

# Nonlinear Mixed Integer Stochastic Programming Model for Sustainable Seafood Production and Distribution Planning Problem

Tutiarny Naibaho

Graduate School of Mathematics,  
University of Sumatera Utara/Quality University  
Medan, Indonesia

Herman Mawengkang

Department of Mathematics,  
University of Sumatera Utara  
Medan, Indonesia

## Abstract—

**I**n the increasing competition of global supply chain system most companies are forced to be more efficient, therefore they need to integrate production and distribution planning. This paper considers a multi-product fish production-distribution planning which produces simultaneously multi fish products from several classes of raw resources. The sustainable production planning problem aims to meet customer demand subject to environmental restrictions. This paper develops a model of the management which performs processing fish into several seafood products and distributes them to several distribution centers. Therefore the model combines simultaneously to optimize production and routing plan. Direct search is used for solving the mixed integer nonlinear programming model.

**Keywords—** Integer Programming, Production planning, routing problem, direct search

## I. INTRODUCTION

Nowadays, most fishery companies, due to the global changes, are organized into networks of production and distribution sites that procure raw fish materials, process them into seafood product, and distribute the finished products to customers. The goal is to deliver the right product to the right place at the right time for the right price. These production-distribution networks are what we call “supply chains.” In supply chain planning, the managers are necessarily to coordinate all material supply, production and distribution. This plan has a clear meaning that the managers need to consider raw materials, storage, and transportation costs while considering supplier, production, storage, and transport capacities.

The production planning includes deciding how many times each process should be run and which quantity of each class of raw materials should be consumed by each process in each period in the planning horizon. The objective is to minimize the costs of production process, products inventory/backorder raw material consumption and workers, considering fulfillment of products demands, machine capacities, and raw material inventory. The other important thing in production planning is to determine lot sizes, that is, calculating the quantity to be produced for each item at each time. A recent result and survey regarding to lot size in production planning can be found in [1]. Reference [2] describe about sustainable optimization of lot sizing.

Afterward the finished seafood products need to be distributed to the customers. In supply chain management the distribution problem comes under the heading of routing problem. This type of model distributes finished products from a central distribution centre, called depot, to multiple geographically dispersed customers using a fleet of capacitated vehicles. The objective of the routing problem is to decide how many products should be distributed to the customers, the assignment of vehicles, and what route should be used [3]

In this paper we consider production and distribution planning problem which arises in marine fisheries industry in Indonesia. Marine fisheries play an important role in the economic development of Indonesia. This industry could also provide employment to people who live at coastal areas, to increase the financial gain of local government, and to conserve sustainability. Fisheries industrial sector can be classified into three different parts, i.e., open sea fishing, fish cultivation and processed fish. This paper is focusing on the latter sector.

In Indonesia, generally the seafood industry can be found at the coastal area. There are a lot of varieties of fish processed can be produced, such as smoked fish, salted fish, crunchy bashed of fish, fish bowl, terrain (fish preserved), etc. The management of this kind of industry is still dominated by the local small traditional business, using conventional management strategy. Consequently, they do not have enough knowledge and experience in managing the supply chain system, in such a way, that they could give benefit for the Government and the people surrounds.

The integrated production and distribution planning system (IPDS) has received a lot of attention among researchers since mid-1980s. An interesting comprehensive review of IPDS can be found in [3], [4], and [5]. Reference [6] modeled the IPDS in a single optimization model that jointly optimize decision variables of different production and distribution function. Reference [7] addressed an integrated lot sizing and inventory routing problem modeled as a mixed integer program with the objective of maximizing the net. They proposed a two-step procedure that first estimated daily delivery quantities and then solved a vehicle routing problem for each day of the planning horizon. Reference [8] addressed an integrated production and distribution planning model for perishable products. They formulated the problem as a large scale integer programming model. Recently, Reference [9] formulated as an optimization model for the integrated of inventory – distribution routing problem in agriculture supply chain.

In every seafood production process, inputs are used to create a processed product or commodity. Inevitably, some inputs are not fully used and are released into the environment in forms that may be considered pollutants or waste. Whenever the level of waste exceeds the environment's ability to absorb and process these discharges, environmental risks develop. Regarding to the importance of the sustainable production planning of fish processed creates a stimulus for the research in the mathematical programming model. Reference [10] proposed a multi objective model for solving sustainable production planning, which take into account environmental constraints. This is a general production model. Reference [11] used an optimization model approach to solving production planning of crude palm oil in order to reduce freshwater usage. The production of seafood particularly fish is a complex problem, due to the influence of processing variables and environmental impacts. An interesting paper in describing sustainability in the integrated production, manufacturing and logistic can be found in [12].

