

# Gravitational Search Algorithm Applied to Optimal Power Flow Problem

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## Abstract—

**T**his paper proposes gravitational search algorithm (GSA) to solve optimal power flow (OPF) in order to minimize the fuel cost while satisfying various operating constraints. The new method has been tested on IEEE standard 30-bus, systems with high precision. The test results have been compared with the same of popular conventional solution methods. The results demonstrate the potential of the proposed approach and show its effectiveness and robustness to solve the OPF problem.

**Keywords—** Evolutionary algorithms, Gravity, Gravitational search algorithm Gravitational search algorithm, optimal power flow, Valve-point loading

## I. INTRODUCTION

Optimal power flow (OPF) problem has been one of the most important operational function and widely studied subject in the power system operation and modern energy management system and so it has received much attention to the researchers in the past few decades. Generally, the OPF problem is a large-scale highly nonlinear non-convex optimization problem. Optimized operation of power system is a challenging problem due to its complex and non-linear nature. There are also many uncertainties in power system, which add its complexity further. Recently, with the introduction of deregulation in power industry, many new complexities have been added to the existing problems. The application of optimization techniques to various power system problems has been an area of research for many years.

Classical mathematical optimization algorithms such as newton based method [1], interior point method [2], quadratic programming [3-4], gradient based methods [5], Lagrangian relaxation (LR) approach [6] linear programming [7-8], dynamic programming [9-10], suffer from certain limitations and are not able to solve the above mentioned problems efficiently. This is because the conventional methods depend upon the derivative of the objective function. Due to the non-differentiable objective functions of OPF, the conventional methods fail to give global optimal results. But with the advent of evolutionary intelligent computational techniques, near global optimal solutions may be obtained. To overcome this problem some intelligent optimization algorithms known as heuristic techniques are applied to solve OPF problem.

Recently, with the advent of computer technology, various optimization techniques have been proposed by many researchers to deal with the OPF problem with various degrees of success. Some of the well popular evolutionary techniques are genetic algorithm (GA) [11-12], 39-40], simulated annealing (SA) [13-15], tabu search (TS) [16-17], differential evolution (DE) [18-20], particle swarm optimization (PSO) [21], artificial bee colony (ABC) [29], ant colony optimization (ACO) [23], immune algorithm [24].

More recently, a new evolutionary computation technique, called gravitational search algorithm (GSA) has been proposed and introduced [25]. The algorithm is inspired by Newton's law of gravity and law of motion and can take care of optimality on rough, discontinuous and multi-modal surfaces. The GSA has three main advantages: it can find near optimal solution regardless the initial parameter values, its convergence is fast and it uses few number of control parameters. Moreover, it can handle integer and discrete optimization. The proposed approach is examined and tested on the standard IEEE 30-bus test system. The potential and effectiveness of the proposed approach are demonstrated. Additionally, the results are compared to those reported in the literature.

The study is organized as follows. First of all, the mathematical formulation of the OPF problem is shown. Afterward, the structure of GSA algorithm is set. Finally, the results and conclusions are displayed for IEEE 30-bus system.

## II. MATHEMATICAL PROBLEM FORMULATION

For the OPF problem the objective function and system constraints are as follows:

### A. Objective function

The main aim of OPF problem is to minimize the total fuel cost or production cost. The generating units with multi-valve steam, turbines exhibit a greater variation in the fuel cost functions. The fuel cost function of generators, considering the valve-point effect, may be expressed as the sum of a quadratic and a sinusoidal function. The total fuel cost in terms of real power output can be expressed as:

$$f_1(x, u) = \min f(P) = \sum_{i=1}^{N_G} f_i(P_{gi}) = \sum_{i=1}^{N_G} (a_i + b_i P_{gi} + c_i P_{gi}^2 + |e_i \sin(f_i(P_{gi}^{\min} - P_{gi}))|) \quad (1)$$

Where,  $a_i, b_i, c_i, e_i$  and  $f_i$  are cost coefficients of the generating unit  $i$ .  $P_{gi}$ ,  $P_{gi}^{\min}$  are operating power and minimum power of generator  $i$ .

