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Fixed Point Theorems in Random Fuzzy Metric Space through Rational Expression

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Abstract—

In this paper we will find some fixed point theorems in random fuzzy metric space & expend random fuzzy2-metric space and random fuzzy3-metric space through rational expression

Keywords— Fixed Point Theorems, Random Fuzzy Metric Space

I. INTRODUCTION

In 1965, the concept of fuzzy set was introduced by Zadeh[39]. After him many authors have developed the theory of fuzzy sets and applications. Fixed Point Theorems in Random Fuzzy Metric Space by Rajesh Shrivastava[40],Rajesh Shrivastav [42] and new application Computer Engineering by R.P. Dubey [41]& Especially, Deng[9], Erceg[11], Kaleva and seikkala[26]. Kramosil and Michalek[28] have introduced the concept of fuzzy metric spaces by generalizing the definition of probabilistic metric space. Many authors have also studied the fixed point theory in these fuzzy metric space are [1], [7], [13], [19], [21], [24], [25], [32] and for fuzzy mappings [2],[3],[4],[5],[22],[31].

In 1994, George and Veeramani [18] modified the definition of fuzzy metric space given by kramosil and Michalek[28] in order to obtain Hausdroff topology in such spaces. Gregori and sapena[20] in 2002 extended Banach fixed point theorem to fuzzy contraction mapping on complete fuzzy metric space in the sense of George and Veeramani [18]. It is remarkable that Sharma, Sharma and Iseki[34] studied for the first time contraction type mapping in 2-metric space. Wenzhi[38] and many other initiated the study of Probabilistic 2-metric spaces. As we know that 2-metric space is a real valued function of a point triples on a set X, whose abstract properties were suggested by the area of function in Euclidean spaces.

Now it is natural to expect 3-metric space which is suggested by the volume function. The method of introducing this is naturally different from 2-metric space theory from algebraic topology.

The concept of Fuzzy-random-variable was introduced as an analogous notion to random variable in order to extend statistical analysis to situations when the outcomes of some random experiment are fuzzy sets. But in contrary to the classical statistical methods no unique definition has been established before the wok of Volker[37]. He presented set theoretical concept of fuzzy-random-variables using the method of general topology and drawing on results from topological measure theory and the theory of analytic spaces. No results in fixed point are introduced in random fuzzy spaces. In [17] paper authors Gupta, Dhagat, shrivastava introduced the fuzzy random spaces and proved common fixed point theorem.

In the present paper we will find some fixed point theorems in random fuzzy metric space, random fuzzy 2-metric space and random fuzzy 3-metric space through rational expression. Also we will find the results for integral type mappings.

To start the main result we need some basic definitions.

II. DEFINITIONS

Definition 2. 1 (Kramosil and Michalek 1975)

A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is a t-norm if it satisfies the following conditions :

I. $*(1,a)=a,\ *(0,0)=0$
II. $*(a,b) = *(b,a)$
III. $*(c,d) \geq *(a,b)$ whenever $c \geq a$ and $d \geq b$
IV. $*(*(a,b),c) = *(a,*(b,c))$ where $a, b, c, d \in [0,1]$

Definition 2. 2: (Kramosil and Michalek 1975)

The 3-tuple (X,M,*) is said to be a fuzzy metric space if X is an arbitrary set * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:

(i) $M(x,y,0) = 0$
(ii) $M(x,y,t) = 1$ for all $t > 0$ iff $x = y$,
(iii) $M(x,y,t) = M(y,x,t)$
(iv) $M(x,y,t)*M(y,z,s) \leq M(x,z,t+s)$,
(v) $M(x,y,.):[0,\infty[\to[0,1]$ is left-continuous,

Where $x, y, z \in X$ and $t, s > 0$.

In order to introduce a Hausdroff topology on the fuzzy metric space, in (Kramosil and Michalek 1975) the following definition was introduced.

Definition 2.3(George and Veermani 1994)

The 3-tuple (X,M,*) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times]0,\infty[$ satisfying the following conditions :

(i) M(x,y,t) > 0

(ii) M(x,y,t) =1 iff x = y,

(iii) M(x,y,t) = M(y,x,t),

(iv) M(x,y,t)*M(y,z,s) ≤ M(x,z,t+s),

(v) M(x,y, .):]0,∞[→[0,1] is continuous, Where x, y, z ϵ X and t, s > 0.

