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Suitability of DOM (Discrete Ordinate Method) to Simulate Radiation Transfer in Participating Medium Interacting with Variable Property Natural Convection Subjected to Collimated/Diffused Irradiation at Boundary

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Abstract—

Most of the published works covering natural convection in enclosures that exist today can be classified into two groups: differentially heated enclosures and enclosures heated from below and cooled from above (Rayleigh Bénard problems). Benchmark solutions related to differentially heated enclosures can be found in many numerical investigations. The present study deals with the interaction phenomenon of combined natural convection with radiation in presence of radiatively active gray medium inside a two dimensional enclosure. The left wall is radiatively semitransparent and is subjected to collimated irradiation. All walls are assumed to be gray diffuse, except the semitransparent wall which is specular in nature. The variation of properties (k, ρ, μ) with temperature has been taken into consideration to make the investigation realistic. The finite volume discretization has been adopted for numerical simulation. The results are depicted through isotherm and streamline patterns. The effects of influencing parameters on Nusselt Number have also been illustrated.

Keywords— ***variable fluid property, natural convection, collimated irradiation, diffused irradiation, gray medium***

I. INTRODUCTION

Natural convection heat transfer in cavities has been a topic for many experimental and numerical studies found in the literature. From practical and industrial point of views, the interest is justified by its many applications , which include heating and cooling of buildings, energy drying processes, solar energy collectors , etc. Studies aimed at clarifying the effect on flow and temperature regime with variations of the shape of the enclosure (aspect ratio), the fluid properties (Prandtl number), the occurrence of transition and turbulence (Rayleigh number) and the correlation of Nu with Ra or Gr. However, thermal radiation always exists and can strongly interact with convection in many situations of engineering interest. The influence of radiation on natural convection is generally stronger than that of forced convection because of the inherent coupling between the temperature and flow fields in natural convection.

As far as natural convection with incompressible fluid is concerned, the work of de-vahl Davis [1] has remained as a benchmark solution till today. H.N.Dixit and V.Babu , [2] done simulation of high Rayleigh number natural convection in a square cavity using the lattice Boltzmann method. Most fluids however have temperature-dependent properties, and under circumstances where large temperature gradients exist across the fluid medium, fluid properties often vary significantly. Under many conditions, ignoring such variations may cause inaccuracies in estimating heat transfer rates. Natural convection problem, involving buoyancy driven flow in a cavity, was first suggested as a suitable validation test case for CFD codes by Jones [3]. The study of fully developed free convection between parallel plates at constant temperature has been initiated by Ostrach [4]. Sinha [5] studied this problem using water as working fluid at low temperatures where the relation between density and temperature is nonlinear. However the other water properties (viscosity and thermal conductivity) have been considered constants. The first exact solutions for free convection in a vertical parallel plate channel with asymmetric heating for a fluid with constant properties was presented by Aung [6].Vajravelu and Sastri [7] reconsidered the problem treated by Sinha using a more accurate relation between water density and temperature, ignoring again the variation of other water properties with temperature. Vajravelu [8], in a subsequent paper, treated the same problem using water and air as working fluids and considering all fluid thermo physical properties (ρ, μ, k, cp)_ as linear functions of temperature. However, the results are valid for room temperatures between 10°C and 25°C. Chenoweth and Paolucci [9] presented exact solutions for a perfect gas using the Sutherland law for viscosity and thermal conductivity and considering the ambient fluid temperature equal to the reference temperature (mean temperature of the two plates).Chenoweth and Paolucci [10] extended the previous work to cases where the ambient fluid temperature is different from the reference temperature. The presented results in both works are valid for the temperature range between 120 K and 480 K. The interaction of natural convection with radiation in presence of participating media finds its numerous practical applications in boiler, furnaces, gas-solid suspensions, fire spreading, building insulation systems and other high temperature applications. An excellent review has been done by Viskanta [11] on interaction of natural convection with radiation from participating media. Chang et.al (12) numerically simulated combined radiation and natural convection in two dimensional enclosures with partition. Larson [13] considered fire spreading processes in buildings using a numerical approach. Desresreyaud and Lauriat [14] numerically investigated natural convection and radiation analysis based on P1 approximation. Tan and Howel [15]), used the exact integral formulation for radiative transport which was subsequently discretized by the product integration method. They also concentrated on the combined mode of transport in the presence of participating medium in a differentially heated square cavity. Mohapatra et.al[16] The differential approximation and discrete ordinate method has been blended together considering their strength to generate the method suitable for solving the radiative transport equation employed in the authors earlier work.

