

Assessment the Quality of Medical Images (CT & MRI) by Using Wavelet Transformation (WT)

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Abstract—

The measurement of digital image quality is important for many image processing applications. In this paper, been apply of the Wavelet Transformation (WT) technique on medical images taken techniques imaging various medical for the purpose of enhancement and the study of priority medical images in terms of diagnosis in the same parts of the body by assessment the quality of these images through the application of a set of quality metrics objectivity's well known such as Image Fidelity (IF), Structural Similarity Index (SSIM), Mean Squared Error (MSE), synthetic content (SC), Peak Signal-to-Noise Ratio (PSNR), Average Difference (AD), etc.

Keywords— Medical Images Quality, Quality Metrics.

I. INTRODUCTION

In general, measurement of image quality usually can be classified into two categories, which are subjective and objective quality measurements. Subjective quality measurement, Mean Opinion Score (MOS), is truly definitive but too inconvenient, the most time taken and expensive. Therefore, objective measurements are developed such as MSE, MAE, PSNR, SC, MD, LMSE, NAE, ...etc. that are least time taken than MOS but they do not correlation well with MOS. In fact, MSE graphics, where the spatial location of pattern elements is critical and PSNR are the most common measures of image quality in image compression systems, despite the fact that they are not adequate as perceptually meaningful measures, especially MSE variants do not correlation well with subjective quality measures. A number of objective image quality measurements have been evaluated against subjective image quality measurement.

II. DIGITAL IMAGE PROCESSING

An image may be defined as a two-dimensional function, $f(x, y)$, where x and y are spatial (plane) coordinates, and the amplitude of $f(x, y)$ at any pair of coordinates (x, y) is called the intensity or gray level of the image at that point. When x, y , and the amplitude values of f are all finite, discrete quantities, we call the image a digital image. The field of digital image processing refers to processing digital images by means of a digital computer. Note that a digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements, pels, and pixels. Pixel is the term most widely used to denote the elements of a digital image. It could represent a digital image in the form of a matrix as following:

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & - & - & - & - & f(0, N-1) \\ f(1,0) & f(1,1) & - & - & - & - & f(1, N-1) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & - & - & - & - & f(M-1, N-1) \end{bmatrix} \dots\dots\dots(1)$$

Where $f(x, y)$ represent the matrix of image, every element in which called (pixel), and these elements are represented in the form of specific integers within fixed range shown in the following equation:

$$0 \leq f(x, y) \leq L_{\max} \dots\dots\dots(2)$$

where (L_{\max}) represent the upper level of intensity in the image (White) and (0) represent the lower level of intensity in the image (Black) and the interval $[L_{\max}, 0]$ known (Gray Level) . the values $f(x, y)$ It represents the physical signal that reaches the two-dimensional sensor and these values are usually a function of a number of variables including the depth of the body (Z) and color package used in imaging any wavelength used for electromagnetic wave, and Exposure Sensing Time (t) .

III. MEDICAL IMAGING

is the technique and process of creating visual representations of the interior of a body for clinical analysis and medical intervention. Medical imaging seeks to reveal internal structures hidden by the skin and bones, as well as to diagnose and treat disease. Medical imaging also establishes a database of normal anatomy and physiology to make it possible to identify abnormalities. Although imaging of removed organs and tissues can be performed for medical reasons, such procedures are usually considered part of pathology instead of medical imaging.