Definition 2.4: (George and Veermani 1994)

In a metric space (X,d) the 3-tuple (X,Md,*) where Md(x,y,t)= t / (t+d(x,y)) and a*b = ab is a fuzzy metric space. This Md is called the standard fuzzy metric space induced by d.

Definition 2.5: (George and Sepene 2002)

Let (X,M,*) be a fuzzy metric space. A mapping f : X→X is fuzzy contractive if there exists 0 < k < 1 such that

$$\frac{1}{M(f(x), f(y),t)} - 1 \le K\left(\frac{1}{M(x,y,t)} - 1\right)$$

For each x ,y ϵ X and t > 0.

Definition 2.6: (George and Sepene 2002)

Let (X, M,*) be a fuzzy metric space. We will say that the sequence $\{x_n\}$ in X is fuzzy contractive if there exists k ϵ (0,1) such that

$$\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 \le K\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right) \text{ for all } t > 0,\ n \in N.$$

We recall that a sequence $\{x_n\}$ in a metric space (X,d) is said to be contractive if there exist 0<k<1 such that $d(X_{n+1}, X_{n+2}) \le kd(X_n, X_{n+1})$ for all n ϵ N.

Definition 2.7: (Kumar and chugh 2001)

Let (X, τ) be a topological space. Let f and g be mapping from a topological space (X,τ) into itself. The mapping f and g are said to be compatible if the following conditions are satisfied:

(i) fx = gx , x ϵ X implies fgx =gfx ,

(ii) The continuity of f at a point x in X implies lim gfxn =fx whenever $\{x_n\}$ is a sequence in X such that lim gxn=lim fxn=fx for some x in X.

Definition 2.8: A binary operation * :[0,1]×[0,1]×[0,1]→[0,1] is called a continuous t-norm if ([0,1],*) is an abelian topological monoid with unit 1 uch that $a_1 * b_1 * c_1 \le a_2 * b_2 * c_2$ whenever $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$ for all a_1,a_2, b_1,b_2 and c_1 ,c_2 are in [0,1].

Definition 2.9: The 3-tuple (X,M,*) is called a fuzzy 2-metric space if X is an arbitrary set,* is a continuous t-norm and M is a fuzzy set in $X^3 \times [0,\infty)$ satisfying the following conditions for all x, y, z, u ϵ X and t_1, t_2, t_3 >0.

(FM'-1) M(x,y,z,0) = 0,

(FM'-2) M(x,y,z,t) = 1, t > 0 and when at least two of the three points are equal,

(FM'-3) M(x, y, z, t) = M(x, z, y, t) =M(y, z, x, t),
(Symmetry about three variables)

(FM'-4) $M(x, y, z, t_1+t_2+t_3) \ge M(x, y, u, t_1)*M(x, u, z, t_2)*M(u,y,z,t_3)$
(This corresponds to tetrahedron inequality in 2-metric space)

The function value M(x, y, z, t) may be interpreted as the probability that the area of triangle is less than t.

(FM'-5) M(x, y, z, .): [0,1)→[0,1] is left continuous.

Definition 2.10: Let (X, M, *) is a fuzzy 2-metric space:

(i) A sequence $\{x_n\}$ in fuzzy 2-metric space X is said to be convergent to a point x ϵ X if

$$\lim_{n\to\infty} M\left(x_n, x, a, t\right) = 1 \text{ For all } a \in X \text{ and } t > 0.$$

(ii) A sequence $\{x_n\}$ in fuzzy 2-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M\left(x_{n+p}, x_n, a, t\right) = 1 \text{ For all } a \in X \text{ and } t > 0, p > 0.$$

(iii) A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.11: A function M is continuous in fuzzy 2-metric space iff whenever $x_n \to x$, $y_n \to y$, then $\lim_{n\to\infty} M(x_n, y_n, a, t) = M(x, y, a, t)$ For a ϵ X and t > 0.

Definition 2.12: Two mapping A and S on fuzzy 2-metric space X are weakly commuting iff

M(Asu, Sau, a, t) ≥ M(Au,Su,a,t) For all u, a ϵ X and t > 0.