Most earlier works on collimated radiation dealt with solar radiation and other atmospheric or astrophysical application. They are therefore limited to one dimensional cases with uniform irradiation of a planar medium. For this simple case, some exact and approximate solutions have been given by Irviene [17] who used the Henyey-Greenstein phase function, a scattering phase function that approximates the anisotropic scattering behavior of a large number of media. The identical problem for Rayleigh scattering was treated by Kubo [18] without, however, reporting any results. Armaly and El-Baz[19]found some approximate solutions for isotropic scattering in a finite thickness slab using the common kernel approximation. Their application was in the area of solar collector. A similar problem was treated by Houf and Incropera [20] who investigated different approximates techniques for solar irradiation of aqueous media. Smith [21] investigated the case of a uniform strip of collimated radiation incident on a semi-infinite medium. Hunt [22] investigated the effect of a cylindrical collimated beam impinging upon a finite layer. A solution was found for the basic case of Bessel-function varying intensity using Green's function. The first ones to apply this theory to laser radiation appear to be Beckett and co-workers [23], who investigated numerically the effect of a cylindrical beam with Gaussian variation penetrating through a finite layer. They showed how a diagnostic laser beam can be used to deduce radiative properties of an optically thick slab, such as single-scattering albedo, extinction, and absorption coefficient. Collimated irradiation on to a rectangular medium was investigated by Crosbie and Schrenker [24] for isotropic scattering, while Kim and Lee(25) demonstrated the accuracy of the high order discrete-ordinate method by applying it to the same problem with anisotropic scattering. The work of M.F.Modest [26] is significant for understanding the phenomena of Collimated radiation. H.F. Nouanegue et al.[27]has investigated the conduction, convection and radiation in the 2D enclosure for heat flux boundary condition at one side.The thermophysical properties of the fluid are considered from [28].For variation of fluid properties with temperature polynomial approximation is adopted.

However it is realized that the previous researchers have not emphasized the effect of the interaction of natural convection with radiation for collimated radiation in the presence of participating media.

II. MATERIALS AND METHODS

Nomenclature

DOM Discrete ordinate Method
I Radiation intensity [W/m^2]
I_b Black body radiation intensity ($=\sigma T^4/\pi$)[W/m^2]
I_c collimated irradiation intensity
I_d diffuse radiation intensity
k Thermal conductivity [W m^{-1} K^{-1}]
g Acceleration due to gravity [LT^{-1}]
L Characteristic length [m]
q_0 collimated irradiation heat flux[W/m^2]
qr, qc Radiation and convection heat flux[W/m2]
$\hat{S}_c$ Radiative Source term
A Enclosure aspect ratio (L/H)
A1 Linear anisotropic scattering coefficient
$\hat{s}$, $\hat{s}'$ Outgoing and incoming direction
T Absolute temperature [K]
Tc Cold wall temperature(Reference temperature)
u ,v X and Y-components of Fluid velocities (m/s)
NuT Total Nusselt Number
Nur Radiative Nusselt Number
Nuc Convective Nusselt Number
Pr Prandtl number
Ra Rayleigh Number
Cp Specific heat at constant pressure

Greek Symbols:

αa Absorption coefficient [1/m]
σs Scattering coefficient [1/m]β
B Total extinction coefficient ($\alpha_a+\alpha_s$) [1/m]

Ω	Solid angle [Sr]
ω	Single scattering albedo [$\sigma s/\beta$]
σ	Stefan Boltzmann's constant [$5.67X10^{-8}$ W $m^{-2}K^{-4}$]
ε	Wall emissivity
τ	Total optical depth (=βL)
θ	Dimensionless temperature(T/T_c)
Ψ	Dimensionless stream function (ψ/ψ_{max})
α	Thermal diffusivity
ρ	Density