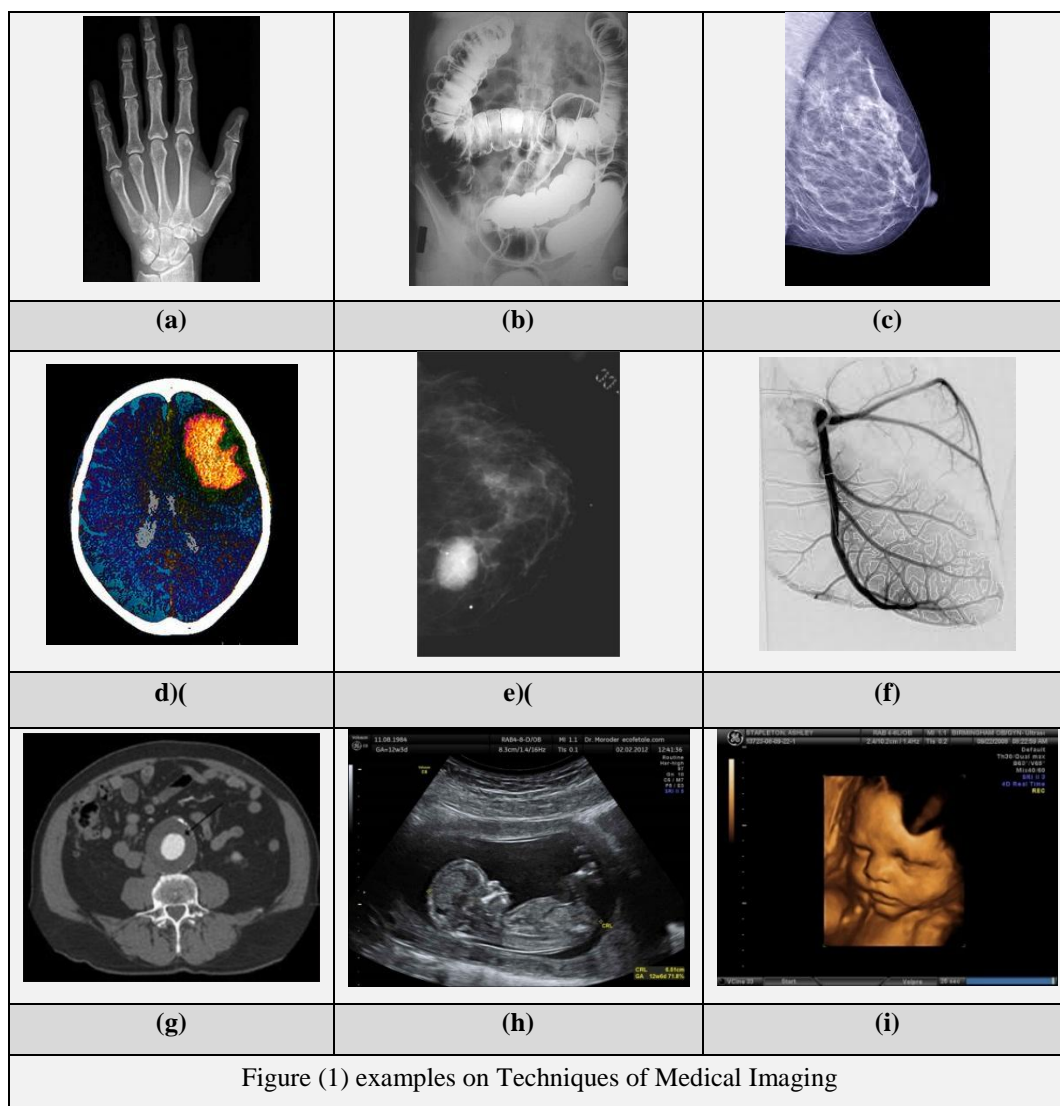
IV. MEDICAL IMAGES

Medical images are a special kind of images. These images are used for the diagnostics of diseases in the patients. A number of modalities exist for obtaining these images. Among popular ones are Computed Topographic Imaging (CT), Magnetic Resonance Imaging (MRI), etc.

V. TECHNIQUES OF MEDICAL IMAGING

- a) X-ray
- b) Fluoroscopy
- c) Mammography
- d) Computed Tomography (CT) Scan
- e) Positron Emission Tomography (PET)
- f) Angiography and Interventional
- g) Magnetic Resonance Imaging (MRI)
- h) Ultrasound
- i) Ultrasound(4-D).

Below, in figure (1), show examples on Techniques of Medical Imaging.



VI. IMAGE QUALITY MEASUREMENTS

It can be classified into two categories are:

A. SUBJECTIVE QUALITY MEASUREMENT

In fact, in image compression system, the truly definitive measure of image quality is perceptual quality. The compressed image quality is specified by Mean Opinion Score (MOS), which is result of perception based on subjective evaluation. The meaning of the 5-level grading scales of MOS is 5- pleasant or excellent quality, 4-good, 3-acceptable, 2-poor quality and 1-unacceptable. MOS is defined as follow:

$$MOS = \frac{1}{S} \sum_{i=1}^S i p(i) \dots \dots \dots (3)$$

Where i is image score $p(i)$ is image score probability and S is number of observer.