Definition 2.13: A binary operation * :$[0,1]^4 \to [0,1]$ is called a continuous t-norms if ([0,1],*) is an abelian topological monoid with unit 1 such that $a_1*b_1*c_1*d_1 \le a_2*b_2*c_2*d_2$ whenever $a_1 \le a_2$,$b_1 \le b_2$, $c_1 \le c_2$ and $d_1 \le d_2$ for all a_1,a_2,b_1,b_2,c_1,c_2 and d_1,d_2 are in [0,1].

Definition 2.14: The 3-tuple (X, M, *) is called a fuzzy 3-metric space if X is an arbitrary set,* is a continuous t-norm and M is fuzzy set in $X^4 \times [0,\infty)$ satisfying the following conditions: for all x, y, z, w, u ϵ X and $t_1, t_2, t_3, t_4 > 0$.

(FM' '-1)M(x, y, z, w, 0) = 0.

(FM' '-2)M(x, y, z, w, t) = 1 for all t > 0,

(only when the three simplex <x, y, z, w > degenerate)

(FM' '-3)M(x, y, z, w, t) = M(x, w, z, y, t) = M(y, z, w, x, t) = M (z, w, x, y, t) = ...

(FM' '-4)$M(x, y, z, w, t_1+t_2+t_3+t_4) \geq M(x, y, z, u, t_1)*M(x, y, u, w, t_2)$

$*M(x, u\, z, w, t_3)*M(u, y, z, w, t_4)$

(FM' '-5)M(x, y, z, w, .):[0,1)→[0,1] is left continuous.

Definition 2.15: Let (X, M, *) be a fuzzy 3-metric space:

(1) A sequence $\{x_n\}$ in fuzzy 3-metric space x is said to be convergent to a point x ε X, if

$$\lim_{n\to\infty} M\left(x_n, x, a, b, t\right) = 1$$

for all a, b ϵ X and t > 0.

(2) A sequence $\{x_n\}$ in fuzzy 3-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M\left(x_{n+p}, x_n, a, b, t\right) = 1$$

for all a, b ϵ X and t > 0 ,p > 0.

(3) A fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.16: A function M is continuous in fuzzy 3-metric space iff whenever $X_n \to X$, $Y_n \to Y$.

$$\lim_{n\to\infty} M\left(x_n,\ y_n,\ a,\ b, t\right) = M(x,\ y, a,\ \ b,\ t)$$

for all a,b ϵ X and t > 0 .

Definition 2.17: Two mapping A and S on fuzzy 3-metric space X are weakly commuting iff

M(ASu, SAu, a, b, t) ≥ M(Au, Su, a, b, t)

for all u, a, b ϵ X and t > 0.

Definition2.18: Throughout this chapter, (Ω, Σ) denotes a measurable space ξ: Ω→X is a measurable selector. X is any one non empty set,* is continuous t-norm, M is a fuzzy set in $X^2 \to [0,\infty)$.

A binary operation *:[0,1]×[0,1]→[0,1] is called a continuous t-norm if ([0,1],*) is an abelian topological monodies with unit 1 such that a*b ≥ c*d whenever a ≥ c and b ≥ d, for all a, b, c, d ϵ [0,1].

Example of t-norm are a* b = a b and a*b = min {a,b}.

Definition2.18 (a): The 3-tuple (X, M, Ω,*) is called a Random fuzzy metric space, if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X^2 \to [0, \infty)$ satisfying the following condition: for all ξ x, ξ y, ξ z ϵ X and s, t > 0

(RMF-1): M (ξ x, ξ y, 0) = 0

(RMF-2): M (ξ x, ξ y, t) = 1, ∀ t > 0, ⇔ x = y

(RMF-3): M (ξ x, ξ y, t) =M (ξ y, ξ x, t)

(RMF-4): M (ξ x, ξ z, t + s) ≥ M (ξ x, ξ y, t) *M(ξ z, ξ y, s)

(RMF-5): M (ξ x, ξ y, ξ a): [0, 1) → [0, 1] is left continuous.

In what follows (X,M,Ω, *) will denote a random fuzzy metric space. Note that M(ξ x, ξ y, t) can be thought of as the degree of nearness between ξ x and ξ y with respect to t. we identify ξ x= ξ y with M(ξ x, y, t) =1 for all t > 0 and M(ξ x, ξ y, t) = 0 with ∞. in the following example , we know that ξ every metric induces a fuzzy metric.