### B. System constraints

System operating constraints are as follows:

$$P_{gi} - P_{di} - V_i \sum_{j=1}^{N_B} V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] = 0, \quad i=1, \dots, N_B \quad (2)$$

$$Q_{gi} - Q_{di} - V_i \sum_{j=1}^{N_B} V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] = 0, \quad i=1, \dots, N_B \quad (3)$$

$$V_{gi}^{\min} \leq V_{gi} \leq V_{gi}^{\max}, \quad i=1, \dots, N_G \quad (4)$$

$$V_{li}^{\min} \leq V_{li} \leq V_{li}^{\max}, \quad i=1, \dots, N_L \quad (5)$$

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, \quad i=1, \dots, N_G \quad (6)$$

$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max}, \quad i=1, \dots, N_G \quad (7)$$

$$T_i^{\min} \leq T_i \leq T_i^{\max}, \quad i=1, \dots, N_T \quad (8)$$

$$Q_{ci}^{\min} \leq Q_{ci} \leq Q_{ci}^{\max}, \quad i=1, \dots, N_C \quad (9)$$

$$S_{li} \leq S_{li}^{\max}, \quad i=1, \dots, N_{TL} \quad (10)$$

where  $G_{ij}$ ,  $B_{ij}$  are the real and imaginary part of the bus admittance matrix, respectively, of the  $(i, j)$  th entry;  $P_{gi}$ ,  $Q_{gi}$  are the active and reactive power generation of generator  $i$ ;  $Q_{gi}^{\min}$ ,  $Q_{gi}^{\max}$  are the minimum and maximum reactive power generation of generator  $i$ , respectively;  $P_{di}$ ,  $Q_{di}$  are the active and reactive load demand of bus  $i$ ;  $V_{gi}^{\min}$ ,  $V_{gi}^{\max}$  are the minimum and maximum generator voltage of bus  $i$ , respectively;  $V_{li}^{\min}$ ,  $V_{li}^{\max}$  are the minimum and maximum voltage of load bus  $i$ , respectively;  $T_i^{\min}$ ,  $T_i^{\max}$  are the minimum and maximum tap setting of transmission line  $i$ , respectively;  $Q_{ci}^{\min}$ ,  $Q_{ci}^{\max}$  are the minimum and maximum reactive power injection of shunt compensator  $i$ , respectively;  $S_{li}^{\max}$  is the maximum apparent power flow in line  $i$ .

### III. GRAVITATIONAL SEARCH ALGORITHM (GSA)

In 2009, Rashedi et al. [26] developed a new algorithm, known as the gravitational search algorithm (GSA), which is based on Newton's law of gravity and law of motion. In the proposed algorithm, agents are considered as objects and their performance is measured by their masses. All these objects attract each other by the gravitational force, and this force causes a global movement of all objects toward the objects with heavier masses. Hence, masses cooperate using a direct form of communication through gravitational force. The heavy masses correspond to good solutions and move more slowly and conversely light masses correspond to poor solutions and move toward heavy masses much faster. This guarantees the exploitation step of the algorithm. In the GSA, each mass (agent) has four specifications: position, inertial mass, active gravitational mass, and passive gravitational mass. Similar to PSO, some velocity and position updating scheme of agents are employed here. The velocity of each agent is updated after calculating the acceleration of each agent using Newton's law of motion. Consequently, the position of each agent is updated using the modified velocity. The position of the mass corresponds to a solution of the optimization problem, and its gravitational and inertial masses represent fitness function.

All masses in the universe have a tendency to accelerate toward each other due to gravitation force. Gravity acts between separated particles without any intermediary and without any delay. According to Newton's law of gravity, each particle attracts every other particle with a "gravitational force" [27-28]. The gravitational force, which acts between two particles, is directly proportional to the product of their masses and inversely proportional to the square of the distance between them [29]:

$$F = G \times \left( \frac{m_1 \times m_2}{r^2} \right) \quad (11)$$

Where,  $F$  is the magnitude of the gravitational force;  $G$  is the gravitational constant;  $m_1$  and  $m_2$  are the mass of the first and second particles, respectively; and  $r$  is the distance between the two particles.