B. OBJECTIVE QUALITY MEASUREMENT

The objective quality measurements are save time more than subjective quality measurement]. The seven simple objective measurements are selected and used for this research study. Definition: $x(i,j)$ denotes the samples of original image, $y(i,j)$ denotes the samples of enhancement image. i and j are number of pixels in row and column directions, respectively.

(1) Mean Square Error (MSE)

The simplest of image quality measurement is Mean Square Error (MSE). The large value of MSE means that image is poor quality. MSE is defined as follow:

$$MSE = \frac{1}{MN} \sum_{i=1}^N \sum_{j=1}^M [x(i, j) - y(i, j)]^2 \dots \dots \dots (4)$$

Where $x(i,j)$: represent the element of the original image in the position (i,j)
 $y(i,j)$: represent the element of the enhancement image in the position (i,j) .

(2) Mean Absolute Error (MAE)

The large value of Mean Absolute Error (MAE) means that image is poor quality. MAE is defined as follow:

$$MAE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N |x(i, j) - y(i, j)| \dots \dots \dots (5)$$

(3) PEAK SIGNAL TO NOISE RATIO (PSNR)

The small value of Peak Signal to Noise Ratio (PSNR) means that image is poor quality. PSNR is defined as follow:

$$PSNR = 10 * \log \left(\frac{(L-1)^2}{MSE} \right) \dots \dots \dots (6)$$

Where typical values for the PSNR in lossy image and video compression are between 30 and 50 dB, provided the bit depth is 8 bits, where higher is better. For 16-bit data, typical values for the PSNR are between 60 and 80 dB.

(4) Structural Content (SC)

The large value of Structural Content (SC) means that image is poor quality. SC is defined as follow:

$$SC = \frac{\sum_{i=1}^M \sum_{j=1}^N [x(i,j) - y(i,j)]^2}{\sum_{i=1}^M \sum_{j=1}^N [x(i,j)]^2} \dots \dots \dots (7)$$

(5) Maximum Difference (MD)

The large value of Maximum Difference (MD) means that image is poor quality. MD is defined as follow:

$$MD = \text{MAX } |x(i, j) - y(i, j)| \dots \dots \dots (8)$$

(6) Laplacian Mean Square Error (LMSE)

This measure is based on the importance of edges measurement. The large value of Laplacian Mean Square Error (LMSE) means that image is poor quality. LMSE is defined as follow:

$$LMSE = \frac{\sum_{i=1}^M \sum_{j=1}^N [L(x(i,j)) - L(y(i,j))]^2}{\sum_{i=1}^M \sum_{j=1}^N [L(x(i,j))]^2} \dots \dots \dots (9)$$

(7) Normalized Absolute Error (NAE)

The large value of Normalized Absolute Error (NAE) means that image is poor quality. NAE is defined as follow:

$$NAE = \frac{\sum_{i=1}^M \sum_{j=1}^N |x(i,j) - y(i,j)|}{\sum_{i=1}^M \sum_{j=1}^N |x(i,j)|} \dots\dots\dots (10)$$

(8) Root Mean Square Error (RMSE)

Defined as the square root of the Mean Square Error (MSE) between the original image x(i,j) and the enhanced image y(i,j) and as the lowest value for (RMSE) indicate that the image has the Better Quality. RMSE is defined as follow:

$$RMSE = \sqrt{\frac{1}{MN} \sum_{i=1}^N \sum_{j=1}^M [x(i,j) - y(i,j)]^2} \dots\dots\dots (11)$$

(9) Peak Mean Square Error (PMSE)

It is defined as the greatest value to the Mean Square Error (MSE) between the original x(i,j) and enhanced images . PMSE is defined as follow:

$$PSME = \frac{1}{MN} \times \frac{\sum_{i=1}^M \sum_{j=1}^N [x(i,j) - y(i,j)]^2}{[\text{MAX}(x(i,j))]} \dots\dots\dots (12)$$

(10) Image Fidelity (IF)