Example: Let (X, d) be a metric space.

Define a*b = a b or ab = min {a, b} and for all x, y ∊ X and t > 0 .

$$M(\xi\ x,\ \xi\ y,\ t) = \frac{t}{t + d(\xi\ x,\ \xi\ y)}$$

Then (X, M, Ω, *) is a fuzzy metric space. We call this random fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition2.18 (b): Let (X, M, Ω, *) is random fuzzy metric space.

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point ξ x ∊ X,

$$\lim_{n\to\infty} M\left(\xi x_n, \xi x, t\right) = 1.$$

(ii) A sequence $\{\xi x_n\}$ in X is said to be Cauchy sequence if $\lim_{n\to\infty} M\left(\xi x_{n+p}, \xi x_n, t\right) = 1, \forall$ t > 0 and p > 0 .

(iii) A random fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Let (X, M, *) is a fuzzy metric space with the following condition.

(RMF-6) $\lim_{n\to\infty} M\left(\xi x, \xi y, t\right) = 1, \forall \xi x, \xi y \in X$

Definition 2.18.(C): A function M is continuous in fuzzy metric space iff whenever $\xi x_n \to \xi x, \xi y_n \to \xi y \Rightarrow \lim_{n\to\infty} M(\xi x_n \xi y_n, t) \to M(\xi x, \xi y, t)$.

Definition 2.18(d): Two mapping A and S on fuzzy metric space X are weakly commuting iff
M (As ξ u, SA ξu , t) ≥ M (A $\xi u, S\xi u, t$).

Some Basic Results 2.2.1.18(e):

Lemma (i)[Motivated by 19] $\xi x,\ \xi y \in X, M(\ \xi x,\ \xi y)$ for all is non decreasing.

Lemma (ii) Let { ξy_n } be a sequence in a random fuzzy metric space (X, M, Ω,*) with the condition.

(RFM-6) If there exists a number q ∊ (0,1) such that $M(\ \xi\ y_{n+2},\ \xi\ y_{n+1}, qt) \ge M(\ \xi\ y_{n+1},\ \xi\ y_n, t), \forall\ t > 0\, and\, n = 1,2,3,\ldots$ then { ξy_n } is a Cauchy Sequence in X.

Lemma (iii) [Motivated by 32] If, for all and for a number q ∊ (0,1) $M(\ \xi x,\ \xi y, qt) \ge M(\ \xi x,\ \xi y, t)$ then $\xi x = \ \xi y.$

Lemma 1, 2, 3 of 2.2.1.18 (e): hold for random fuzzy 2-metric space and random fuzzy 3-metric space also.

Definition 2.18(f): A binary operation *:[0,1]×[0,1]×[0,1]→[0,1] is called a continuous t-norm if ([0,1],*) is an abelian topological monodies with unit 1 ξ x, $\xi y \in X, t > 0$ such that $a_1*b_1*c_1 \ge a_2*b_2*c_2$ whenever $a_1 \ge a_2$, $b_1 \ge b_2$, $c_1 \ge c_2$ for all a_1, a_2, b_1 , b_2 ,c_1,c_2 are in [0,1].

Definition 2.18(g): The 3-tuple (X, M,Ω,*) is called a random fuzzy 2-metric space if X is an arbitrary set, * is continuous t-norm and M is fuzzy set in $X^3 \times [0,\infty)$ satisfying the followings (RFM'-1): M($\xi x, \xi y, \xi z, 0$)=0

(RFM'-2): M ($\xi x, \xi y, \xi z, t$) =1, $\forall\ t > 0 \Leftrightarrow x = y$

(RFM'-3): M ($\xi x, \xi y, \xi z, t$) =M ($\xi x, \xi z, \xi y, t$) =M ($\xi y, \xi z, \xi x, t$) , symmetry about three variable

(RFM'4): M $(\xi x, \xi y, \xi z, t_1, t_2, t_3) \ge M(\xi x, \xi y, \xi u, t_1) * M(\xi x, \xi u, \xi z, t_2) * M(\xi u, \xi y, \xi z, t_3)$

(RFM'-5): M ($\xi x, \xi y, \xi z)$:[0,1)→[0,1] is left continuous, $\forall\ \ \xi x, \xi y, \xi z, \xi u \in X, t_1, t_2, t_3 > 0$.

