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Robust H_2 Output Feedback Controller Design for Damping Power System Oscillations

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Abstract—

This paper addresses the design problem of robust H_2 output feedback controller design for damping power system oscillations. Sufficient conditions for the existence of output feedback controllers with norm-bounded parameter uncertainties are given in terms of linear matrix inequalities (LMIs). Furthermore, a convex optimization problem with LMI constraints is formulated to design the output feedback controller which minimizes an upper bound on the worst-case H_2 norm for a range of admissible plant perturbations. The technique is illustrated with applications to the design of stabilizer for a single-machine infinite-bus (SMIB) power system. The LMI based control ensures adequate damping for widely varying system operating.

Keywords— H_2 controller, linear matrix inequality, power system stabilizer, single-machine infinite-bus systems

I. INTRODUCTION

Power system stabilizers (PSSs) have been used for many years as supplementary controllers to provide extra damping for synchronous generators in electrical power system. In spite of availability of various innovative designs, the fixed gain, lead-lag compensation-type of stabilizer has been the most popular with the electrical utilities. PSSs enhance the power system stability limit by enhancing the system damping of low frequency oscillations associated with the electro-mechanical modes [1]. Conventional power system stabilizers (CPSSs) are used to damp out small signal oscillations and they are designed based on a model which is linearized around a particular operating point. Conventional design tunes the gain and time constants of the PSS, which are mostly lead-lag compensators, using modal frequency techniques. Such designs are specific for a given operating point; they do not guarantee robustness for a wide range of operating conditions.

In the last few years, robust control technique has been applied to power system controller design to guarantee robust performance and robust stability, due to uncertainty in plant parameter variations. Some of those efforts have been contributed to design robust controllers for damping power system oscillations [2-4]. The main advantage of H_∞ optimization methods is that the model uncertainties can be accounted for at the design stage. Normally, the problem is formulated as a weighted mixed sensitivity design and solved by a Riccati approach. In power system damping control design, the main objective is to improve the damping ratio of the electromechanical modes over widely varying operating conditions.

Design methods based on the H_∞ norm of the closed-loop transfer function have gained popularity, because unlike H_2 methods (best known as LQG), they offer a single frame- work in which to deal both with performance and robustness. On the other hand, since an H_2 cost function offers a more natural way of representing certain aspects of the system performance, improving the robustness of H_2-based design methods against perturbations of the nominal plant is a problem of considerable importance for practical applications [5]. In the robust H_2 approach, the controller is designed to minimize an upper bound on the worst-case H_2 norm for a range of admissible plant perturbations.

With the development of numerical algorithms for solving linear matrix inequality (LMI) problems in the last few years, the LMI approach have emerged as a useful tool for solving a wide variety of control problems [6, 7]. An LMI-based controller design for damping power system oscillation has been discussed in several recent papers [8-11]. In this paper, we consider the design problem of robust H_2 output feedback controller for damping power system oscillations. We obtain sufficient conditions for the existence of output feedback controllers with norm-bounded parameter uncertainties in terms of linear matrix inequalities (LMIs). The efficiency of an LMI-based design approach as a practical design tool is illustrated with applications the design of stabilizers for a SMIB power system.

II. CONTROLLER DESIGN

Consider the linear plant ***P*** with input *u*, disturbance *w*, performance output *z* and the measurement signal *y*. The input is generated by dynamic output feedback, using the controller ***K***. The signal *z* is the performance associated with the H_2 criterion. The state space representation of the controlled system with parameter uncertainties can be written as follows:

$$\begin{aligned} \dot{x} &= (A+\Delta A)x + (B_1+\Delta B_1)w + (B_2+\Delta B_2)u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{1}$$

where $\boldsymbol{A}$, $\boldsymbol{B_1}$, $\boldsymbol{B_2}$, $\boldsymbol{C_1}$, $\boldsymbol{C_2}$, $\boldsymbol{D_{12}}$, and $\boldsymbol{D_{21}}$ are known constant real matrices of appropriate dimensions, ΔA, ΔB_1, and ΔB_2 are matrix-valued functions representing time-varying parameter uncertainties in the system models.