Image fidelity (inferred by the ability to discriminate between two images) and image quality (inferred by the preference for one image over another) are often assumed to be directly related. IF is defined as follow:

$$IF = 1 - \frac{\sum_{i=1}^M \sum_{j=1}^N [x(i,j) - y(i,j)]^2}{\sum_{i=1}^M \sum_{j=1}^N [x(i,j)]^2} \dots\dots\dots (13)$$

(11) Normalized Cross – Correlation (NK)

NK is defined as follow:

$$NK = \frac{\sum_{i=1}^M \sum_{j=1}^N [x(i,j) - y(i,j)]}{\sum_{i=1}^M \sum_{j=1}^N [x(i,j) - y(i,j)]^2} \dots\dots\dots (14)$$

(12) Average Difference (AD)

It is defined as the average difference between the original and enhanced images. AD is defined as follow:

$$AD = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N [(x(i,j) - y(i,j))] \dots\dots\dots (15)$$

(13) Correlation Quality (CQ)

CQ is defined as follow:

$$CQ = \frac{\sum_{i=1}^M \sum_{j=1}^N x(i,j) * y(i,j)}{\sum_{i=1}^M \sum_{j=1}^N x(i,j)} \dots\dots\dots (16)$$

(14) Universal Quality Index (UQI)

This measure differs from the traditional metrics to calculate the error rate in the image, as all of Wang and Bovik found that the image-processing model is a mixture consisting of three factors is the loss of the correlation, distortion of light, and the contrast distortion. UQI is defined as follow:

$$UQI = \frac{4\bar{x}\bar{y}\sigma_{xy}}{(\bar{x}^2 + \bar{y}^2)(\sigma_x^2 + \sigma_y^2)} \dots\dots\dots (17)$$

Where \bar{x} and \bar{y} : represent the average all of the original and enhanced images respectively, σ_x and σ_y represent the standard deviation all of the original and enhanced images respectively, σ_{xy} represent covariance between two images.

(15) Structural Similarity Index (SSIM)

Natural images are highly structured and their pixel values exhibit strong dependencies. SSIM is an Image Quality Assessment (IQA) algorithm based on these structural dependencies within an image. The human visual system is highly adapted to extract structural information from the viewing field. The SSIM algorithm separates the luminance component l(x, y), contrast component c(x, y) and the structural component s(x, y) from the original image (x) and the enhanced image (y) and compares these components. SSIM index is defined as follow:

$$SSIM(x, y) = l(x, y) . c(x, y) . s(x, y) \dots\dots\dots (18)$$

(16) Feature Similarity Index (FSIM)

FSIM is based on the theory that HVS understands an image based on its low-level features such as edges, and a good IQA metric could be obtained by comparing these low-level features of the original image and the enhanced image. At points of high phase congruency of the Fourier waves of different frequencies of the image, highly informative features can be extracted. FSIM utilizes this property of the Fourier transform of the reference and distorted images for the quality assessment. In FSIM, the phase congruency (PC) and the image gradient magnitude (GM) are computed for the quality assessment of the enhanced image with respect to the original image. FSIM is defined as follow:

$$FSIM = \frac{\sum_{i=1}^M \sum_{j=1}^N (\text{Similarity Image})}{\sum_{i=1}^M \sum_{j=1}^N (PC_m)} \dots\dots\dots (19)$$

(17) IQA based on Spectral Residuals (SR-SIM)

The spectral residual visual saliency (SRVS) is used in the computation of this metric. The hypothesis behind this approach is that an images perceived quality is related to its visual saliency map. SR-SIM is defined as follow:

$$SR - SIM = \frac{\sum_{x \in \Omega} S_V(x) [S_G(x)]^\alpha \cdot R_m(x)}{\sum_{x \in \Omega} R_m(x)} \dots\dots\dots (20)$$

(18) Feature Similarity Index by using Riesz Transforms (RFSIM)

(RFSIM), is proposed based on the fact that the human vision system (HVS) perceives an image mainly according to its low-level features. RFSIM is defined as follow:

$$RFSIM = \prod_{i=1}^5 \frac{\sum \sum \frac{2x(i,j) \cdot y(i,j) + c}{x^2(i,j) + y^2(i,j) + c} \cdot M(x,y)}{\sum \sum M(x,y)} \dots\dots\dots (21)$$