According to Newton's second law, when a force  $F$  is applied to a particle having mass  $m$ , its acceleration  $a$  is given by:

$$a = \frac{F}{m} \tag{12}$$

Therefore based on (11), it is clear that there is an attracting gravitational force among all particles of the universe, where the effect of the bigger and closer particle is higher. The concept is presented in Fig. 1. An increase in the distance between two particles means decreasing the gravitational force between them. In this figure,  $F_{1j}$  is the force that acting on  $m_1$  from  $m_j$  and  $F_1$  is the overall force that acts on  $m_1$  and causes the acceleration vector  $a_1$ .

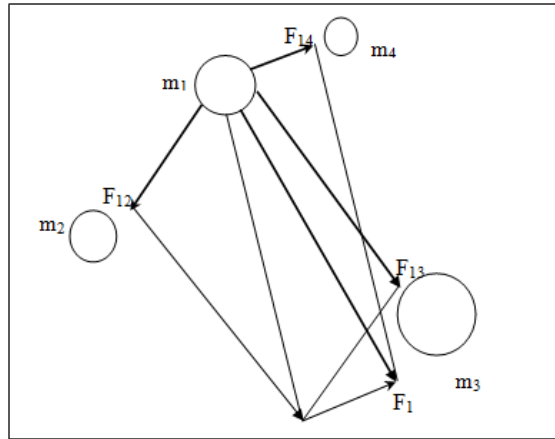


Fig. 1 Force of Attraction among Various Masses

The actual value of the “gravitational constant” depends on the actual age of the universe. This is represented as follows:

$$G(t) = G(t_0) \times \left(\frac{t_0}{t}\right); \quad \beta < 1 \tag{13}$$

Where,  $G(t)$  is the value of the gravitational constant at time  $t$ .

There are three kinds of masses defined in theoretical physics:

Active gravitational mass  $m_a$  is a measure of the strength of the gravitational field due to a particular object. The gravitational field of an object with a large active gravitational mass is stronger than the object with less active gravitational mass. Passive gravitational mass  $m_p$  is a measure of the strength of an object's interaction with the gravitational field. Within the same gravitational field, an object with a larger passive gravitational mass experiences a larger force than an object with a smaller gravitational mass. Inertial mass  $m_i$  is a measure of an object's resistance to change its state of motion when a force is applied. An object with larger inertial mass changes its motion more slowly than an object with smaller inertial mass.

Accordingly, Newton's laws can be rewritten in the following ways. The gravitational force  $F_{ij}$ , exerted by the  $j^{th}$  object on the  $i^{th}$  object, is proportional to the product of the active gravitational mass and passive gravitational mass of the  $j^{th}$  and the  $i^{th}$  object and inversely proportional to the square of distance between them. Acceleration  $a_i$  is proportional to  $F_{ij}$  and inversely proportional to inertia mass of the  $i^{th}$  object. More precisely, (11) and (12) may be modified as follows:

$$F_{ij} = G \times \left( \frac{m_{aj} \times m_{pi}}{r^2} \right) \tag{14}$$

$$a_i = \frac{F_{ij}}{m_{ii}} \tag{15}$$

Where,  $m_{aj}$  and  $m_{pi}$  are the active gravitational mass of the  $j^{th}$  object and the passive the gravitational mass of the  $i^{th}$  object, respectively;  $m_{ii}$  represents the inertial mass of the  $i^{th}$  object.

Theoretically inertial mass, passive gravitational mass, and active gravitational mass are distinct; however, no experiment has ever clearly demonstrated any difference between them. The theory of general relativity is based on the assumption that inertial and passive gravitational mass are equivalent [30]. Standard general relativity is based on the fact that inertial mass and active gravitational mass are the same [30]. This equivalence is sometimes called the strong equivalent principle.