Subscripts:

c	Convection transfer
r	Radiation transfer
w, m	wall, medium

The present work delineates the interaction phenomenon of combined natural convection and collimated irradiation in presence of radiatively active medium inside a two dimensional enclosure. One of the vertical sides of the enclosure is maintained at constant temperature. The enclosure is considered to have insulated horizontal walls. The left wall which is radiatively semitransparent and is exposed for collimated incidence. The enclosure is filled with working fluid and all walls are assumed to be gray, diffuse, except the semitransparent wall which is specular in nature. In addition the flow is steady, laminar and two dimensional. The variation of properties (k, ρ, μ) with temperature has been considered in the work. The fluid under consideration is having (Prandtl = 0.71) and the Rayleigh number is taken in the range10^4 Ra 10^6. The thermo-physical property variation effect has been synthesized. The phenomena in presence of both collimated irradiation and diffused irradiation has been critically analyzed.

III. DISCUSSION /ANALYSIS

Physical model representation of the system under investigation is shown in Fig. 1. The transport equations with varying thermo-physical properties neglecting viscous dissipation are as follows.

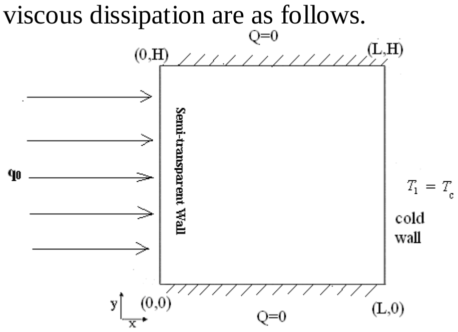


Fig-1 Schematic diagram of the Enclosure

Governing Equations

The equation for continuity is

$$u\frac{\partial(\rho u)}{\partial x} + v\frac{\partial(\rho v)}{\partial y} = 0 - - - \quad \text{--(1)}$$

Equations of momentum in x, y, direction are,

$$u\frac{\partial(\rho u)}{\partial x} + v\frac{\partial(\rho u)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial(\mu u)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial(\mu u)}{\partial y}\right) + F_X \quad \text{-------------------------(2)}$$

$$u\frac{\partial(\rho v)}{\partial x} + v\frac{\partial(\rho v)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\frac{\partial(\mu v)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial(\mu v)}{\partial y}\right) + F_Y \quad \text{-------------------------(3)}$$

$$c_p\left(u\frac{\partial(\rho T)}{\partial x} + v\frac{\partial(\rho T)}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial(kT)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial(kT)}{\partial y}\right) - (\nabla . q_r) \quad \text{----------------------(4)}$$

Energy equation;

where

$$\nabla . q_r = k(4\pi I_b - G)$$

In the constant property analysis, the density variation is adopted as per Bousinesq approximation whereas the variation of thermo-physical properties i.e. ρ (t), μ (t), k (t) is considered in accordance with polynomial approximation

$$\varphi(t) = A_1 + A_2 t + A_3 t^2$$

The equation of transfer for an absorbing, emitting and anisotropically scattering gray medium at any location and direction(r, $\hat{S}$) is given by

$$\hat{s}\nabla I(r,\hat{s}) = Ib(r) - \beta I(r,\hat{s}) + \frac{\sigma_s}{4\pi}\int_{4\pi} I(r,\hat{s})\Phi(\hat{s},\hat{s}')d\Omega \quad \text{--(5)}$$

The intensity at any point, can be considered to be sum of diffuse part and collimated part.

$$I(r,\hat{s}) = I_c(r,\hat{s}) + I_d(r,\hat{s}) \quad \text{--(6)}$$

The collimated part of irradiation is

$$\hat{s}\nabla_\tau I_c(r,\hat{s}) = -\beta I_c(r,\hat{s}) \quad \text{---(7)}$$

The equation of transfer for the non collimated radiation is

$$\frac{1}{\beta}\,\hat{s}.\nabla I_d(r,\hat{s}) = \hat{s}.\nabla_\tau I_d(r,\hat{s})$$

$$= -I_d(r,\hat{s}) + \frac{\omega}{4\pi}\int_{4\pi} I_d(r,\hat{s})\Phi(\hat{s},\hat{s}')d\Omega + (1-\omega)I_b(r) + \omega S_c(r,\hat{s}) \text{ -----------------------------------(8)}$$