(19) Visual Information Fidelity (VIF_p)

The image information measure that quantifies the information that is present in the original image, and also quantify how much of this reference information can be extracted from the distorted image. Combining these two quantities, we propose a visual information fidelity (VIF_p) metric for image quality assessment. VIF_p is defined as follow:

$$VIF_p = \frac{\sum_{j \in \text{subbands}} I(\vec{C}^{N,j}; \vec{F}^{N,j} | \vec{S}^{N,j})}{\sum_{j \in \text{subbands}} I(\vec{C}^{N,j}; \vec{E}^{N,j} | \vec{S}^{N,j})} \dots\dots\dots (22)$$

(20) Signal –to- Noise Ratio (SNR)

It is defined as the ratio of the power of signal and noise. SNR is defined as follow:

$$SNR = 10 * \log_{10} \frac{\sum_{i=1}^M \sum_{j=1}^N [x(i,j)]^2}{\sum_{i=1}^M \sum_{j=1}^N [x(i,j) - y(i,j)]^2} \dots\dots\dots (23)$$

(21) Visual Signal-to-Noise Ratio (VSNR)

It is defined as efficient metric for quantifying the visual fidelity of natural images based on near threshold and supra-threshold properties of human vision. VSNR is defined as follow:

$$VSNR = 10 * \log_{10} \left[\frac{C^2(x)}{VD^2} \right] \dots\dots\dots (24)$$

(22) Personal Correlation Coefficient (PCC)

It is defined as follow:

$$PCC = \sum_{i=1}^M \sum_{j=1}^N \frac{x(i,j) * y(i,j)}{\text{Std}[x(i,j)] * \text{Std}[y(i,j)]} \dots\dots\dots (25)$$

VII. WAVELET TRANSFORM (WT)

In conventional Fourier transform, we use sinusoids for basis functions. It can only provide the frequency information. Temporal information is lost in this transformation process. In some applications, we need to know the frequency and temporal information at the same time, such as a musical score, we want to know not only the notes (frequencies) we want to play but also when to play them. Unlike conventional Fourier transform, wavelet transforms are based on small waves, called wavelets. It can be shown that we can both have frequency and temporal information by this kind of transform using wavelets. Moreover, images are basically matrices. For this reason, image processing can be regarded as matrix processing. Due to the fact that human vision is much more sensitive to small variations in color or brightness, that is, human vision is more sensitive to low frequency signals. Therefore, high frequency components in images can be

compressed without distortion. Wavelet transform is one of a best tool for us to determine where the low frequency area and high frequency area is. These kinds of applications will be discussed later.

VIII. CATEGORIES OF WAVELET TRANSFORM

there are many categories of Wavelet Transform of them :

a) Haar Wavelet

Is the simple category from other wavelet categories .is a sequence of rescaled "square-shaped" functions which together form a wavelet family or basis. Wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal function basis.

b) Daubechies Wavelet

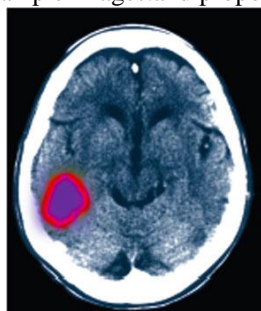
Named after of the researcher (Ingrid Daubechies).based on the work of Ingrid Daubechies, are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function (called the father wavelet) which generates an orthogonal Multiresolution analysis.

c) Biorthogonal Wavelet

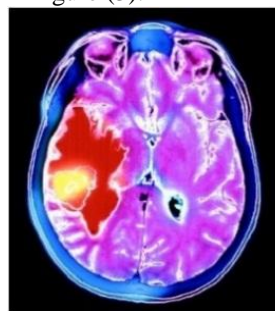
Is a wavelet where the associated wavelet transform is invertible but not necessarily orthogonal. Designing biorthogonal wavelets allows more degrees of freedom than orthogonal wavelets. One additional degree of freedom is the possibility to construct symmetric wavelet functions.

IX. SAMPLES IMAGES AND ALGORITHM

Below, in Figure (2), sample images.and proposed scheme of Algorithm in Figure (3).