In this section, based on the law of gravity, an optimization algorithm known as the GSA [26] is described. In this algorithm, the performance of the objects is measured by their masses. Using gravitational force masses communicate with each other. The heavy masses correspond to good solutions and move more slowly than lighter ones. This step improves the exploitation ability of the algorithm. In the GSA, the mass of each object has four specifications: position, inertial mass, active gravitational mass, and passive gravitational mass. Each mass is responsible to find out a solution of the optimization problem and its position corresponds to a solution of the problem. The algorithm is navigated by the proper adjustment of the gravitational and inertia masses which are determined using a fitness function. Masses obey the following laws.

**Law of gravity:** Each particle attracts every other particle and the gravitational force between two particles is directly proportional to the product of their masses and inversely proportional to the distance between them. After performing experiment several times, it has been observed that  $r$  provides better results than  $r^2$ . Therefore the term  $r^2$  has been changed to  $r$  in (14).

**Law of motion:** The present velocity of any mass is equal to the sum of the fraction of its previous velocity and the variation in the velocity. Variation in the velocity i.e. acceleration of any mass is equal to the force acted on the system divided by its inertial mass.

The position of the  $i^{th}$  agent may be defined as:

$$X_i = (x_i^1, x_i^2, \dots, x_i^d, \dots, x_i^n) \quad i \in N \quad (16)$$

where  $x_i^d$  represents the position of the  $i^{th}$  agent in the  $d^{th}$  dimension. At any specific time 't', the force acting on the  $i^{th}$  object due to the  $j^{th}$  object may be represented as:

$$F_{ij}^d(t) = G(t) \times \left( \frac{m_{aj}(t) \times m_{pi}(t)}{r_{ij}(t) + \varepsilon} \right) \times (x_j^d(t) - x_i^d(t)) \quad (17)$$

where  $m_{aj}$  is the active gravitational mass related to the  $j^{th}$  agent,  $m_{pi}$  is the passive gravitational mass related to the  $i^{th}$  agent,  $G(t)$  is gravitational constant at time  $t$ ,  $\varepsilon$  is a constant term whose magnitude is very small.  $r_{ij}(t)$  is the Euclidian distance between the  $i^{th}$  and the  $j^{th}$  agents which can be represented as:

$$r_{ij} = \|x_i(t), x_j(t)\|_2 \quad (18)$$

The total force that acts on the  $i^{th}$  agent in a dimension  $d$  is a randomly weighted sum of the  $d^{th}$  components of the forces exerted from other agents:

$$F_i^d(t) = \sum_{\substack{j=1 \\ j \neq i}}^N rand_j \times F_{ij}^d(t) \quad (19)$$

where  $rand_j$  is a random number in the interval [0, 1].

According to Newton's law of motion, the acceleration  $a_i^d(t)$  of the agent the  $i^{th}$  at time  $t$ , in the  $d^{th}$  direction, is given by:

$$a_i^d(t) = \frac{F_i^d(t)}{m_{ii}(t)} \quad (20)$$

where  $m_{ii}$  is the inertial mass of the  $i^{th}$  agent.

Furthermore, the updated velocity of an agent is considered as a fraction of its current velocity added to its acceleration. Therefore, its position and its velocity could be calculated as follows:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \quad (21)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (22)$$

The random number  $rand_i$  gives a randomized characteristic to the search.

The gravitational constant  $G$  is initialized at the beginning and reduces with time to control the search accuracy. In other words,  $G$  is a function of the initial value  $G_0$  and time  $t$ :

$$G(t) = G(G_0, t) \quad (23)$$

Gravitational and inertia masses are simply calculated by the fitness evaluation. A heavier mass means a more efficient agent. Assuming the equality of the gravitational and inertia masses, the values of masses are calculated using the map of fitness. The gravitational and inertial masses are updated by the following equations:

$$m_{ai} = m_{pi} = m_{ii} = m_i \quad i \in N \quad (24)$$

$$m_i(t) = \frac{M_i(t)}{\sum_{j=1}^N M_j(t)} \quad (25)$$

$$\text{where } M_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \quad (26)$$

where  $fit_i(t)$  represent the fitness value of the  $i^{th}$  agent at time  $t$ , and,  $worst(t)$  and  $best(t)$  are defined as follows for a minimization problem:

$$best(t) = \min(fit_1(t), fit_2(t), \dots, fit_N(t)) \quad (27)$$

$$worst(t) = \max(fit_1(t), fit_2(t), \dots, fit_N(t)) \quad (28)$$

One way to perform a good compromise between exploration and exploitation is to reduce the number of agents with lapse of time. Hence, in this algorithm only a set of agents with bigger mass apply their force to the other. However, this property may reduce the exploration ability and increase the exploitation capability. Initially the exploration ability of the algorithm should be more powerful in order to avoid trapping in a local optimum. As the time progress, exploration ability of the algorithm must fade out and exploitation ability should fade in. To improve the performance of GSA by controlling exploration and exploitation ability, the  $K_{best}$  agents will attract the others.  $K_{best}$  is a function of time, whose initial value is  $K_0$  at the beginning and decreases with time. Therefore, all agents apply the force at the beginning, and as time passes, the term  $K_{best}$  decreases linearly. At the end there will be just one agent applying force to the others. Therefore, (19) could be modified as:

$$F_i^d(t) = \sum_{\substack{j \in kbest \\ j \neq i}}^N rand_j \times F_{ij}^d(t) \quad (29)$$

where  $kbest$  is the set of first  $K_0$  agents with the best fitness value and biggest mass.

In GSA, value of  $G$  in (23) is modified as follows:

$$G(t) = G_0 \times e^{-\alpha t / Iter_{max}} \quad (30)$$

where  $G_0$  is starting value of gravitational constant, and  $Iter_{max}$  is the total number of iterations (the total age of system).  $\alpha$  is a constant term.

The different steps of the proposed algorithm are the followings:

- Step 1:** Each of the control variables of an agent is initialized randomly using a uniform random number distribution within its feasible range.
- Step 2:** The fitness value of each agent in the population are calculated.
- Step 3:** Based on fitness value,  $best(t)$ ,  $worst(t)$  and  $M_i(t)$  are calculated.
- Step 4:** Gravitational force applied by each agent to the others is evaluated.
- Step 5:** Acceleration and velocity are updated using (20) and (21), respectively.
- Step 6:** Position of each agent is updated using (22).
- Step 7:** If termination condition is not met, go to Step 2.

#### IV. SIMULATION RESULTS AND DISCUSSION

In order to access the efficiency of the proposed GSA-based approach, it has been applied to IEEE 30-bus for fuel cost minimization. The programming of proposed method is developed in MATLAB 7.1 and run on PC with 2.5 GHz core duo processor of 2 GB RAM. For implementing it, population size of 50 and the maximum number of iterations of 100 is taken in this simulation study. The IEEE 30-bus test system has six generators at the buses 1, 2, 5, 8, 11 and 13 and four transformers with off-nominal tap ratio at lines 6–9, 6–10, 4–12 and 28–27. In addition, buses 10, 12, 15, 17, 20, 21, 23, 24 and 29 have been chosen as shunt VAR compensation buses [31]. The total system demand is 2.834 p.u. at 100 MVA base. The maximum and minimum voltages of all load buses are considered to be 1.05–0.95 in p.u. The operating limits of tap ratio of the regulating transformers are within 0.95-1.1. The optimal settings of control variables, obtained by the proposed GSA algorithm, are presented in Table 1 with minimum fuel cost obtained from the proposed approach equal to 799.4369 \$/h. order to assess the potential of the proposed algorithm, a comparison between the results of fuel cost obtained by the proposed GSA algorithm and those reported in the literature has. It is worth mentioning that the comparison has been carried out with the same control variable limits, initial conditions, and other system data. Table 1 clearly shows that the proposed GSA based approach outperforms the DE [31] and LTLBO [32] techniques. To illustrate the convergence of the proposed GSA algorithm, cost value over 100 iterations are plotted in Fig. 2. It is found that the proposed GSA algorithm converges rapidly towards the optimal solution.

In order to check the robustness of the proposed GSA approach, 50 different trials with different initial populations are made. Table 2, presents the statistical results obtained by the different algorithms. It is clear from the simulation results that the difference amongst the minimum, maximum and mean objective values obtained by GSA is very insignificant. Thus proofs the robustness of the proposed GSA method.