This paper concerns with the modeling the integrated production and distribution planning for seafood products. In the model we impose requirement regarding to meet the sustainability criteria, i.e., economic, social, and environment. We propose a mixed integer nonlinear programming (MINLP) model to formulate the problem as the demand for the seafood is assumed stochastic. A direct neighborhood search approach is developed for solving the model.

This article is organized as follows: section two briefly reviews the structure of the nonlinear integer programming model. Section three presents the problem background. The MINLP model of the problem can be found in Section four. In Section five we present the algorithm. We conclude this paper in Section six.

## II. STRUCTURE OF THE PROBLEM

The general form of the nonlinear integer programming (NLIP) problem to be used in this paper is given under label (1), and we assume that a bounded feasible solution exists to the problem.

$$\begin{aligned} & \underset{x \in R^n}{\text{minimize}} && f^0(\mathbf{x}^N) + \mathbf{c}^T \mathbf{x}^L && (1) \\ & \text{Subject to} && \mathbf{f}(\mathbf{x}^N) + \mathbf{A}_1 \mathbf{x}^L = \mathbf{b}_1 && (m_1 \text{ rows}) \\ & && \mathbf{A}_2 \mathbf{x}^N + \mathbf{A}_3 \mathbf{x}^L = \mathbf{b}_2 && (m_2 \text{ rows}) \\ & && l \leq \mathbf{x} \leq \mathbf{u} && (m = m_1 + m_2) \\ & && x_j \text{ integer, } j \in J_1 \end{aligned}$$

There are  $n$  variables and  $m$  constraints,  $m < n$ .

Some (assumed small) proportion of the variables  $x$  are assumed to be nonlinear in either the objective function and/or the constraints, and some (also assumed small) proportion of the variables are required to be integer-valued. We refer to a variable as *nonlinear* if it appears nonlinearly in the problem formulation in either the objective function or the constraints.

The solution procedure involves a sequence of *major iterations*, in which the first-order Taylor series approximation terms replace the nonlinear constraint functions to form a set of linear constraints, and the higher order terms are adjoined to the objective function with Lagrange multiplier estimates.

The set of linear constraints (excluding bounds) is then written in the form:

$$\mathbf{A}\mathbf{x} = [\mathbf{B} \quad \mathbf{S} \quad \mathbf{N}] \begin{bmatrix} \mathbf{x}_B \\ \mathbf{x}_S \\ \mathbf{x}_N \end{bmatrix} = \mathbf{b} \quad (2)$$

$\mathbf{B}$  is  $m \times m$  and non-singular,  $\mathbf{x}_N$  are "non-basic" variables which are held at one or other of their bounds.  $\mathbf{x}_B$  and  $\mathbf{x}_S$  are referred to as basic and superbasic variables respectively, and in order to maintain feasibility during the next step they must satisfy the equation

$$\mathbf{B}\Delta\mathbf{x}_B + \mathbf{S}\Delta\mathbf{x}_S = 0 \quad (3)$$

or, since the basis is non-singular, we may write

$$\Delta\mathbf{x}_B = -\mathbf{B}^{-1} \mathbf{S}\Delta\mathbf{x}_S \quad (4)$$

Apart from the choice involved in deciding which nonbasic to slide up or down toward its other bound in order to improve the objective function, we have freedom to alter the superbasics. A step to the interior of the feasible set is possible since the superbasics need not be at a bound and are normally between bounds.

Because of equation (4), the superbasics are seen as the driving force, since the step  $\Delta\mathbf{x}_S$  determines the whole step  $\Delta\mathbf{x}$ . The key to the success of the algorithm is the assumption that the dimension of  $\mathbf{x}_S$  remains small. This can be assured if the proportion of nonlinear variables is small, but also in many instances in practice even when all the variables are nonlinear. Similar assumptions will be made about the structure of nonlinear integer programs. It will be assumed that the proportion of integer variables in the problem is small.

## III. PROBLEM BACKGROUND

Fish and its processed products are the most affordable source of animal protein in the diet of most people. In Indonesia, most of the fish processed industries are found at the coastal area. In these industries fish are processed traditionally. There are eight kinds of fish product to be produced by the community, namely, dried fish, salted fish, BBQ fish, pindang fish, smoked fish, fish preserved, pressed fish, and fish bowl.