Brain CT Scan jpg 294 * 376



Brain MRI Scan jpg 342 * 390

Figure (2) Images samples

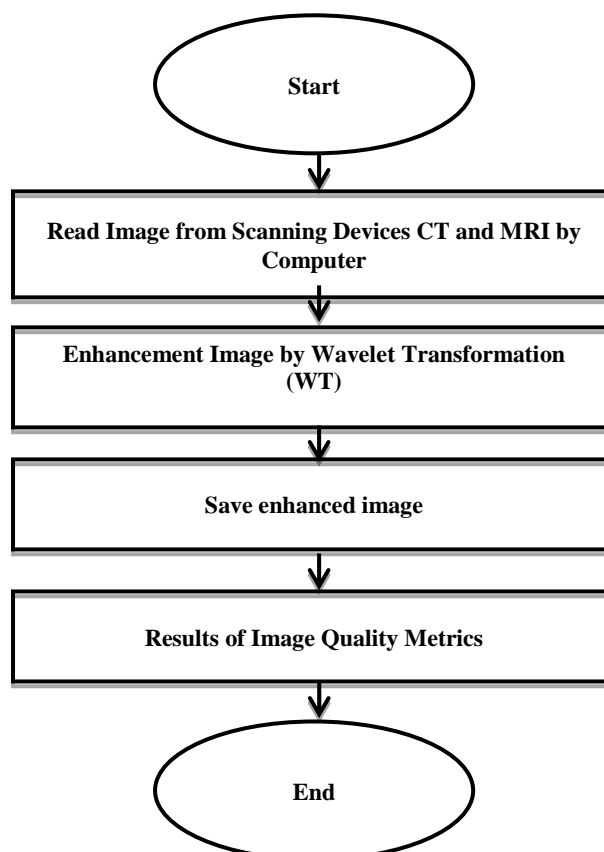


Figure (3) scheme proposed of Algorithm

The Results

Tables from (1) to (3) shows the results of application of Wavelet Transform (WT)with Types (Haar, db2, bior 2.2)on the images in figure (2) by Matlab2013a program.

Table (1)
 The Results of the Brain Image Quality Metrics (CT & MRI) After Apply Wavelet Transform Category - Haar

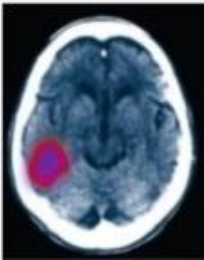
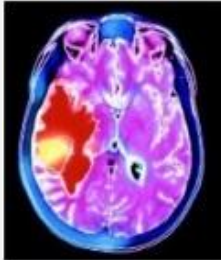
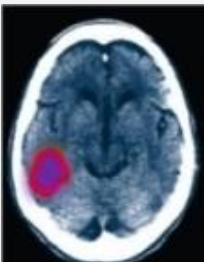
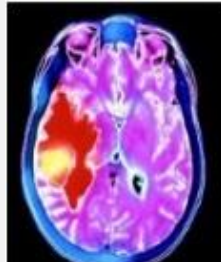
Case 1	Imaging Techniques	Wavelet Type
Brain	CT & MRI	Haar
Image	CT	MRI
Original		
Approximation		
IF	0.9857	0.986
SSIM	0.7694	0.7602
RFSIM	0.4981	0.5366
SRSIM	0.9707	0.9581
FSIM	0.9337	0.9228
VIFp	0.9263	0.911
MSE	243.724	220.55
RMSE	15.612	14.851
PMSE	62200	56200
LMSE	3.3862	0.3187
SNR	18.4487	18.5318
VSNR	24.496	20.942
PSNR	24.262	24.696
NK	0.9848	0.9852
SC	1.0163	1.0158
AD	3.0323	3.3442
MD	140	134
NAE	0.0637	0.0736
MAE	6.0739	6.7479
CQ	176.202	168.954
UQI	0.9379	0.8835
PCC	108840	131350

TABLE (2)

The Results of the Brain Image Quality Metrics (CT & MRI) After Apply Wavelet Transform Category – db2