TABLE 1: SIMULATION RESULTS OBTAINED USING DE, LTLBO AND GSA

Approaches	DE [31]	LTLBO [32]	GSA
$P_{g1}$ (MW)	176.2592	177.46	177.1266

$P_{g2}$ (MW)	48.5602	48.6837	48.6883
$P_{g5}$ (MW)	21.3402	21.3146	21.2921
$P_{g8}$ (MW)	22.0553	20.8867	21.0193
$P_{g11}$ (MW)	11.7785	11.8086	11.8602
$P_{g13}$ (MW)	12.0217	12.0000	12.0000
$V_1$ (p.u.)	1.0999	1.1000	1.1000
$V_2$ (p.u.)	1.0890	1.0817	1.0876
$V_5$ (p.u.)	1.0659	1.0509	1.0613
$V_8$ (p.u.)	1.0697	1.0555	1.0690
$V_{11}$ (p.u.)	1.0965	1.0826	1.1000
$V_{13}$ (p.u.)	1.0996	1.0574	1.0999
$TC_{6-9}$	1.0429	1.0461	1.0397
$TC_{6-10}$	0.9179	0.9583	0.9000
$TC_{4-12}$	1.0190	0.9996	0.9780
$TC_{27-28}$	0.9896	0.9891	0.9613
$Q_{C10}$ (Mvar)	4.5453	5.0000	5.0000
$Q_{C12}$ (Mvar)	4.4158	5.0000	5.0000
$Q_{C15}$ (Mvar)	4.1734	5.0000	5.0000
$Q_{C17}$ (Mvar)	2.5171	5.0000	5.0000
$Q_{C20}$ (Mvar)	2.0916	4.0000	4.2913
$Q_{C21}$ (Mvar)	4.1990	5.0000	5.0000
$Q_{C23}$ (Mvar)	2.5527	3.0000	2.7006
$Q_{C24}$ (Mvar)	4.3812	5.0000	5.0000
$Q_{C29}$ (Mvar)	2.7503	3.0000	2.3013
Fuel cost (\$/hr)	799.2891	799.4369	798.9157
Transmission loss (MW)	8.6150	8.7558	8.5865

TABLE 2 STATISTICAL COMPARISON OF VARIOUS ALGORITHMS

Approaches	Average fuel cost	Worst fuel cost	Best fuel cost
LTLBO [32]	799.7186	800.2578	799.4369
NPSO [33]	800.9024	801.37	800.6815
Fuzzy-GA [34]	801.627	802.1158	801.0554
DE-PS [35]	800.7962	801.2417	800.1475
ABC [36]	800.8715	801.8674	800.66
MDE [37]	802.382	802.404	802.376
GSA	798.9993	799.1212	798.9157

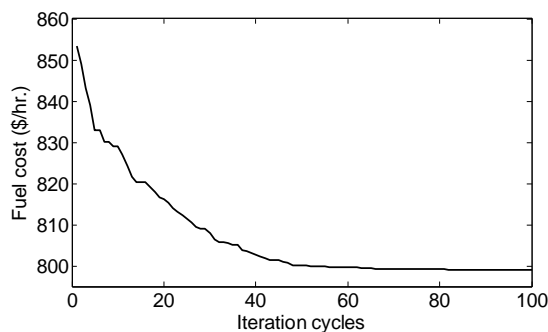


Fig. 2 Fuel Cost Convergence Obtained By Gsa Algorithm

## V. CONCLUSIONS

In this paper, the constrained OPF problem is formulated as an optimization problem with minimization of the fuel cost as the objective function. A gravitational search algorithm is proposed to identify the optimal control variable setting of the OPF problem. The simulation results on IEEE 30-bus test system shows that the proposed algorithm is effective for fuel cost minimization of OPF problem. Moreover, the simulation results show that the proposed GSA approach outperforms other optimization techniques available in the literature.

