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Medium Domination Number of Certain Graphs

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Abstract –

In this paper the author have found out the medium domination number of Helm graph, Friendship graph.

Keywords – Dominating set, minimum dominating set, domination number, total domination number, medium domination number.

I. INTRODUCTION

A set S of nodes of G is a dominating set of G if each node of G is dominated by some node in S. If $S$is a dominating set of a graph G and no proper subset of G is a dominating set of G, then S is called a minimum dominating set. A minimum dominating set in a graph G is a dominating set of minimum number. The number of a minimum dominating set is called the domination number of G and is denoted by $\gamma(G)$. The total domination number of G, denoted as $\gamma_t(G)$ is the cardinality of the smallest total dominating set of G. For $G = (V, E)$ and for every $x, y \in V$ if x and y are adjacent they dominate each other then dom$(x, y) \geq 1$. In any graph $G = (V, E)$ and for every $x, y \in V$, the total dominating number of nodes is denoted as $TDV(G) = \sum_{\forall x,y \in V} dom(x, y)$.

Example: 1.1

For the graph G given below,

dom(v_1, v_2) = 1, dom(v_1, v_3) = 2, dom(v_1, u) = 1, dom(v_1, v_4) = 1,

dom(v_2, v_3) = 1, dom(v_2, v_4) = 2 , dom(v_2, u) = 1, dom(v_3, v_4) = 1,

dom(v_3, u) = 1, dom(v_4, u) = 0 .

$TDV(G)$ = 1+2+1+1+1+2+1+1+1+0 = 11.

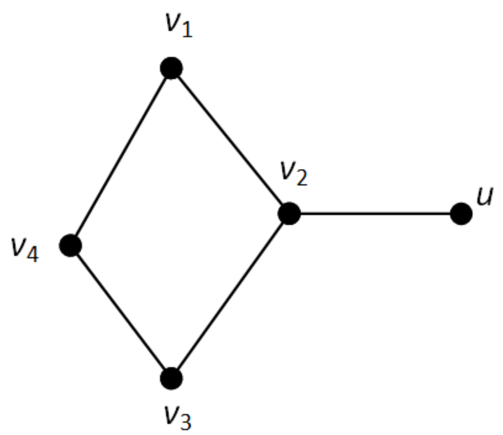

Figure 1.1

II. MEDIUM DOMINATION NUMBER

Definition: 2.1

In any connected simple graph G of order p, the medium domination number of G is defined as

$\gamma_m(G) = \frac{TDV(G)}{pC_2}$.

Example: 2.2
For the graph G given above,
Total Domination number $TDV(G) = 11$
Medium domination number $\gamma_m(G) = \frac{11}{5C_2} = \frac{11}{10}$

Observation: 2.3
Let G be a graph with p nodes, q edges and for $\deg(v_i) \geq 2$; $TDV(G) = q + \sum_{v_i \in V} degv_i\, C_2$.

Theorem: 2.4
For a Helm graph H_n, $\gamma_m(H_n) = \frac{8n+6+\sum_{i=4}^{n} i}{(2n+1)C_2}$, $n \geq 3$.

Proof. The Helm graph H_n has 2n+1 nodes and 3n edges.
Let $P(n) = \frac{8n+6+\sum_{i=4}^{n} i}{(2n+1)C_2}$.
We prove this theorem by the method of induction.
Basis Step :P (3) is true.
H_3 contains seven nodes namely, v_1, v_2, v_3, u_1, u_2, u_3, u with u as a middle node and nine edges v_1v_2, v_1v_3, v_2v_3, v_1u, v_2u, v_3u, u_1v_1, u_2v_2, u_3v_3 .u is adjacent to three nodes v_1, v_2 and v_3 . Here the three nodes v_1, v_2 and v_3 are of even degree and other nodes are of odd degree. The total number of nodes that dominate each couple of nodes is the sum of number of edges and the summation of $\deg v_i C_2$.
TDV(G) is split-up into three parts, namely the number of edges, the odd degree nodes and the even degree nodes. By 2.3,

$$TDV(G) = q + \sum_{v_i \in V} degv_i\, C_2.$$

Now
$$TDV(H_3) = q + \textstyle\sum_{odd\, v_i}(degv_i\, C_2) + \sum_{even\, v_i}(degv_i C_2)$$

$$= 9 + 1(3C_2) + 3(4C_2) \quad = 9+3+18 \quad = \quad 30$$

Therefore

$$\gamma_m(H_3) = \frac{TDV(H_3)}{(2n+1)C_2}$$

$$= \frac{30}{(2\times3+1)C_2} = \frac{30}{7C_2}$$

Induction Step: Assume $P\ (n - 1)$ is true.

