Cryptanalysis of Knapsack Cipher: A Survey

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Abstract—

*T*he transformation of cipher text to plaintext is knows as cryptanalysis. The topic is the one of the most researched ones since 1932. As the problem is NP Hard, soft computing techniques provide a permanent means to reach an almost a great solution in limited time. It was shown that the knapsack based cryptosystem is highly insecure. The work paves way for the use of Knapsack Cipher in the witching field of cryptanalysis. As per the paper, the results are more efficient and convenient than Genetic Algorithms (GA).

Keywords— Knapsack, Cryptography, Cryptanalysis, Knapsack Cipher, Genetic Algorithm.

I. INTRODUCTION

Cryptography and Cryptanalysis have always enchanted the computing fraternity. In Cryptography, we convert the original information which is known as the plaintext, into the encrypted data, the cipher text. Encryption converts plaintext into cipher text, and decryption converts cipher text back to the plaintext [1]. On the other hand cryptanalysis is method of retrieving a plaintext from a cipher text [2]. The work reviews the available techniques of cryptanalysis of knapsack cipher. Knapsack cipher is deep-seated on Knapsack problem which is an NP complete problem. This spellbinding algorithm is worth exploring, owing to its uses in public key cryptography. A specified set of values m_1 up to m_n and a sum S. It is favourable to find the coefficient C_o up to C_i such that,

$$S = C_o m_o + C_1 m_1 + \ldots\ldots + C_n m_n \text{ [3]} \qquad (1)$$

The values C_i can be 0 or 1 containing that the elements are selected in an outcome sets or not. In the cryptosystem the plaintext is equivalent to the number of items in a pile and cipher text would be a resultant sum.

An extensive literature review has been carried in order to determine various types of ciphers and to legitimatize the use of soft computing techniques for cryptanalysis. Primarily, the study actuate the three most prominent cryptosystem namely Merkle-Hellman, Diffie-Helman, Cryptosystem and Shamir's.

In 1976, Whitefield Diffie and Martin Hellman proposed the first public key exchange system over an insecure channel exploiting symmetric which is known as Diffie-Hellman Key Exchange [4]. In 1977, Rivest, Shamir and Adleman discovered a public key cryptosystem exploiting asymmetric algorithm [5]. Later in 1978, Merkle and Martin Hellman proposed a public key cryptosystem firmly-fixed on a Knapsack and Subset sum problem which in turn is NP complete [6].

The review paper has been organized as follows: Section 2 present background. Section 3 presents the literature review and Section 4 concludes. As stated earlier, the work analysis different techniques used in cryptanalysis of knapsack cipher.

II. BACKGROUND

Knapsack problem is one of the most significant problems in computer science. The problem finds its applications in various fields. In knapsack problem, vector `C' is to be determined, given vector `W' and sum `S' so that the equation is:

$$\sum C_i W_i = S \qquad (2)$$

It is a NP-complete problem since the solution is very difficult to find and once the solution is found, it is easy to verify. The density of knapsack problem is specified as under:

$$D = \frac{n}{\log_2 A}, \text{ where `A' is the maximum amongst } C_{i's} \text{ [7]} \qquad (3)$$

Merkle and Hellman proposed one of the most important analysis of knapsacks. In hatred of its being NP-complete, it can be broken by the technique proposed by Shamir [6]. It is interesting to note that there are plenty versions of knapsack problem, almost all of them were broken except one [8]. There are many attacks which are relying on low density of knapsack system; therefore it becomes necessary to analyse the minimum density which a given knapsack must have, so that it becomes hard to break it.

Shamir proposed a pioneering work to break knapsack. The attack was on Merkle-Hellman knapsack cryptosystem where in the sender chooses a super-increasing sequence in a way that any number in the sequence is greater than the sum of the number preceding it. We need two positive integer `W' and `P' such that $a_i = b_i W \bmod P$ and then choose one of the permutation of so formed array. Here super increasing sequence would act as the private key and a's would act as the public key. In the original system the value of n was 100. The message can be encrypted by grabbing an element from the message and multiply it with the corresponding elements with a_i, the receiver computes $M = C_i W^{-1} \bmod P$. since the value of `P' is greater than sum of the all b_i's, therefore

$$b_j \leq 2^{ni} \leq 1 \qquad (4)$$

The low density attack on knapsack system is relying on the following premise.

If $N \geq \frac{1}{2}\sqrt{n}$, then the system can be broken easily if the density is less than 0.904.

Cryptanalysis of Hwang cryptosystem uses the following method. Here we compute $a_i = b.W$, and vector that requires to be find out is a subset of given D. The methodology uses the verity that the ability of regression using Neural Networks has already been established.

III. LITERATURE REVIEW

In order to place everything in the correct prospect, an immense literature review has been carried out. The review which has been done is to find out the existing techniques toward the topic and to find the gaps therein.

The papers have been selected in accordingly to the guidelines proposed by Kichenham [9]. The summary of the review are presented in the Table below. The table shows the techniques used in the work and the validation/verification used by the author(s):

Table 1 Literature Review

Ref.no.	Technique Used	Result
[10]	Binary Firefly Algorithm	As per the paper, the results are more effective than Genetic Algorithms (GA).
[11]	Binary Particle Swarm Optimization (PSO)	The paper states that for the sample taken, PSO are able to producing better results than GAs.
[12]	Differential Evolution Optimization	Here results have been compared with GA.
[13]	Mathematical computation	In this paper, the cryptosystem has been cracked using mathematic algorithms.
[14]	Two different congestion optimization techniques for cryptanalysis have been used.	As per the paper the samples taken for the verification and validation are adequate to prove the robustness of the proposed system.
[15]	A comprehensive review and history of knapsack-based cryptosystems and their possible cryptanalysis attacks.	The work primarily elucidate the following along with the basics. a) Integer programming approaches with focus on the polyhedral studies of convex hull of the integer set. b) Directions in exploiting integer programming in the cryptanalysis of knapsack ciphers.
[16]	This paper extent the knapsack based two-lock cryptosystem.	It has been shown that the knapsack based two-lock cryptosystem is extremely insecure.
[17]	The work shows a large class of diverse problems have a bi composite structure which makes it possible to solve them by using dissection.	The technique is a mixture of dissection and parallel collision search. The results obtained are fascinating.

IV. CONCLUSION

The survey shows the impact of Neural Network and Genetic algorithm in cryptanalysis of knapsack cipher where the security of original text is highly important. The GA algorithms are not stellar comparing to the other techniques used in cryptanalysis of knapsack chipper. It may be mentioned here that the above work is being extended to include different types of the said problem. The next phase of this project would use Diploid Genetic Algorithms (DGA), the review which has been already carried out. DGAs have been applied successfully to some of the NP Hard problems. The current problem could be reduced to a search problem and is hence a rival of the application of DGA.

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