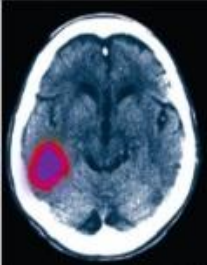
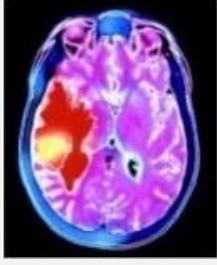
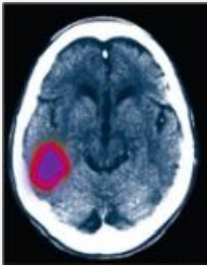
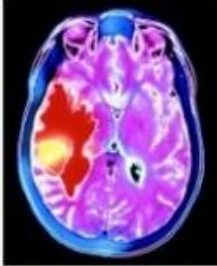
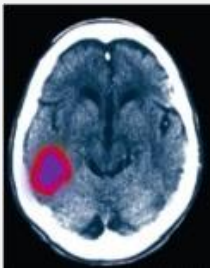
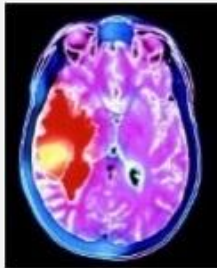
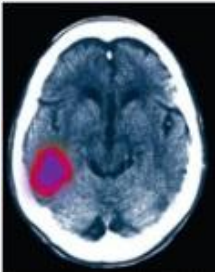
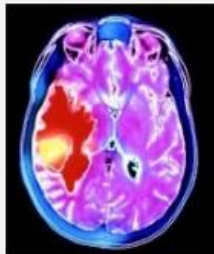
Case 2	Imaging Techniques	Wavelet Type
Brain	CT & MRI	db2
Image	CT	MRI
Original		
Approximation		
IF	0.9921	0.9867
SSIM	0.8153	0.7967
RFSIM	0.6803	0.6177
SRSIM	0.9793	0.9649
FSIM	0.9471	0.9241
VIFp	0.9632	0.9491
MSE	135.307	208.887
RMSE	11.632	14.453
PMSE	34503	53266
LMSE	1.038	0.406
SNR	21.004	18.768
VSNR	24.023	20.437
PSNR	26.818	24.932
NK	0.9864	0.979
SC	1.0195	1.0295
AD	2.305	3.1408
MD	129	136
NAE	0.0459	0.0691
MAE	4.3762	6.3403
CQ	176.489	167.894
UQI	1.7981	0.8928
PCC	109610	131480

TABLE (3)

The Results of the Brain Image Quality Metrics (CT & MRI) After Apply Wavelet Transform Category – bior 2.2

Case 3	Imaging Techniques	Wavelet Type
Brain	CT & MRI	bior 2.2
Image	CT	MRI
Original		
Approximation		
IF	0.9954	0.9841
SSIM	0.8246	0.7827
RFSIM	0.7446	0.6251
SRSIM	0.9828	0.9666
FSIM	0.9585	0.9208
VIFp	0.9692	0.9489
MSE	78.1588	250.7541
RMSE	8.8407	15.8352
PMSE	19930	63942
LMSE	0.8246	0.6275
SNR	23.3879	17.9744
VSNR	25.5599	20.7069
PSNR	29.201	24.1383
NK	0.9921	0.9764
SC	1.0113	1.0322
AD	1.6466	3.1601
MD	142	137
NAE	0.0347	0.0698
MAE	3.3033	6.4036
CQ	177.5062	167.442
UQI	0.9529	0.8821
PCC	110000	131100

X. CONCLUSIONS

Results shown in Tables (1), (2) and (3) after applying three categories of Wavelet Transformation (WT) are (Haar, db2, bior 2.2) showed the following conclusions:

- 1- After applying Wavelet Transformation category of (Haar) :Image quality metrics showed that the image of the brain taken Magnetic Resonance Imaging technique (MRI) has a far better quality from image of the brain taken by Computed Tomography technique (CT) because the objective image quality metrics given better results for the image of the brain taken by technique (MRI).
- 2- After applying wavelet transformation of the two categories (db2 & bior 2.2): image quality metrics showed that the image of the brain taken by Computed Tomography technique (CT) has a very far better quality from image of the brain taken by Magnetic Resonance Imaging technique (MRI) because the objective image quality metrics given better results for the image of the brain taken by technique (CT).

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