Let us take the Helm graph $H_{n-1}, n > 3$.

H_{n-1} has $(2n-1)$nodes $u, v_1, v_2, v_3, \ldots, v_{n-1}, u_1, u_2, \ldots, u_{n-1}$ and $3(n-1)$ edges, $v_1v_2, v_2v_3, \ldots, v_{2n-2}v_{2n-1}, v_1u_i, 1 \leq i \leq n-1$ and $uv_i, 1 \leq i \leq n-1$. In H_{n-1}, both odd and even degree nodes are $(n-1)$ and middle node u has degree $(n-1)$.

The total number of nodes that dominates each couple of nodes is

$$TDV(H_{n-1}) = 8(n-1) + 6 + \sum_{i=4}^{n-1} i$$

To prove $P\ (n)$ is true.

Consider the Helm graph H_n .

Here H_n has $(2n+1)$ nodes $u, v_1, v_2, v_3, \dots, v_n, u_1, u_2, \dots, u_n$ and $3n$ edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_iu_i, 1 \le i \le n$ and uv_i, $1 \le i \le n$. In H_n, the number of both even and odd degree nodes are n and middle node u has degree n . By the graphs H_3 and H_{n-1} comparisons, for each $(n-1)$ stage to n^{th} stage, it was found that two additional nodes and 3 edges are included.

Therefore the total number of nodes and edges added in the next stage will be
$3 + 1(4C_2) + \{nC_2 - (n-1)C_2\}$.

Therefore

$$TDV(H_n) = TDV(H_{n-1}) + 3 + 4C_2 + \{nC_2 - (n-1)C_2\}.$$

$$= 8(n-1) + 6 + \sum_{i=4}^{n-1} i + 3 + 6 + \left\{\frac{n(n-1)}{2} - \frac{(n-1)(n-2)}{2}\right\}$$

$$= 8n - 2 + \sum_{i=4}^{n-1} i + 9 + n - 1$$

$$= 8n + 6 + \sum_{i=4}^{n-1} i - n + n$$

$$TDV(H_n) = 8n + 6 + \sum_{i=4}^{n-1} i$$

$$\gamma_m(H_n) = \frac{TDV(H_n)}{(2n+1)C_2}$$

$$\gamma_m(H_n) = \frac{8n+6+\sum_{i=4}^{n-1} i}{(2n+1)C_2}, n \ge 3.$$

Theorem: 2.5

For a Friendship graph F_n, $\gamma_m(F_n) = \frac{2n^2+4n}{(2n+1)C_2}$, where n is the number of triangles .

Proof. The Friendship graph F_n has $2n+1$ nodes and $3n$ edges. Let u be the middle node and $v_1, v_2, \dots, v_{2n}$ be the remaining nodes of F_n .

Let $P(n) = \frac{2n^2+4n}{(2n+1)C_2}$.

We prove this theorem by the method of induction.

Basis Step :P (3) is true.

In F_2, there are five nodes v_1, v_2, v_3, v_4, u and six edges $v_2, v_2u, v_1u, v_3v_4, v_3u$, v_4u. The middle node u is adjacent to other nodes v_1, v_2, v_3, v_4 . Here all nodes are of even degree. The total number of nodes that dominate each couple of nodes is the sum of number of edges and the summation of $\deg v_i C_2$.