Boundary conditions for equation (7) and equation(8) for the gray medium are

$$I_c(r_w,\hat{s}) = [1-\rho(r_w)\delta[\hat{s}-\hat{s}_c(r_w)] \quad \text{- ------------------------------------(9)}$$

$$I_d(r_w,\hat{s}) = \varepsilon I_{bw}(r_w) - \frac{\rho_{r_w}}{\pi}[H_c(r_w) + \int_{\hat{n}.\hat{s}<0} I_d(r_w,\hat{s}')\,|\hat{n}.\hat{s}'|\,d\Omega'] \quad \text{---------------------------(10)}$$

Boundary Conditions

For Momentum equations;

u(x,0) =0 , v(x,0) = 0 , u(x, L) =0, v(x, L) = 0.

u(0, y)=0 , v(0, y) = 0, u(L, y)=0 , v(L, y) = 0

For energy equation:

$q_0(0,y)$=1353watt/m^2, t(x, L) = t_c = 300K,

$q_c+q_r=0$ at $0<X<L$, for y=0,H

In the present study, the total average Nusselt number is calculated as

$$Nu_T = Nu_r + Nu_c = \frac{q_c+q_r}{k\Delta T/L} \quad \text{--(11)}$$

Based on control volume method, 2-D analysis of fluid flow and heat transfer for the square enclosure is done on fluent software. The non-uniform meshing has been adopted. Mesh in the core is coarse compared to the size of mesh near the walls. All computations are conducted with 68X68 control volumes considering grid independent test. Solution of N-S equation as per Simple algorithm is opted and PRESTO is chosen for solving pressure equation. Second order upwind scheme for solving momentum equation is used. Convergence criterion is set as 10^{-7} for continuity, x-momentum and y-momentum equations and 10^{-6} for energy equation. Prandtl number is kept fixed and is taken as 0.71 for all computations. Results are analyzed from obtained isotherms and streamline pattern and calculated Nusselt number.

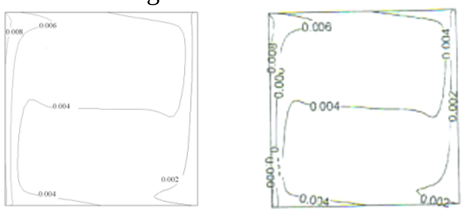
Numerical results have been obtained varying different influencing parameters. Before analyzing the results obtained for combined natural convection and radiation, with one wall exposed to collimated irradiation, the FLUENT CFD software solutions for few relevant cases are validated against benchmark solutions. The effect of collimated radiation in presence of radiatively participating medium has been investigated assuming properties to be constant. Thereafter, the effect of variation in properties has also been examined on combined radiation and convection. The sole effect of collimated irradiation has been addressed by comparing with phenomenon in presence of diffused irradiation. The properties of the fluid are considered to be constant. The sole effect of any parameter is realized by keeping other parameters fixed. The temperature and flow field are presented in order to bring clarity in understanding of complex conjugate heat transfer phenomena. From the governing equations it is apparent that Rayleigh number, emissivity, scattering albedo, optical thickness are the influencing parameters .The properties of the working fluid at a temperature of 300K are taken as the reference values. Rayleigh number is varied within 10^4 to 10^6 in order to study laminar flow regime. All surfaces of the enclosure are considered to be gray and of the same emissivity. The results obtained for natural convection in a differentially heated enclosure using FLUENT software has been validated against the published works of De Vahl Davis [1] and Dixit and Babu [2] and has been presented in the table 1.

Table No.1 The Effect of (Ra) on (Nu) in pure natural convection within differentially heated square enclosure.

Rayleigh Number	Nusselt No DeVahlDav-is [1]	Nusselt No Present code	Nusselt No Dixit and Babu[2]
Ra=10^3	1.116	1.100	1.128
Ra=10^4	2.242	2.227	2.286

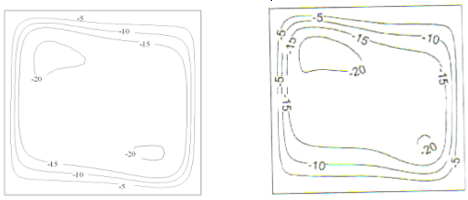
$Ra=10^5$	4.531	4.486	4.563
$Ra=10^6$	9.035	8.701	8.800

The software has also been used to obtain solution for combined Natural convection and radiation within differentially heated square enclosure with heat flux boundary condition. The work has been validated against Nouanegue et.al [27] and presented in fig-2.The validation is in excellent agreement.