## REFERENCES

- [1] W. H. E. Liu, A. D. Papalexopoulos and W. F. Tinney, "Discrete shunt controls in a Newton optimal power flow", *IEEE Trans Power Syst*, Vol. 17, No. 4 :pp. 1509–18,1992.
- [2] W. Yan, S. Lu and D. C. Yu, "A hybrid genetic algorithm–interior point method for optimal reactive power flow", *IEEE Trans Power Syst*, Vol. 21, No. 3, pp. 1163–9, 2006.



- [3] V. H. Quintana and M. Santos-Nieto, "Reactive-power dispatch by successive quadratic programming", *IEEE Trans Energy Convers*, Vol. 4, No. 3, pp. 425–35, 1989.
- [4] K. Aoki, A. Nishikory and R. T. Yokoyana, "Constrained load flow using recursive quadratic programming", *IEEE Transactions on Power Systems*, Vol. PWRS-2, No. 1, pp. 8-16, 1987.
- [5] B. T. Burchett, "Aerodynamic parameter identification for symmetric projectiles: An improved gradient based method" *Aerospace Science and Technology*, Vol. 30, No. 1, pp. 119-127, 2013.
- [6] W. G. Wood, "Spinning reserve constrained economic dispatch", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-101, No. 2, pp. 381-388, 1982.
- [7] D. W. Wells, "Methods for economic secure loading of a power system", *Proceeding IEE*, Vol. 115, No. 8, pp. 1190-1194, 1968.
- [8] C. M. Shen and M. A. Laughton, "Power system load scheduling with security constraints using dual linear programming", *Proceeding IEE*, Vol. 117, No. 1, pp. 2117-2127, 1970.
- [9] R. E. Bellman and S. E. Dreyfus, "*Applied dynamic programming*", Princeton University Press, Princeton, NJ, 1962.
- [10] F. C. Lu and Y. Y. Hsu, "Reactive power/voltage control in a distribution substation using dynamic programming", *IEE Proceeding Generation, Transmission and Distribution*, Vol. 142, No. 6, pp. 639-645, 1995.
- [11] Y. Wei, L. Fang, C. Y. Chung and K. P. Wong, "A hybrid genetic algorithm-interior point method for optimal reactive power flow", *IEEE Trans Power Syst*, Vol. 21, No.3, pp.1163–1169, 2006.
- [12] L. Zhihuan, L. Yinhong and D. Xianzhong, "Non-dominated sorting genetic algorithm- II for robust multi-objective optimal reactive power dispatch", *Gener, Transm Distrib IET*, Vol. 4, No. 9, pp. 1000–1008, 2010.
- [13] L. Keyan, S. Wanxing and L. Yunhua, "Research on reactive power optimization based on adaptive genetic simulated annealing algorithm", *Int conference power syst tech, PowerCon*, pp.1–6, 2006.
- [14] L. Guo, X. Ding, G. Chen, J. Song, Q. Cui and W Liu, "A combination strategy for reactive power optimization based on model of soft constrain considered interior point method and genetic-simulated annealing algorithm", *Int conference information science manag engineering*, pp. 151–154, 2010.
- [15] K. P. Wong and Y. W. Wong, "Short term hydrothermal scheduling, Part I: simulated annealing approach", *IEE Proceeding Generation, Transmission, Distribution*, Vol. 141, No. 5, pp. 497-501, 1994.
- [16] L. Wennan, L. Yihua, X. Xingtao and I. Maojun, "Reactive power optimization in area power grid based on improved Tabu search algorithm", *Int conference on electric utility deregulation restructuring power tech*, pp. 1472–1477, 2008.
- [17] Y. Zou, "Optimal reactive power planning based on improved Tabu search Algorithm", *Int conference electrical control engi (ICECE)*, pp.3945–8, 2010.
- [18] W. Shouzheng, M. Lixin and S. Dashuai, "Hybrid differential evolution particle swarm optimization algorithm for reactive power optimization", *Int Power energy engineering conference (APPEEC) Asia-Pacific*, pp. 1–4, 2010.