The parameter uncertainties considered here are assumed to be norm bounded and of the form [12]:

$$[\Delta A \quad \Delta B_1 \quad \Delta B_2] = D\Sigma(t)[E_1 \quad E_2 \quad E_3] \tag{2}$$

where $\boldsymbol{D}$, $\boldsymbol{E_1}$, $\boldsymbol{E_2}$, and $\boldsymbol{E_3}$ are known constant real matrices of appropriate dimensions, which represent the structure uncertainties, and $\Sigma(t) \in R^{i\times j}$ is an unknown matrix function with Lebesque measurable elements and satisfies

$$\Sigma^{\mathrm{T}}(t)\Sigma(t) \le I \tag{3}$$

in which ***I*** denotes the identity matrix of appropriate dimension.

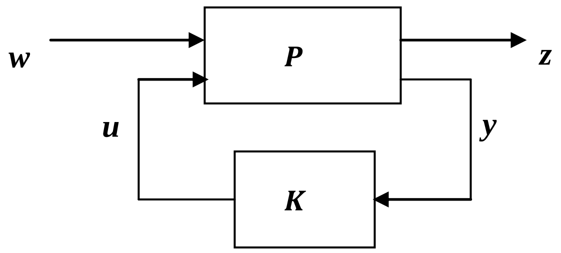

Fig. 1 Generalized Plant

The illustration of the controlled system is shown in Fig. 1. Furthermore, ***w*** represents an exogenous signal and $H_{zw}(s)$ denotes the transfer function from ***w*** to ***z***. The basic problem to be formulated is the stabilizability problem with respect to the strictly proper controller ***K*** with state space realization

$$\begin{aligned} \dot{x}_K &= A_K x_K + B_K y \\ u &= C_K x_K \end{aligned} \tag{4}$$

such that the closed-loop system

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}w \\ z &= \bar{C}\bar{x} \end{aligned} \tag{5}$$

$$\text{where} \quad \bar{A} = \begin{bmatrix} \hat{A} & \hat{B}_2 C_K \\ B_K C_2 & A_K \end{bmatrix}, \ \bar{B} = \begin{bmatrix} \hat{B}_1 \\ B_K D_{21} \end{bmatrix}, \ \bar{C} = \begin{bmatrix} C_1 & D_{12} C_K \end{bmatrix} \tag{6}$$

$$\begin{aligned} \hat{A} &= A + D\Sigma(t)E_1 \\ \hat{B}_1 &= B_1 + D\Sigma(t)E_2 \\ \hat{B}_2 &= B_2 + D\Sigma(t)E_3 \end{aligned} \tag{7}$$

Satisfies the following control objective:

1) The closed-loop system (5) is asymptotically stable.
2) The performance function $\|H_{zw}\|_2$ is minimized.

Associated to this closed-loop system we introduce the following matrix functions:

$$H(Y,L) = \hat{A}Y + Y\hat{A}^T + \hat{B}_2 L + L^T \hat{B}_2^T \tag{8}$$

$$G(F,X) = \hat{A}^T X + X\hat{A} + FC_2 + C_2^T F^T \tag{9}$$

$$Z(X,Y,F,L) = \hat{A} + Y\hat{A}^T X + L^T \hat{B}_2^T X + YC_2^T F^T \tag{10}$$

The first two are linear, whereas the third one is nonlinear. In the sequel, when no confusion is possible, they are simply written as ***H***, ***G*** and ***Z***, respectively.

Theorem 1: The system (1) can be stabilized by a dynamic output feedback (5) if, and only if, there exist symmetric matrices $X, Y \in R^{n\times n}$ and matrices $M \in R^{n\times n}$, $L \in R^{m\times n}$, $F \in R^{n\times r}$ as solution to the nonlinear inequalities [13],

$$\begin{bmatrix} H & Z+M \\ Z^T + M^T & G \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} Y & I \\ I & X \end{bmatrix} > 0 \tag{12}$$