Definition2.18 (h): Let (X, M, Ω) be a random fuzzy 2-metric space.

(1) A sequence {} in fuzzy 2-metric space X is said to be convergent to a point $\xi x_n\ \xi x \in X, \lim_{n\to\infty} M(\xi \chi_n, \xi x, \xi a, t) = 1$,for all $\xi a \in X\ and\ t, p > 0.$

(2) A sequence {} in random fuzzy 2-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(\xi \chi_{n+p}, \xi \chi_n, \xi a, t) = 1 \text{ For all } \xi a \in X\ and\ t, p > 0.$$

(3) A random fuzzy 2 metric space in which every Cauchy sequence is convergent is said to be complete.

Definition2.18 (i): A function M is continuous in random fuzzy 2-metric space, iff whenever for all $\xi a \in X\ and\ t > 0.$

$$\xi x_n \to \xi\ x, \xi y_n \to \xi\ y, then\ \lim_{n\to\infty} M(\xi x_n, \xi y_n, \xi a, t) = M(\xi x, \xi y, \xi a, t), \forall\ \xi a \in X\ and\ t > 0.$$

Definition: 2.18(j): Two mapping A and S on random fuzzy 2-metric space X are weakly commuting iff M ($AS\xi u, SA\xi u, \xi a, t$) ≥ M ($A\xi u, S\xi u, \xi a, t$),$\forall\ \xi u, \xi a\ \in X\ and\ t > 0.$

Definition: 2.18(K): A binary operation *: $[0, 1]^4 \to [0, 1]$ is called a continuous t-norm if ([0, 1],*) is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 * d_1 \ge a_2 * b_2 * c_2 * d_2$ whenever $a_1 \ge a_2, b_1 \ge b_2, c_1 \ge c_2\ and\ d_1 \ge d_2$ for all $a_1, a_2, b_1, b_2, c_1, c_2\ and\ d_1, d_2\ are\ in\ [0,1].$

Definition: 2.18(l): The 3-tuple (X, M, Ω,*) is called a fuzzy 3-metric space if X is an arbitrary set,* is a continuous t-norms monoid and M is a fuzzy set in $X^4 \times [0,\infty]$ satisfying the following conductions:

(RFM"-1): M ($\xi x, \xi y, \xi z, \xi w, 0$) =0

(RFM"-2): M ($\xi x, \xi y, \xi z, \xi w, t$) = 1, $\forall$ t >0,

Only when the three simplex < x, y, z, w > degenerate

(RFM"-3): M ($\xi x, \xi y, \xi z, \xi w, t$) = M ($\xi x, \xi w, \xi z, \xi y, t$) = M ($\xi z, \xi w, \xi x, \xi y, t$) =- - - - -

(RFM"-4): M ($\xi x, \xi y, \xi z, \xi w, t + t_2 + t_3$) ≥ M ($\xi x, \xi y, \xi z, \xi u, t_1) * M(\xi x, \xi y, \xi u, \xi w, t_2) * M(\xi x, \xi u, \xi z, \xi w, t_3) * M(\xi u, \xi y, \xi z, \xi w, t_4)$

(RFM"-5):M($\xi x, \xi y, \xi z, \xi w$):[0,1] →[0,1] is left continuous, $\forall\ \xi x, \xi y, \xi z, \xi u, \xi w\ \in X\ , t_1, t_2, t_3, t_4 > 0.$

Definition 2.18 (m): Let (X, M, Ω,*) be a Random fuzzy 3-metric space:

(1) A Sequence { ξX_n } in fuzzy 3-metric space X is said to be convergent to a point $\xi x \in X$, if $\lim_{n\to\infty} M(\xi \chi_n, \xi x, \xi a, \xi b, t) = 1$, for all $\xi a, \xi b \in X\ and\ t > 0.$

(2) A Sequence $\{\xi X_n\}$ in random fuzzy 3-metric space X is called a Cauchy sequence ,if

$$\lim_{n\to\infty} M(\xi x_{n+p}, \xi x_n, \xi a, \xi b, t) = 1, \text{ for all } \xi a, \xi b \in X \text{ and } t, p > 0$$