$TDV(G)$ is split-up into three parts, namely the number of edges, the odd degree nodes and the even degree nodes. By 2.3,

$$TDV(G) = q + \sum_{v_i \in V} degv_i\, C_2.$$

Now

$$TDV(F_2) = q + \sum_{odd\, v_i} (degv_i\, C_2) + \sum_{even\, v_i} (degv_i C_2)$$

$$= q + \sum_{even\, v_i} (degv_i C_2) \quad [since \sum_{odd\, v_i} (degv_i\, C_2)\ ]$$

$$= 6 + 6(2C_2) \ = 12$$

Therefore

$$\gamma_m(F_2) = \frac{TDV(F_2)}{(2n+1)C_2} \quad = \frac{12}{(2\times3+1)C_2} \quad = \frac{12}{5C_2}$$

Induction Step: Assume $P\ (n\ -\ 1)$ is true.

F_{n-1}has$(2n-1)$nodes u,v_1,$v_2, v_3, \dots, u_{2n-2}$ and $3(\ n-1\ )$edges, $v_1v_2, v_3v_4, \dots, v_{2n-3}v_{2n-2}, v_iu$, $1 \le\ i\ \le\ 2n\ -\ 1$. In F_{n-1} there are $(2n-2)$nodes are of even degree and middle node u of degree $(\,2n - 2\,)$.
The total number of nodes that dominates each couple of nodes is $TDV(F_{n-1}) = 2(n-1)^2 + 4(n-1)$
To prove $P\ (n)$ is true.
Here F_n has $(\,2n\ +\ 1\,)$ nodes $v_1, v_2, \dots, v_{2n}, u$ and $3n$ edges $v_1v_2, v_3v_4, \dots, v_{2n-1}v_{2n}, v_iu$, $1\ \le\ i\ \le\ 2n$. In F_n, there are $2n$ nodes are of even degree and middle node u of degree $2n$. By the graphs F_2 and F_{n-1} comparisons, for each $(n-1)$ stage to n^{th}stage, it was found that two additional nodes and 3 edges are included.
Therefore the total number of nodes and edges added in the next stage will be 3 + $2(2C_2)$ + $\{2nC_2 - (2n - 2)C_2\}$.
Therefore

$$TDV\ (F_n)\ =\ TDV\ (F_{n-1})\ +\ 3\ +\ 2(2C_2)\ +\ \{2nC_2\ -\ (2n\ -\ 2)C_2\}.$$

$$= 2(n-1)^2 + 4(n-1) + 5 + \left\{\frac{2n(2n-1)}{2} - \frac{(2n-2)(2n-3)}{2}\right\}$$

$$= 2n^2 + 3 + 4n - 3\ = 2n^2 + 4n$$

$$\gamma_m(F_n) = \frac{TDV(F_n)}{(2n+1)C_2}$$

$$\gamma_m(F_n) = \frac{2n^2+4n}{(2n+1)C_2}.$$

III. CONCLUSION

In this paper, the author have studied about Medium Domination Number of certain graphs like Helm, Friendship graphs.

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REFERENCES

[1] . Anitha.K and L.JethruthEmelda Mary, “The Medium Domination Number of a Graph”, Vol : 03, 857-860 , International Journal of Com-puting Algorithm, 2014.
[2] . Bondy.J.A and Murty.U.S.R, “Graph Theory with Application”, North - Holland , New York.
[3] . Dundar.P and N.Tacmin, “Domination and Total Domination Number of a Graphs ”, Master thesis, Faculty of Science, Ege University, 2006.
[4] . DuyguVargor and Pinar Dundar, “The Medium Domination Number of a Graph ”, International Journal of Pure and Applied Mathematics Vol: 70, No: 3, 297-306, 2011.

[5] . Parvathi.N and Ramachandran.M, "The Medium Domination Number of a Jahangir Graph $J_{2,m}$ ", National Conference on Mathematical Techniques and its Applications(NCMTA), Chennai:SRM University,2014.
[6] . Ramachandran.M and N.Parvathi, "The Medium Domination Number of a Graph $J_{m,n}$," Indian Journal of science and Technology, Vol 8(5), 400-406, 2015.
[7] . Vargor.D and P.Dundar, "The Medium Domination Number of a Graph," Int. J. pure Appl. Math. Sci. 70: 297-306, 2011.