(a) present work (b) Nouanegue et.al(27)

isotherms for A=1, $\omega=0.0$, $\varphi=90^0$, $\varepsilon=1.0$, $Ra=10^8$



(c) Present work (d) Nouanegue et.al [27]

Streamlines for A=1, $\omega=0.0$, $\varphi=90^0$, $\varepsilon=1.0$, $Ra=10^8$

Fig- 2 Isotherms and Streamline pattern for combined natural convection and radiation with heat flux boundary condition(left wall) in square enclosure when $\omega=0.0$, $\tau=1$, $\varepsilon=1.0$, $Ra=10^8$

To examine the solving capability of the software for phenomenon with collimated irradiation, the computational test domain [Fig-3] in agreement with the work of Kim and Lee(25) has been considered. The solution obtained for isotropic scattering medium and the variation of transmitted flux along the bottom wall has been validated against the work of Kim and Lee (25) for transmitted fluxes for collimated incidence

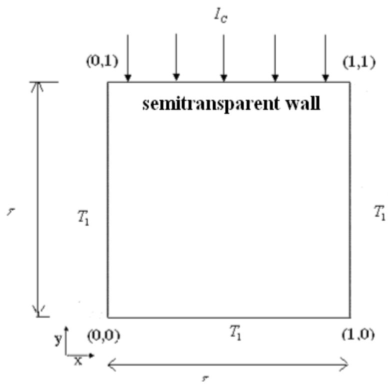


Fig.3 Computational Test Domain for the validation purposepurpose [i . e, in accordance with Kim & Lee(25)]

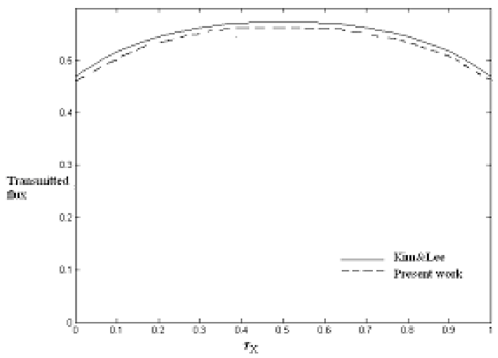


Fig.4 Variation of Transmitted Fluxes with τ along the bottom wall , for isotropic scattering media ($\varepsilon=1,\omega=1,\tau_X=\tau_Y=1$)

Combined effect of varying fluid properties:

In this section the combined effect of varying properties i.e., thermal conductivity (k), density (ρ) and viscosity (μ) has been explained through variation in Nusselt number. The combined effect of thermal conductivity and density gives the maximum variation to the Nusselt number. At low Rayleigh number effect of varying properties dominates over effect of constant properties and the Nusselt number increase by 37.84 %(by comparing the values in Table-2). However at higher Rayleigh number also, effect of varying properties dominates over effect of constant properties and the Nusselt number increased by 38.97 % .

Table 2 Effect of Rayleigh no. on Nusselt no.for constant and variable properties of the medium, when τ =1, ε =0.5, ω =0.5,

Variable parameter	Nu_T	Nu_T
	Constant property	Variable property(k, ρ, μ)
Ra = 10^4	5.62503	7.753622
Ra = 10^5	8.78046	12.329848
Ra= 10^6	14.517314	20.17474

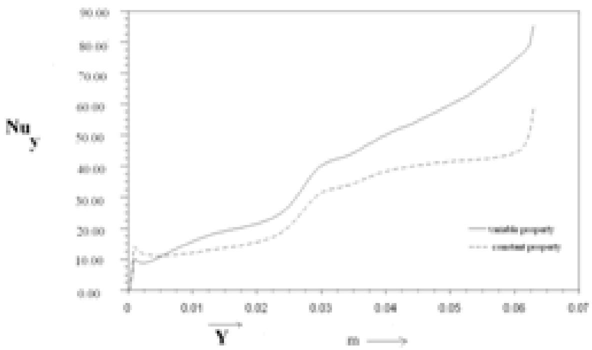
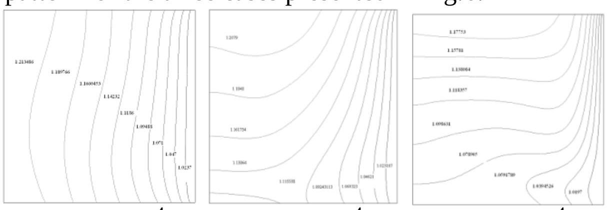