(3) A random fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.18 (n): A function M is continuous in random fuzzy 3-metric space if $\xi x_n \to \xi x, \xi y_n \to \xi y$ then

$$\lim_{n\to\infty} M(\xi x_n, \xi y_n, \xi a, \xi b, t) = M(\xi x, \xi y, \xi a, t), \forall \xi a, \xi b \in X \quad \text{and } t > 0.$$

Definition 2.18 (o): Two mappings A and S on random fuzzy 3-metric space X are weakly commuting iff $M\left(AS\xi u, SA\xi u, \xi a, \xi b, t\right) \geq M\left(A\xi u, S\xi u, \xi a, \xi b, t\right) \forall u, a, b \in X and t > 0$.

III. PREPOSITIONS

Preposition 3.1(Gregori and Sepene 2002)
Let (X, d) be a metric space. The mapping f: X→X is contractive (a contraction) on the metric space (X, d) with contractive constant k iff f is fuzzy contractive, with contractive constant k, on the standard fuzzy metric space (X, Md,*), induced by d.

Preposition 3.2(Gregori and Sepene 2002)
Let (X, M,*) be a complete fuzzy metric space in which fuzzy contractive sequence are Cauchy. Let T: X→ X be a fuzzy contractive mapping being k the contractive constant. Then T has a unique fixed point.

Preposition 3.3(Gregori and Sepene 2002)
Let (X, Md,*) be the standard fuzzy metric space induced by the metric d on X. The sequence $\{x_n\}$ in X is contractive in (X, d) iff $\{x_n\}$ is fuzzy contractive in (X, Md,*).

Preposition 2.2.2.1 and 2.2.2.3 imply that Preposition 2.2.2.2 is a generalization of Banach fixed point theorem to fuzzy metric space as defined by George and Veermani.

It is to be noted that all the prepositions are true for (RFM)

Now, we state and prove our main theorem as follows,

IV. MAIN RESULTS

Theorem 4.1: Let (X, Ω, M, *) be a complete Random fuzzy metric space in which fuzzy contractive sequences are Cauchy and T,A and B be mapping from (X, Ω, M, *) into itself $\xi : \Omega \to X$ is a measurable selector satisfying the following conditions:

$$T(X) \subseteq A(X) and T(X) \subseteq B(X) \qquad (2.3.1.1)$$

$$\frac{1}{M\left(T(\xi x), T(\xi y), t\right)} - 1 \leq k\left(\frac{1}{Q(\xi x, \xi y, t)} - 1\right) \qquad (2.3.1.2)$$

With 0 < k < 1 and

$$Q(\xi x, \xi y, t) = \min\left\{\begin{array}{l} M(A\xi x, B\xi y, t), M(B\xi x, A\xi y, t), M(A\xi x, T\xi x, t), \\ \dfrac{M(B\xi x, A\xi y, t) M(A\xi x, T\xi x, t)}{M(A\xi x, B\xi y, t)}, \dfrac{M(B\xi x, T\xi x, t) M(B\xi y, T\xi y, t)}{M(A\xi y, T\xi y, t)} \end{array}\right\}$$

The pairs T, B and T, A are compatible A, T and B are W-continuous. (2.3.1.3)

Then A,T and B have a unique common fixed point. (2.3.1.4)

Proof: Let $\xi x_0 \in X$ be an arbitrary point. Since $T(x) \subseteq A(x) and T(x) \subseteq B(x)$,we can construct a sequence $\{x_n\}$ in X such that