The fish processed industry under investigation is located at the eastern coastal area of North Sumatra province of Indonesia. The industry run by the community of that area has to make a production plan for these eight fish processed

products to fulfill market demand over each period of time  $t, t = 1, \dots, T$ . In this case each period equals to three months. Therefore there will be four periods in a year. A limited amount of fish raw material can be temporarily stored in the production facility with unit holding cost of  $\rho_{jt}$  ( $t = 1, \dots, T$ ).

In order to distribute the seafood product, there is a set of  $n$  distribution centers built by the community located around the production facility, such that the products are to be delivered to them. Each distribution center  $i$  ( $i = 1, 2, \dots, n$ ) has a nonnegative and deterministic demand  $D_{jt}^i$  of  $j$  kind fish product in planning period  $t$  of the planning horizon. A limited amount of inventory can be stored in distribution center  $i$  with unit holding cost of  $\rho_{jt}^i$ .

For this is a sustainable production and distribution planning problem, the model formulated includes a variable about waste of fish, and number of workers from local people.

Now for the part of routing problem. As in the conventional vehicle routing problem (VRP), there is a fleet of capacitated homogeneous vehicles which is responsible to deliver the seafood products from the facility to the distribution centers. The fleet is hired by the management. The incurred costs are calculated based of number of trips vehicles make. There are some other requirements imposed in the model, i.e., each vehicle can make at most one delivery per planning period and each distribution center can be visited at most once per planning period.

In this production planning problem we will decide:

- The quantity of each fish processed product scheduled to be produced in each period
- The additional resource to be used
- The number of regular additional and laying-off workers in each period
- The quantity of waste fish,
- The distribution centres that must be visited, and
- The quantities of seafood product to be delivered to the assigned centre.

The objective is to minimize the total costs.

Model parameter and decision variables used throughout this paper are defined as follows.

#### Sets

- $T$  = number of periods
- $N$  = set of products
- $M$  = set of resources
- $L$  = set of distribution centers
- $V$  = set of vehicles

#### Variables

- $X_{jt}$ : Quantity of product  $j \in N$  in period  $t \in T$  (ton)
- $z_{jvt}^l$ : Quantity of product  $j \in N$  delivered to distribution center  $l \in L$  in period  $t \in T$  by vehicle  $v \in V$  (ton)
- $u_{it}$ : Additional amount of resource  $i \in M$  to purchase in  $t \in T$  (unit)
- $k_t$ : Number of workers required in period  $t \in T$  (man-period)
- $k_t^-$ : Number of workers laid-off in period  $t \in T$  (man-period)
- $k_t^+$ : Number of additional workers in period  $t \in T$  (man-period)
- $I_{jt}^0$ : Inventory level of product  $j \in N$  at the production facility in period  $t \in T$
- $I_{jt}^l$ : Quantity of product  $j \in N$  to be stored in distribution center  $l \in L$  period  $t \in T$  (units)
- $B_{jt}$ : Under-fulfillment of product  $j \in N$  in period  $t \in T$  (units)

#### Binary variables

- $C_{jvt}$ : If distribution of product  $j \in N$  is served by vehicle  $v \in V$  in period  $t \in T$
- $H_{vt}$ : If vehicle  $v \in V$  is used to serve distribution centers in period  $t \in T$

#### Parameters

- $\alpha, \beta, \gamma, \delta, \mu, \rho, \lambda, \eta, \tau$  are all costs
- $D_{jt}$ : Demand for product  $j \in N$  in period  $t \in T$  (units)
- $U_{jt}$ : Upper bound on  $u_{jt}$
- $r_{ij}$ : Amount of resource  $i \in M$  needed to produce one unit of product  $j \in N$

- $f_{it}$ : Amount of resource  $i \in M$  available at time  $t \in T$  (units)
- $a_j$ : Number of worker needed to produce one unit of product  $j \in N$
- $w_{jt}^p$ : Waste of fish product  $j \in N$  in period  $t \in T$  (units)
- $UI_{jt}^0$ : Upper bound on inventory of product  $j \in N$  at the production facility in period  $t \in T$  (units)
- $UI_{jt}^l$ : Upper bound on inventory of product  $j \in N$  at the distribution center  $l \in L$  in period  $t \in T$  (units)
- $g$ : Vehicle capacity