Fig.5 comparison of Nusselt number (Nu_T)at cold wall for constant and variable (k, ρ, μ) fluid property at *Ra*= 10^6 , when τ =1, ε =0.5, ω =0.5,

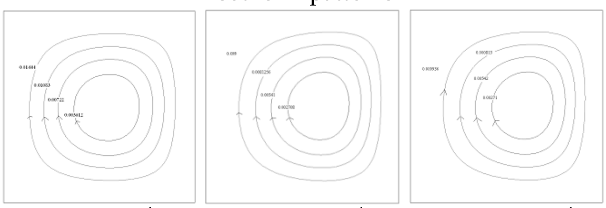
Comparison of diffused and collimated irradiation

In this section, isotherm and streamline patterns obtained [fig.6] are compared when the left wall of the enclosure is subjected to either the collimated irradiation or diffuse irradiation. There is a rise in Nusselt number for variable property case compared to other two cases. For comparison purpose, the fluid properties are considered to be constant for the case with diffused irradiation boundary condition.The maximum velocity of fluid within the enclosure is more for diffused irradiation compared to collimated radiation as shown in Fig.8. The isotherms are closely packed near the cold wall for the case of variable property with collimated radiation. It can be observed that variable property effects produce a flow pattern in which the temperature and velocity profile extend further in to the cold wall from the semitransparent wall. The effect of which is found to be, increased convective heat transfer at the cold wall. This can be realized through the isotherm pattern and streamlines pattern for the three cases presented in Fig.6.



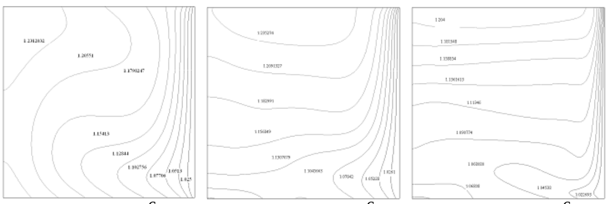
(a) *Ra*= 10^4 (b) *Ra*=10^4 (c) *Ra*=10^4

Isotherm patterns



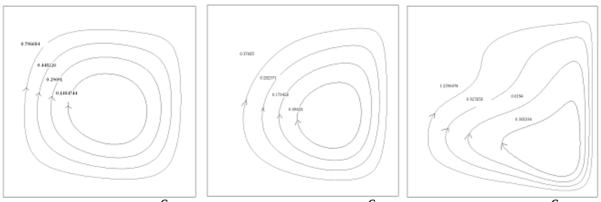
(a) *Ra*=10^4 (b) *Ra*= 10^4 (c)*Ra*=10^4

Streamline patterns



(a) *Ra*= 10^6 (b) *Ra*=10^6 (c) *Ra*=10^6

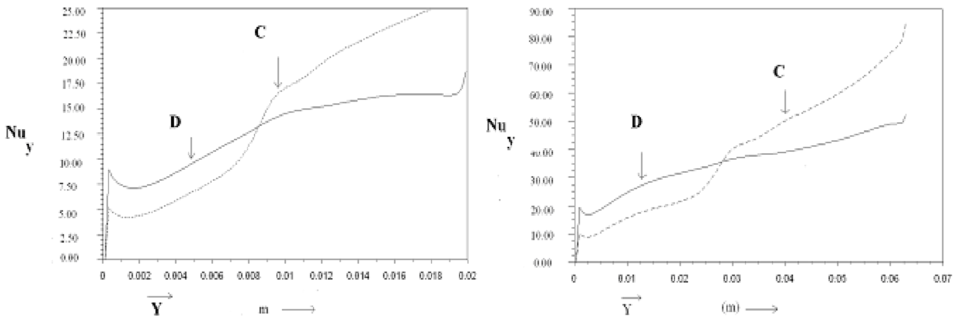
Isotherm patterns



(a) $Ra=10^6$ (b) $Ra= 10^6$ (c)$Ra= 10^6$

Streamline patterns

Fig-6 Isotherms and Streamline pattern for (a) Constant property diffuse irradiation, (b) Constant property collimated irradiation (c) variable property collimated irradiation, at $Ra=10^4$, $Ra=10^5$, $Ra=10^6$