- [19] H. Chao-Ming, C. Shin-Ju, H. Yann-Chang and Y Sung-Pei, "Optimal active-reactive power dispatch using an enhanced differential evolution algorithm", *IEEE Int conference industrial electronics applications (ICIEA)*, pp. 1869–74, 2011.
- [20] H. I. Shaheen, G. I. Rashed, and S.J. Cheng, "Optimal location and parameter setting of UPFC for enhancing power system security based on Differential Evolution algorithm", *Electrical Power and Energy Systems*, Vol. 33, No.1, pp. 94–105, 2011.
- [21] B. Zhao, C. X. Guo and Y. J. Cao, "A multiagent-based particle swarm optimization approach for optimal reactive power dispatch", *IEEE Trans Power Syst*, Vol. 20, No. 2, pp. 1070–1078, 2005.
- [22] D. Karaboga and B. Basturk, "A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm", *J Glob Optimiz*, Vol. 39, No. 3, pp. 459–71, 2007.
- [23] S. J. Huang, "Enhancement of hydroelectric generation scheduling using ant colony system based optimization approaches", *IEEE Transactions on Energy Conversion*, Vol. 16, No. 3, pp. 296-301, 2001.
- [24] S. A. Taher and M. K. Amooshahi, "New approach for optimal UPFC placement using hybrid immune algorithm in electric power systems", *Electrical Power and Energy Systems*, Vol. 43, No. 1, pp. 899–909, 2012.
- [25] S. Mondal, A. Bhattacharya and S. Halder nee Dey, "Multi-objective economic emission load dispatch solution using gravitational search algorithm and considering wind power penetration", *Electrical Power and Energy Systems*, Vol. 44, pp. 282–292, 2013.
- [26] E. Rashedi, H. Nezamabadi-pour and S. Saryazdi, "GSA: A gravitational search algorithm", *Inform Sci*, Vol. 179, No. 13, pp. 2232-2248, 2009.
- [27] C. Dai, W. Chen, Y. Zhu and X. Zhang, "Reactive power dispatch considering voltage stability with seeker optimization algorithm", *Int J Electr Power Syst Res*, Vol. 79, No. 10, pp. 1462-1471, 2009.
- [28] R. D. Zimmerman, C. E. Murillo-Sanchez and D. Gan, "Matlab power system simulation package (version 3.1b2)", 2006, available at: <http://www.pserc.cornell.edu/matpower/>.
- [29] D. Holliday, R. Resnick and J. Walker, "Gravitation," in *Fundamentals of physics*, John Wiley and Sons, pp. 411-442, 1993.
- [30] I. R. Kenyon, "Gravitational radiation," In *General Relativity*, Oxford University Press, pp. 124-143, 1990.
- [31] Roy, P.K., Ghoshal, S.P. and Thakur, S.S. (2010b) 'Optimal power flow using biogeography based optimization,' *International Journal of Power and Energy Conversion*, Vol. 2, No. 3, pp. 216–249.

- [32] Mojtaba Ghasemi, Sahand Ghavidel, Mohsen Gitizadeh a, Ebrahim Akbari bAn improved teaching–learning-based optimization algorithm using Lévy mutation strategy for non-smooth optimal power flow, *Electrical Power and Energy Systems* 65 (2015) 375–384.
- [33] Selvakumar AI, Thanushkodi K. A new particle swarm optimization solution to nonconvex economic dispatch problems. *IEEE Trans Power Syst*, 2007;22: 42–51.
- [34] Hsiao Y-T, Chen C-H, Chien C-C. Optimal capacitor placement in distribution systems using a combination fuzzy-GA method. *Int J Electric Power Energy Syst* 2004;26:501–8.
- [35] Gitizadeh M, Ghavidel S, Aghaei J. Using SVC to economically improve transient stability in long transmission lines. *IETE J Res* 2014;60:319–27.
- [36] Rezaei Adaryani M, Karami A. Artificial bee colony algorithm for solving multiobjective optimal power flow problem. *Int J Electric Power Energy Syst* 2013; 53:219–30.
- [37] Sayah S, Zehar K. Modified differential evolution algorithm for optimal power flow with non-smooth cost functions. *Energy Convers Manage* 2008;49: 3036–42.