#### IV. THE MODEL

Minimize

$$\begin{aligned} & \sum_{j \in N} \sum_{t \in T} \alpha_{jt} x_{jt} + \sum_{i \in M} \sum_{t \in T} \beta_{it} u_{it} + \sum_{t \in T} \mu_t k_t + \sum_{t \in T} \gamma_t k_t^- + \sum_{t \in T} \delta_t k_t^+ \\ & + \sum_{j \in N} \sum_{t \in T} \eta_{jt} w_{jt}^p + \sum_{j \in N} \sum_{t \in T} \rho_{jt}^0 I_{jt}^0 + \sum_{j \in N} \sum_{t \in T} \lambda_{jt} B_{jt} \\ & + \sum_{v \in V} \sum_{t \in T} \tau_{vt} H_{vt} + \sum_{j \in N} \sum_{t \in T} \sum_{l \in L} I_{jt}^l \end{aligned} \quad (5)$$

Subject to

$$\sum_{j \in N} r_{jt} x_{jt} \leq f_{it} + u_{it} \quad \forall i \in M, \forall t \in T \quad (6)$$

$$u_{it} \leq U_{it} \quad \forall i \in M, \forall t \in T \quad (7)$$

$$\sum_{j \in N} a_j x_{jt} \leq k_t \quad \forall t \in T \quad (8)$$

$$0.10x_{jt} \leq w_{jt}^p \leq 0.20x_{jt}, \quad \forall j \in N, \forall t \in T \quad (9)$$

$$\sum_{j \in N} \sum_{t \in T} w_{jt}^p \leq C^p \quad (10)$$

$$I_{jt}^l = I_{jt-1}^l + \sum_{v \in V} Z_{jvt}^l - D_{jt} \quad \forall j \in N, t \in T \quad (11)$$

$$I_{jt}^0 \leq UI_{jt}^0 \quad \forall j \in N, t \in T \quad (12)$$

$$I_{jt}^l \leq UI_{jt}^l \quad \forall j \in N, l \in L, t \in T \quad (13)$$

$$k_t = k_{t-1} + k_t^+ - k_t^- \quad t = 2, \dots, T \quad (14)$$

$$x_{jt} + B_{jt-1} + I_{jt}^0 - B_{jt} = D_{jt} \quad \forall j \in N, \forall t \in T \quad (15)$$

$$Z_{jvt}^l \leq g \cdot C_{jvt} \quad \forall j \in N, v \in V, l \in L, t \in T \quad (16)$$

$$\sum_{j \in N} Z_{jvt}^l \leq g \quad \forall v \in V, l \in L, t \in T \quad (17)$$

$$\sum_{j \in N} C_{jvt} \leq 1 \quad \forall v \in V, t \in T \quad (18)$$

$$\sum_{v \in V} C_{jvt} \leq 1 \quad \forall j \in N, t \in T \quad (19)$$

$$\sum_{j \in N} C_{jvt} \leq f \cdot H_{vt} \quad \forall v \in V, t \in T \quad (20)$$

$$x_{jt}, u_{it}, k_t, k_t^-, k_t^+, Z_{jvt}^l, I_{jt}^0, I_{jt}^l, B_{jt} \geq 0 \quad \forall j \in N, \forall i \in M, \forall t \in T, \forall l \in L, \forall v \in V \quad (21)$$

$$C_{jvt}, H_{vt} \in \{0, 1\} \quad \forall j \in N, v \in V, t \in T \quad (22)$$

All of these decisions are formulated in expression (5) of the model as an objective function. Constraint (6) expresses that the amount of resource  $i \in M$  needed to produce product  $j \in N$  at least should have the same amount of resources available at time  $t \in T$  together with the additional resource needed. However, the additional resource needs to have an upper bound (expression (7)). In constraint (8), we have the number of workers needed to produce one unit product  $j \in N$ . The amount of fish waste should be between 10%-20% can be found in (9). Constraint (10) expresses that the process of fish waste should be within the capacity  $C^p$ . Constraints (11) to (13) describe about the inventories at the production facility and distribution center. Constraint (14) ensures that the available workers in any period equal the number of worker from the previous period plus any change in the number of worker level during the current period. The change in the number of worker level may be due to either adding extra workers or laying-off redundant workers. Constraint (15)

determines either the quantity of product to be stored in inventory or to purchase from outside to fulfill the shortfall in meeting market demand. Constraints (16) and (17) express the capacity for delivering to the distribution centers. Constraint (18) is modeled in order to meet the requirement that there will be a delivery to a distribution center in the planning period. To guarantee that each vehicle is used mostly once we impose Constraint (19).