From the Lyapunov theorem, the closed-loop matrix $\bar{A}$ is asymptotically stable if, and only if, there exists $\bar{P} = \bar{P}^T > 0$ of dimension $2n \times 2n$ such that

$$\bar{A}^T \bar{P} + \bar{P}\,\bar{A} < 0 \tag{13}$$

By defining $T \in R^{2n\times 2n}$ and letting $\bar{P}$ and $\bar{P}^{-1}$ be partitioned according to $n \times n$ dimensioned blocks

$$T = \begin{bmatrix} Y & I \\ V^T & 0 \end{bmatrix},\ \bar{P} = \begin{bmatrix} X & U \\ U^T & \hat{X} \end{bmatrix},\ \bar{P}^{-1} = \begin{bmatrix} Y & V \\ V^T & \hat{Y} \end{bmatrix} \tag{14}$$

then $Y = (X - U\hat{X}^{-1}U^T)^{-1} > X^{-1}$ so that (12) must hold. The regularity assumption can be made with no loss of generality when working with strict inequalities since one can always induced a small regular perturbation on matrix ***V*** to achieve regularity. With the following change of variables:

$$L = C_K V^T,\quad F = UB_K,\quad M = VA_K^T U^T \tag{15}$$

and multiplying (13) on the left by T^T and on the right by T, one gets the necessary and sufficient condition (11).

From the above theorem, provided matrix $\bar{A}$ in (6) is Hurwitz, the square of this norm can be expressed in terms of the solution to a Lyapunov equation such that this minimization problem with respect to the triplet of variable (A_K, B_K, C_K) is given by

$$\min\left\{ trace[\bar{B}^T \bar{P}\,\bar{B}] :\ \bar{A}^T \bar{P} + \bar{P}\,\bar{A} + \bar{C}^T \bar{C} = 0 \right\} \tag{16}$$

The equality constraint in the problem above can be replaced by an inequality using the nondecreasing property of Lyapunov equation solutions. So, using the Schur complement, the partitions for matrices $\bar{P}$, $\bar{P}^{-1}$ and the change of variables (15), the problem (16) is given by LMI formulation as follows:

$$\min\ trace(W)$$

$$\text{s.t.} \begin{bmatrix} Y & I & \hat{B}_1 \\ I & X & X\hat{B}_1 + FD_{21} \\ \hat{B}_1^T & \hat{B}_1^T X + D_{21}^T F^T & W \end{bmatrix} \geq 0 \tag{17}$$

$$\begin{bmatrix} H & YC_1^T + L^T D_{12}^T \\ C_1 Y + D_{12} L & -I \end{bmatrix} \leq 0 \tag{18}$$

$$\begin{bmatrix} G & C_1^T \\ C_1 & -I \end{bmatrix} \leq 0 \tag{19}$$

where ***X***, ***F***, ***Y***, ***L*** and ***W*** are the optimization variables. The controller matrices are recovered from the following equations

$$UV^T = I - XY \tag{20}$$

$$Z = \hat{A} + \begin{bmatrix} Y & L^T \end{bmatrix} \begin{bmatrix} \hat{A} & \hat{B}_2 \\ C_2 & 0 \end{bmatrix}^T \begin{bmatrix} X \\ F^T \end{bmatrix} \tag{21}$$

$$M = -Z - (YC_1^T + L^T D_{12}^T)C_1 \tag{22}$$

$$A_K = U^{-1} M^T (V^T)^{-1},\ B_K = U^{-1} F,\ C_K = L(V^T)^{-1} \tag{23}$$

III. THE SYSTEM MODEL

In this study, a single-machine infinite-bus (SMIB) power system [11, 14] as shown in Fig. 2 is considered. The IEEE type-ST1 excitation system with stabilizer is shown in Fig. 3. Neglecting the effect of damper winding, stator transient and resistance, the synchronous machine together with its excitation system is modeled using the following 4th order non-linear dynamic equations:

$$\dot{\omega} = \frac{1}{M}(T_m - T_e + D(\omega - 1)) \tag{24}$$

$$\dot{\delta} = \omega_b(\omega - 1) \tag{25}$$

$$\dot{E}_q' = \frac{1}{T_{d0}'}\left\{E_{fd} - (x_d - x_d')i_d - E_q'\right\} \tag{26}$$

$$\dot{E}_{fd} = \frac{1}{T_A}\left\{K_A(V_{ref} - v_t + u) - E_{fd}\right\} \tag{27}$$