(a) (b)

Fig.7 comparison of Nusselt number (Nu_T) at cold wall,for(C)collimated and(D)diffuse irradiation with variable property,at(a)Ra=10^4,(b)Ra=10^6whenε=0.5,ω= 0.5, τ =1

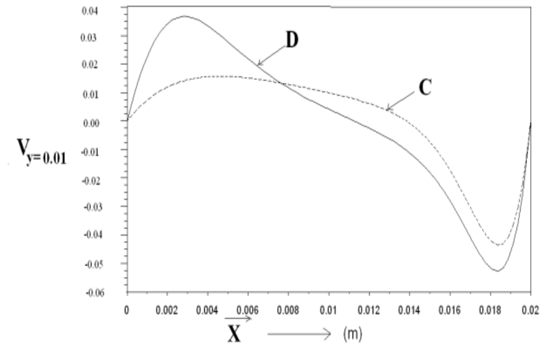


Fig.8 Midplane velocity distribution for variable property(C)collimated and (D)diffused irradiation at Ra=10^4,when ε=0.5, ω=0.5, τ =1

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IV. CONCLUSIONS

A two dimensional numerical study of coupled natural convection with radiation transfer in an enclosed cavity heated by the collimated irradiation through the semitransparent wall has been carried out. The following conclusions are made out of the investigation.

The collimated irradiation at left semitransparent wall makes the left wall subjected to negligible convective heat flux whereas right cold wall experiences both the radiative heat flux and convective heat flux in a competitive manner. The medium mobility is almost negligible towards the left wall and top portion of the cavity.

The phenomenon with collimated irradiation results higher Nusselt (i.e. about 10% more) at cold wall compared to the same in presence of diffuse irradiation. However, the left wall experiences fluid motion in its near vicinity resulting convective heat flux. The variation of Nu_y along the cold wall for collimated irradiation is found to be more compared to the same for diffuse irradiation.

The effect of varying properties (caused due to temperature variation within the domain) is noticeable when compared with results obtained assuming constant properties and the variation in the Nu value exceeds 35%. There is also significant deviation in both the flow features and temperature distribution. It is suggested to consider the effect of variable properties to make the modeling pragmatic.

The Discrete ordinate method is found to be accurate, in solving problem related to collimated irradiation. It is suggested to consider DOM(Discrete ordinate method)to deal with this type of problem..

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REFERENCES

[1] G. De Vahl Davis, Natural Convection of air in a square cavity: a benchmark Numerical solution, *Int. J. Numeri. Meth. Fluids* 3,249-264, 1983.

[2] H. N. Dixit and V. Babu, simulation of high Raleigh number natural convection in a square cavity using the lattice Boltzman method. Int. journal of heat mass transfer 49,pp. 727-739,2006.

[3] I. P. Jones,.A Comparison Problem for Numerical Methods in Fluid Dynamics, The 'Double-Glazing' Problem', in *Numerical Methods in Thermal Problems, Proceedings ofthe First International Conference*, 338-348, 1979.

[4] S.Ostrach, "Laminar Natural Convection Flow and Heat Transfer of Fluids With and Without Heat Sources in Channels with Constant Wall Temperature," *Report No. NACA TN 2863*,1952.

[5] P.C.Sinha, "Fully Developed Laminar Free Convection Flow between vertical Parallel Plates," *Chem. Eng. Sci., 24*, pp. 33–38,1969.

[6] W.Aung, "Fully Developed Laminar Free Convection between Vertical Plates heated Asymmetrically," Int. J. Heat Mass Transfer, 15, pp. 1577–1580,1972.

[7] K.Vajravelu and K.Sastri, "Fully Developed Laminar Free Convection Flow between Two Parallel Vertical Walls," Int. J. Heat Mass Transfer, 20, pp. 655–660, 1977.