### V. THE ALGORITHM

After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let

$$x = [x] + f, \quad 0 \leq f \leq 1$$

be the (continuous) solution of the relaxed problem,  $[x]$  is the integer component of non-integer variable  $x$  and  $f$  is the fractional component.

Stage 1.

Step 1. Get row  $i^*$  the smallest integer infeasibility, such that  $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate  $\sigma_{ij} = v_{i^*}^T \alpha_j$

With corresponds to

$$\min_j \left\{ \frac{d_j}{\alpha_{ij}} \right\}$$

Calculate the maximum movement of nonbasic  $j$  at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic  $j$  (if available). Eventually the column  $j^*$  is to be increased from LB or decreased from UB. If none go to next  $i^*$ .

Step 4.

Solve  $B\alpha_{j^*} = \alpha_{j^*}$  for  $\alpha_{j^*}$

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic  $j^*$  from its bounds.

Step 6. Exchange basis

Step 7. If row  $i^* = \{\emptyset\}$  go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasis by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasis. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

### VI. CONCLUSIONS

In this paper, we develop a mixed integer nonlinear programming model for the integrated of production and distribution planning problem of a fish processed industry at coastal area with deterministic demand. The model is adequate for solving the planning problem faced by the management of the industry. The model includes the computation of worker which is very useful for the industry in order they will be able to schedule a number of local people, and to conserve sustainability. We also propose an algorithm for solving the mixed integer programming problem.

### ACKNOWLEDGMENT

Special thanks to the Ministry of Higher Education and Research Technology for supporting this research under a scheme of Fundamental Research with contract no.26/UN5.2.3.1/PPM/SP/2015.

### REFERENCES

- [1] A. Andriolo, D. Battini, R. W. Grubbsrom, A. Persona, and F. Sgarbossa, "A century of evolution from Harris' basic lot size model: Survey and research agenda," *Int. J. Production Economics*, 155 (2014) 16-38.
- [2] D. Battini, A. Persona, F. Sgarbossa, "A sustainable EOQ model: Theoretical formulation and applications," *Int. J. Production Economics*, 149 (2014) 145-153.
- [3] M. Rieman, R. T. Neto, and E. Bogendorfer, "Joint optimization of production planning and vehicle routing problems: A review of existing strategies," *Pesquisa Operacional* (2014) 34(2): 189-214.
- [4] Z.-L. Chen, "Integrated production and outbound distribution scheduling: review and extensions," *Operations Research*, vol. 58, no. 1, pp. 130-148, 2010.
- [5] B. Fahimnia, R. Zanjirani Farahani, R. Marian, and L. Luong, "A review and critique on integrated production-distribution planning models and techniques," *Journal of Manufacturing Systems*, vol. 32, no. 1, pp. 1-19, 2013
- [6] A. M. Sarmiento and R. Nagi, "A review of integrated analysis of production-distribution systems," *IIE Transactions*, vol. 31, no. 11, pp. 1061-1074, 1999.

- [7] J. F. Bard and N. Nananukul, "Heuristics for a multiperiod inventory routing problem with production decisions," *Computers and Industrial Engineering*, vol. 57, no. 3, pp. 713-723, 2009.
- [8] S. M. Seyedhosseini and S. M. Ghoreyshi, "An integrated model for production and distribution planning of perishable products with inventory and routing considerations," *Mathematical Problems in Engineering*, Vol. 2014, Article ID 475606.
- [9] Li Liao, Jianfeng Li, and Yaohua Wu, "Modeling and optimization of inventory-distribution routing problem for agriculture products supply chain," *Discrete Dynamic in Nature and Society*, Vol. 2013, Article ID 409869.
- [10] Radulescu M, Radsulescu S, and Radulescu C. Z., "Sustainable production technologies which take into account environmental constraints," *European Journal of Operational Research*, vol. 193(3), pp. 730-740, 2009.
- [11] S. Prasertsan, C. Bunyakan, and J. Chungsiriporn, "Cleaner production of plam oil milling by process optimization," *PSU-UNS International Conf. On Engineering and Environment – ICEE-2005*, Novi Sad, 2005.
- [12] Y. Bouchery, A. Ghaffari, Z. Jemai, and Y. Dallery, "Including sustainability criteria into inventory models," *European Journal of Operations Research*, 222 (2012) 229-240.