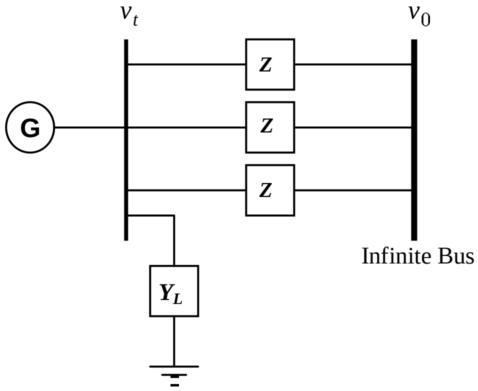

Fig. 2 A SMIB Power System

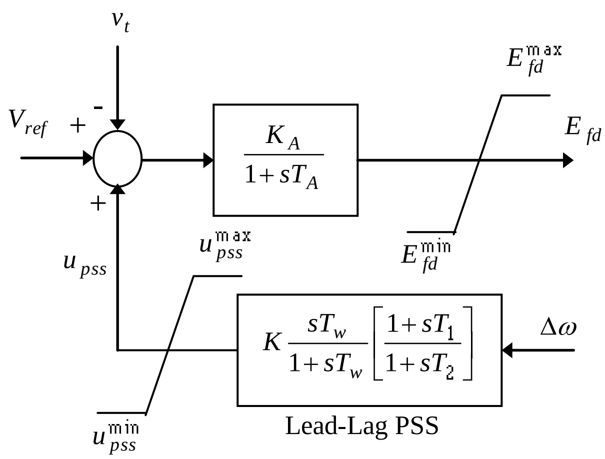

Fig. 3 IEEE Type-ST1 Excitation System with PSS

To permit analysis and control of the power system, the model is linearised around the operating point. The state variables of this model are $\Delta\omega, \Delta\delta, \Delta E_q', \Delta E_{fd}$, respectively, angular speed, rotor angle, voltage behind transient, and excitation voltage. In this study, we use only $\Delta\omega$ as a feedback input signal. The power input to the generator shaft is assumed constant, the network is represented by a set of algebraic equation and the load is modeled by constant impedance.

IV. SIMULATION RESULTS AND DISCUSSIONS

A typical single-machine infinite-bus (SMIB) power system is chosen for analysis of the proposed controller. Details of system data and CPSS constants are given in [11, 14].

The operating condition: P_e = 1.0 pu, Q_e = 0.015 pu and v_t = 1.05 pu are chosen as the nominal operating condition and other operating points are regarded as perturbations of the nominal system. Four different loading conditions representing nominal, heavy, light, and leading power factor (PF) are considered as given in Table I. The eigenvalues of the nominal system are 0.295±j4.96 and –10.4±j3.28. It is observed that the electromechanical mode (characterized by the pair of eigenvalues 0.295±j4.96) is negatively damped and the eigenvalues for this mode should be shifted leftward to more desirable locations into the left half s-plane.

Table I Operating Conditions

Case	P (pu)	Q (pu)
1. Nominal	1.0	0.015
2. Heavy	1.2	0.20
3. Light	0.7	0.10
4. Leading PF	0.7	-0.20

The technique described in Section 2 was applied to the design of stabilizer (proposed PSS) for the study system. By employing Matlab LMI Control Toolbox [7] to solve the LMIs (17)-(19), we obtain the corresponding feasible solution for the 4th order controller K in the form (20)-(23) with the transfer function

$$K = \frac{7127\,(s+4.773)(s^2+21.01s+120.7)}{(s+31.11)(s+6.047)(s^2+18.21s+545.8)} \quad (28)$$

For evaluation purposes, the performance of the system with the proposed controller was compared to the CPSS and H_∞ PSS (standard HPSS). A small disturbance of 10% step increase in the reference voltage (V_{ref}) was applied to the SMIB power system at four different operating conditions. Simulation results shown in Figs. 4-7 illustrate the performance and robustness of the proposed PSS under different operating conditions. It can be seen that the proposed PSS yields the better dynamic performance and more robust against model uncertainties.

For completeness and verifications, all controllers were tasted at the following disturbances and loading conditions.

(a) Nominal loading (P,Q) = (1.0,0.015) pu with 3LG fault,
(b) Heavy loading (P,Q) = (1.2,0.3) pu with 3LG fault.

A 3LG (Three-phase-to ground) fault is assumed; one of the transmission lines (as shown in Fig. 2) met a 3LG fault and the circuit breaker operated. The simulation results for cases (a) and (b) as shown in Fig. 8 and Fig. 9, respectively. Fig. 9 shows that the CPSS fails to stabilize the system with disturbance (b), the proposed PSS provide good damping characteristics and system is stable under this disturbance. It is clear that the proposed PSS exhibits better damping properties and guarantees robust stability of the power systems.

