurements set would be considered unreliable [17]. Therefore, the other significant objective is to identify an additional set of meters with a minimal possible cost for achieving SE under different contingencies. Recently, the hazards of cascade blackout [18] become more serious because the international connection of power grids and their numerous loads. Therefore, the complicated monitoring and control issues of modern power systems, turns power engineers to make use of PMU capabilities to avoid or to mitigate the impact of the line outage or the loss of the PMU itself. For the above purpose, new tools like the Cellular Genetic Algorithm (CGA) have been developed to obtain the reliability goal by Miljanic et al. [19]. The CGA is used for computing the minimum number of PMUs and their placement in case of limited availability of communication channels and branch outage as well. The contingencies, which are taken into account in this study, are single meter loss and Single-Branch outage. Mazhari et al. [20] address the same target by using the Cellular Learning Automata (CLA) for optimizing the computing of PMUs placement under the same contingencies of Miljanic's study. However, the maintenance of maximum measurement redundancy during the optimal placement process has been added as a constraint to develop a "multi-objective" approach [18]. Again, graph theory plays a significant role in analyzing methods due to its ability to represent the topological configuration of power systems. Thus, Anderson et al. [19] use graph theory for the topological analysis of the placement problem by considering the number of available channels and branch outages. The Binary Integer Programming method has its role in state estimation reliability studies, where the researcher of [20] has used this method for achieving a complete topological observability using minimum number of PMUs. This work takes into consideration the existence of conventional measurements and the failure of a PMU or a single line. This paper focuses on using 'unequal' constraints of adjacent busses to reduce the need for more PMUs in this study, the unequal constraints that are represented by "larger than" and "lower than" is used for evaluation of placement optimality. The authors claim that the efficiency of using the unequal constraints has been proven by their test results, which provide a complete observability by using a number of PMUs less than that used by other relative studies.

IEEE 30, 57 and 118-bus system have been used for testing this method [22]. Enshaee et al. [23] implement the Binary Integer Programming method has been used again in a significant extension of earlier reliability studies. This research's importance comes from the treatment of several placement requirements such as achieving full observability; maximize measurement redundancy and addressing three cases of contingency, which are: single-line outage, single-PMU loss, and limited channel case. The following formula is used in this paper as an objective function for maximum redundancy [25]:

$$\min \sum C \times N \sum w \sum m \sum N \sum f N \quad (3)$$

Where: C is the cost of the installed PMU, x is a vector which refers to the PMU status of the i^{th} bus (installed or not), w is the weighting factor, m is the maximum number of observing the i^{th} bus while the number of times that the i^{th} bus can be observed by the PMUs is represented by f. This research has used the IEEE standard networks for testing its

method [23]. State vector (x) should be computed as accurately as possible, which means a minimum value for the variance of the error (σ^2). However, many studies have followed Koglin's term of "interesting quantities" to identify the most significant electrical quantities in power system that need to be monitored accurately [17], [24]. Accordingly, the accuracy requirement can be determined as [24]:

$$\min \{ \sum S \beta I \} \quad (4)$$

Where: β is the estimated variance of component No. i , i is the specified accuracy limit for component No. i (represents the upper limit of interesting quantity's variance) [24], I is the total number of interesting quantities. Considering PMU placement, Jiang and Vijay [25] classify the buses into two groups according to their local redundancy and their accuracy and then the PMUs placement is achieved by ranking these groups of buses. In this method, some measurements are classified as critical measurements, which cannot be excluded. The implementation of this approach on IEEE 14-bus and IEEE 118-bus indicates that SE accuracy can be enhanced by increasing PMU numbers. Another paper in [26] aims to improve the state estimation accuracy by choosing the optimal numbers and locations of PMUs. In this research, the diagonal of the gain matrix inverse (G^{-1}) has been used as an indication for SE accuracy (where, G^{-1} is equal to covariance matrix). Additionally, this method is carried out on a part of the Swedish transmission grid, which is called (SvK). Artificial intelligence-based methods have been suggested to solve this multi-objective problem of achieving an accurate state estimation in addition to minimum placement of PMUs. Rosli et al research [27] gives an Example about that trend. Where, the authors have used the Particle Swarm Optimization (PSO) method for finding an optimal placement of PUMs accompanied with a high accuracy of estimated state. PSO is one of the heuristic methods, which simulate the systematic movement of 'birds' in their 'flock' by maintaining their "position" and "velocity". In this paper [27], the position is represented by the covariance matrix, which implies the difference between the true values and estimated values. This paper results refer to the direct relation between the PMUs numbers and the accuracy level of SE. However, they indicate that the inconsistency of PSO method may affect the quality of computation; therefore, the authors for developing this method recommend more research. Recently, the Gauss-Newton (GN) algorithm has been used in two studies by the same team [28], [29] for finding the optimal PMUs placement for a hybrid state estimation by considering the accuracy and the convergence of SE process. GN method is used in these studies for solving the iterative SE algorithm efficiently. Both studies use IEEE 30-bus and IEEE 118-bus to test the GN algorithm. Distribution System Considerations should distribution system have its own objective for PMUs placement? The answer of this question relates to whether the PMUs placement in distribution system requires different constraints compare to transmission system constraints. Actually, the studies which are mentioned above, have been tested on transmission system or on standard networks like IEEE 57- and 118-bus systems which definitely have different characteristics with the traditional distribution grid. There are differences between distribution and transmission system such as: the unbalance nature of distributed loads (because the existence of single-phase and two phase loads), and the tendency for using the distributed generators (DGs) in the distribution side in addition to the radial configuration of distribution grids [30]. Therefore, specific requirements and different strategies should be followed for PUMs placement in the distribution system. Otherwise, the traditional methods, which are implemented in transmission system, would be inaccurate or inefficient when applied in the distribution system. One of the earlier studies in this field is the research of Baran et al. [31], which introduces the need of an accurate state estimation for distribution system due to the requirements of real-time monitoring and control of the distribution grid. The authors have used meter-ranking method, which is based on the interesting quantities criterion. The meter ranking aims to classify the meters in a particular power system according to their importance for the monitoring operation. Accordingly, the most important meters are selected. The authors [31] used radial shaped network for their method's implementation. In Yang and Roy research [32], a comparison between a meshed transmission system and the radial distribution system is implemented to show the performance of their method that is based on a three-phase state estimation. The authors avoided the mistakes of previous studies that dealt with single-phase scheme that may fail in the usual case of the unbalance distribution grid. Accordingly, they compute the minimal number of PMUs to obtain full observability based on three-phase state estimation. They state that there is more than one optimal measurement set; consequently, their works are to choose the best set that achieves best state estimation performance. For this purpose, the authors have used two heuristic methods: greedy algorithm and integer programming optimization, which have succeeded in finding the global optimal solution [32]. In considering the case of active distribution systems which contain distributed generation sources, Pegoraro et al. [32] have used the Genetic Algorithm for optimizing PMUs and smart meter locations within accuracy limits for different load configuration of the active distribution grid. On the same trends, the study of [34] has suggested a hybrid intelligent method (Nelder-Mead Simplex search with Ant Colony Optimization) for finding the optimal PMUs numbers regarding the best state estimation criterion. In this paper, the best distribution SE is achieved by minimizing the difference between the measured values and calculated values. Recently, the research in [35] reused the Genetic algorithm as an intelligent technique for finding the optimal PMU placement for the state estimation of active distribution systems. In this research, both the accuracy constraint the measurements deficiency is studied in addition to the effect of the DGs on the power flow. For achieving the optimal placement, Monte Carlo method is used in addition to Genetic algorithm. Despite the cost of PMUs, more measuring devices and PMUs should be used for ensuring the required accuracy for robust SE [35].

IV. DISCUSSION AND SUGGESTIONS FOR FUTURE STUDIES OF PMUS PLACEMENT

According to the publications that have been reviewed, the main attributes of a PMU placement are devices' numbers, reliability, accuracy, and modern distribution network requirements. However, the issue of optimal PMU placement in modern power networks needs to be addressed by regarding the different requirements and the different

objectives of the PMUs employment. That is obvious from the recent serious attempts for implementing studies, which deal with multi-objective placement approaches; however, these studies still reflect the authors' field of research and thus, they focus on a specific application [36]. Regarding the reliability requirement, it can be noticed from the above review that the current studies have dealt with specific contingency cases and all of them have been implemented on power system which has no more than 118 busses. Therefore, for the sake of reliability, more constraints and operating conditions should be considered in future studies such as the case of multiple failures that could result from human error or natural disaster. Hence, testing the proposed algorithm for large-scale networks, which simulates the real power systems is necessary. Accordingly, expanded investigations that involve large-scale systems should be implemented to explore the influence of networks' size and the PMUs numbers on SE accuracy. In addition, futures studies need to take into account the effect of meter accuracy and the influence of local environmental parameters. Finally, the applications of distribution system state estimation have become the most attractive field within power system monitoring operations; consequently, PMUs placement in distribution grid has started to consider new functions and objectives. The modern PMUs placement papers have taken into consideration the enhancement of distribution grid power quality, i.e. the Harmonics and voltage Sag concerns, in addition to the considerations of the fault location strategies and the effect of the distribution network reconfiguration [34] - [36].

V. CONCLUSION

The optimal PMUs placement problem is a multidimensional and multi-functional problem because PMUs can be installed for many objectives. Therefore, this paper has focused on the main objectives of PMU employment rather than the methods of finding the optimal placement, which the previous studies have focused on. What is important from this research point of view is to know the main requirements and the possible constraints of PMU employment and hence, the required approach for finding the optimal placement of these expensive devices can be selected properly. The diversity of methods and algorithms that have been implemented during the last three decades for PMUs placement in addition to the studies that include mixed or hybrid measurement sets turn the classification task to be larger than this paper limits. However, in addition to the State Estimation applications, the issue of PMUs placement needs more detailed studies for discussing the different applications of the Phasor Measurement Units in power systems.

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