[8] K. Vajravelu, , "The Effect of Variable Fluid Properties on the Free Convective Flow of Air/Water Confined Between Two Parallel Vertical Walls," Acta Mech., 31, pp. 199– 211,1979.

[9] D.R.Chenoweth and S.Paolucci, "Gas Flow in Vertical Slots With Large Horizontal Temperature Differences," Phys. Fluids, 28, pp. 2365–2374.1985.

[10] D.RChenoweth, and S.Paolucci, , "Gas Flow in Open Vertical Slots With Large Horizontal Temperature Differences and Arbitrary External Temperature," Phys. Fluids, 29, pp. 3187–3198,1986.

[11] B. W. Webb and R. Viskanta , Radiation- induced buoyancy driven flow in rectangular enclosures: experiments and analysis, ASME J . Heat transfer 109,427-433,1987.

[12] L.C. Chang, K. T. Yang and J. R. Liyod , Radiation –Natural convection interaction in two-dimensional complex enclosures, ASME J. Heat Transfer 105,89-95,1983.

[13] D.W.Larson Enclosed radiation and turbulent Natural convection induced by a fire .In Numerical Methods in Heat Transfer(EditedbyR.W.Lewis, K.Morganand,O.C. Zienkiewicz)

[14] G. Desreyaud and g .Lauriat Natural Convection of a radiating fluid in a vertical layer .ASME J. Heat Transfer 107,710-712,1985.

[15] Zhiqiang Tan and John R. Howell, Combined radiation and natural convection in a two dimensional Participating square medium, Int. Journal of heat mass transfer, vol34.no.3.pp785-793,1991.

[16] S.K Mahapatra, B.K.Dandapat and A Sarkar, "*Analysis of combined conduction and radiation heat transfer in presence of participating medium by the development of hybrid method.* " journal of Quantitative Spectroscopy and Radiative Transfer, vol-102,no-2: 277-292,2006,

[17] W. M Irvine" Multiple scattering by large particles II. Optically Thick Layers," The Astrophysical journal, vol.152:.823-834. 1968.

[18] S.Kubo,"Effects of Anisotropic Scattering on steady one-dimensional Radiative Heat Transfer through an Absorbing-emitting medium. "J. phys. soc. Japan, vol.41, no.3:.894-898, 1976.

[19] B. F Armally and H. S. El-Baz: "Radiative Transfer Through an Isotropic ally Scattering Finite Medium: Approximate Solution, " AIAA Journal, vol. 15 no. 8: .1180-1185,1976.

[20] Houf , W. G., and F. P. Incropera: "An Assessment of Techniques for Predicting Radiation Transfer in Aqueous Media." Journal of Quantitative Spectroscopy and Radiative Transfer, vol. 23 :101-115. 1980.

[21] Smith, M. G.:"The Transport Equation with Plane Symmetry and Isotropic Scattering, *" Proc. Camb. Phil. Soc.,* vol.60: 909,1964.

[22] Hunt. G.E.: "The Transport Equation of Radiative Transfer with Axial Symmetry," SIAM J. Appl. Math., vol 16 no 1: 228-237,1968.

[23] Beckett, P.J. Foster, V. Huston, and R. L Moss "Radiative Transfer for a Cylindrical Beam Scattered Isotropic ally" Journal of Quantitative Spectroscopy and Radiative Transfer. Vol. 14: 1115-1125, 1974.

[24] A. L Crosbie and R. G. Schrenker"Multiple scattering in a two dimensional Rectangular Medium exposed to collimated Radiation, "journal of Quantitative Spectroscopy and Radiative Transfer, vol.33,no. 2:101-125.1985.

[25] T.K. Kim and H.S. Lee Radiative Transfer in two dimensional Anisotropic Scattering media with collimated Incidence," journal of Quantitative Spectroscopy and Radiative Transfer, vol.42:225 –238, 1989.

[26] M .F Modest Radiative Heat Transfer .McGraw-Hill, New York, 1993.

[27] H.F.Nouanegue, Muftuoglu,E.Bilgen. Heat transfer by natural convection, conduction and radiation in an inclined square enclosure bounded with a solidwall, Int.J. of Thermal Sciences48(2009)871-880, 2008

[28] C.P.Arora, "Refrigeration and air conditioning", second edition,PP-931.2000.