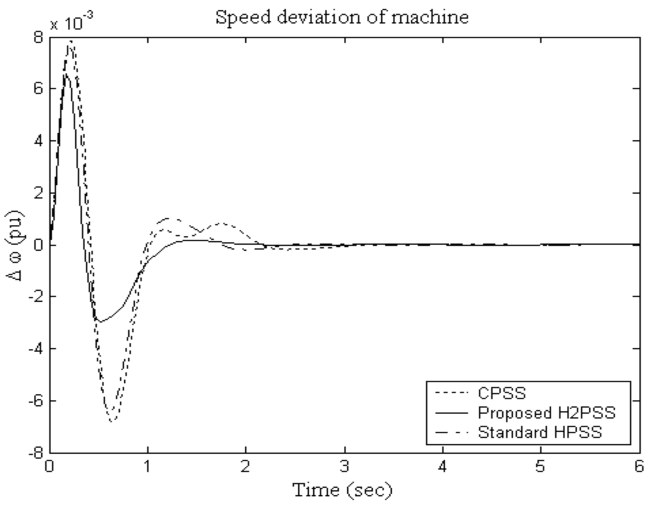

Fig. 4 Responses with 10% Step in V_{ref} for Case 1

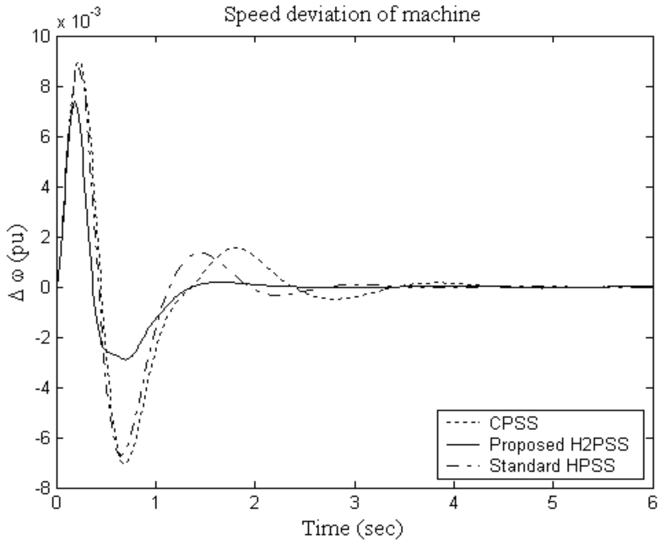

Fig. 5 Responses with 10% Step in V_{ref} for Case 2

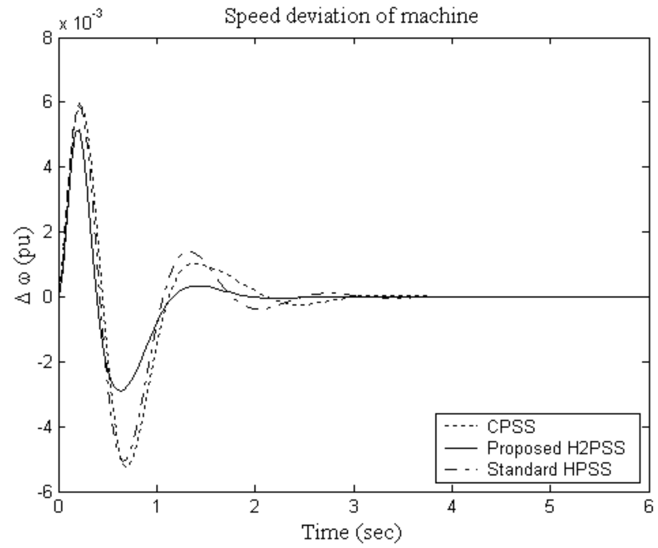

Fig. 6 Responses with 10% Step in V_{ref} for Case 3

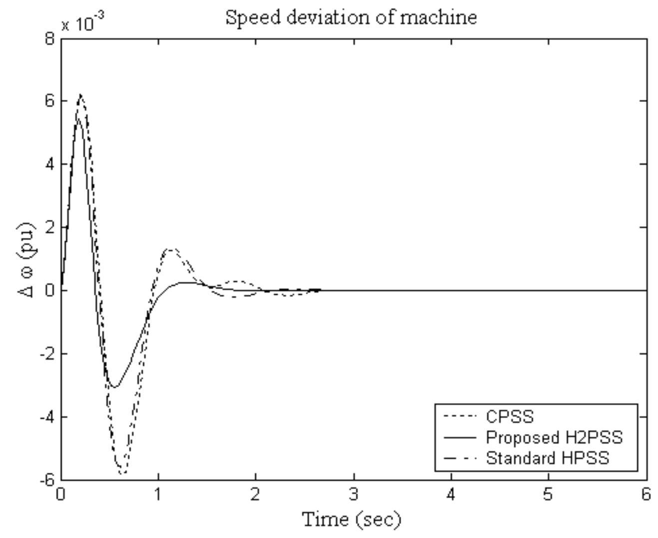

Fig. 7 Responses with 10% Step in V_{ref} for Case 4

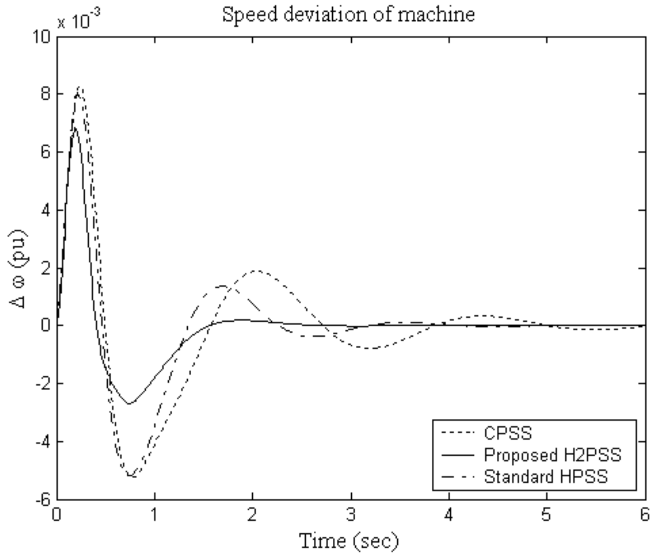

Fig. 8 Responses with Fault Disturbance for Case (a)

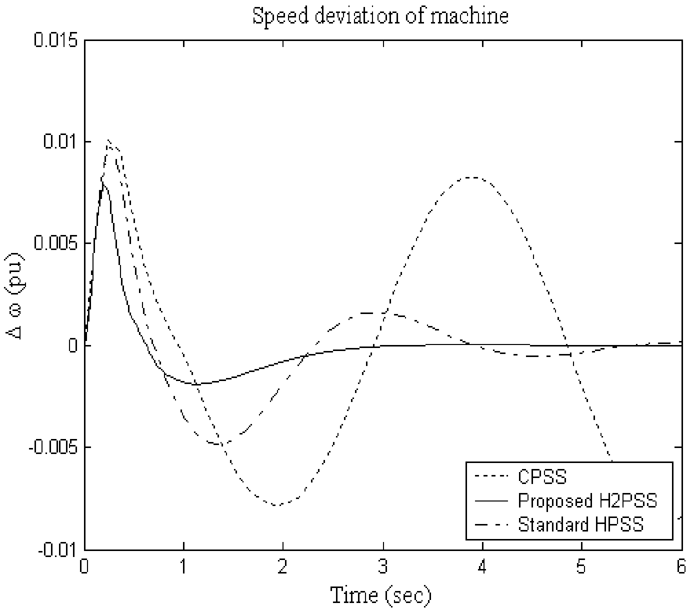

Fig. 9 Responses with Fault Disturbance for Case (b)

V. CONCLUSIONS

This paper has presented the design problem of robust H_2 output feedback controller for damping power system oscillations. Sufficient conditions for the existence of output feedback controllers with norm-bounded parameter uncertainties are given in terms of linear matrix inequalities. In the design, the proposed PSS uses only a rotor speed deviation ($\Delta\omega$) of generator as the feedback input signal. Moreover, the practical realization in power systems can be easily implemented. The performance of the proposed stabilizer on a SMIB power systems are seen to be robust over a wide range of operating conditions. Finally, simulation results show the effectiveness and robustness of the proposed stabilizer to enhance the damping of low frequency oscillations.

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