$$T\xi x_{n-1} = A\xi x_n = B\xi x_n \qquad (2.3.1.5)$$

$$Q(\xi x_n, \xi x_{n+1}, t) = \min\left\{\begin{array}{l} M(A\xi x_n, B\xi x_{n+1}, t), M(B\xi x_n, A\xi x_{n+1}, t), M(A\xi x_n, T\xi x_n, t), \\ \dfrac{M(B\xi x_n, A\xi x_{n+1}, t) M(A\xi x_n, T\xi x_n, t)}{M(A\xi x_n, B\xi x_{n+1}, t)}, \dfrac{M(B\xi x_n, T\xi x_n, t) M(B\xi x_{n+1}, T\xi x_{n+1}, t)}{M(A\xi x_{n+1}, T\xi x_{n+1}, t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{l} M(T\xi x_{n-1}, T\xi x_n, t), M(T\xi x_{n-1}, T\xi x_n, t), M(T\xi x_{n+1}, T\xi x_n, t), \\ \dfrac{M(T\xi x_{n-1}, T\xi x_n, t) M(T\xi x_{n+1}, T\xi x_n, t)}{M(T\xi x_{n-1}, T\xi x_n, t)}, \dfrac{M(T\xi x_{n-1}, T\xi x_n, t) M(T\xi x_n, T\xi x_{n+1}, t)}{M(T\xi x_n, T\xi x_{n+1}, t)} \end{array}\right\}$$

$$= \min\left\{M(T\xi x_{n-1}, T\xi x_n, t), M(T\xi x_n, T\xi x_{n+1}, t)\right\}$$

We now claim that $M(T\xi x_{n-1}, T\xi x_n, t) < M(T\xi x_n, T\xi x_{n+1}, t)$

Otherwise we claim that $M(T\xi x_{n-1}, T\xi x_n, t) \geq M(T\xi x_n, T\xi x_{n+1}, t)$

i.e $Q(\xi\chi_n, \xi\chi_{n+1}, t) = M(T\xi\chi_n, T\xi\chi_{n+1}, t)$ (2.3.1.6)

$$\therefore \quad \frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, t)} - 1\right) \quad \text{[by2.3.1.2]}$$

This is a contradiction.

Hence, $$\frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, t)} - 1\right) \quad (2.3.1.7)$$

$\therefore$ {Tξx_n} is a fuzzy contractive sequence in (X,Ω, M,*) So {Tξx_n} is a Cauchy sequence .

As X is a complete fuzzy metric space, {Tξx_{n-1}} is convergent. So, {Tξx_{n-1}} converges to some point z in X.

$\therefore$ {Tξx_{n-1}},{Rξx_n},{Sξx_n} converges to z. By W-continuity of R,S and T, there exists a point ξu in ξX such that $\xi x_n \to \xi u \ as\ n \to \infty$ and so $lim\ R\xi x_n = lim\ S\xi x_n = lim\ T\xi x_{n-1} = z\ implies$

$$A\xi u = B\xi u = T\xi u = \xi z \quad (2.3.1.8)$$

Also by compatibility of pair T,A and T,B and Tu =Au = Bu = z implies

$$T\xi z = TA\xi u = AT\xi u = A\xi z \text{ and } T\xi z = TB\xi u = BT\xi u = S\xi z$$

Therefore, $T\xi z = A\xi z = B\xi z$ (2.3.1.9)

We now claim that $T\xi z = \xi z$.

If not $$\frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, t)} - 1\right)$$

$$Q(\xi z, \xi u, t) = \min\left\{\begin{matrix} M(A\xi z, B\xi u, t), M(B\xi z, A\xi u, t), M(A\xi z, T\xi z, t), \\ \frac{M(B\xi z, A\xi u, t)M(A\xi z, T\xi z, t)}{M(A\xi z, B\xi u, t)}, \frac{M(B\xi z, T\xi z, t)M(B\xi u, T\xi u, t)}{M(A\xi u, T\xi u, t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(T\xi z, \xi z, t), M(T\xi z, \xi z, t), M(T\xi z, T\xi z, t), \\ \frac{M(T\xi z, \xi z, t)M(T\xi z, T\xi z, t)}{M(T\xi z, \xi z, t)}, \frac{M(T\xi z, T\xi z, t)M(\xi z, T\xi z, t)}{M(\xi z, \xi z, t)} \end{matrix}\right\}$$

$$= \min\{M(T\xi z, \xi z, t)M(T\xi z, \xi z, t), 1, 1, 1\}$$

$$= M(T\xi z, \xi z, t)$$

$$\therefore \quad \frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, t)} - 1\right)$$

This is a contradiction.

Hence $T\xi z = \xi z$

So ξz is a common fixed point of A, T and B.

Now suppose ξv$\neq$ ξz be another fixed point of A,T and B, T.

$$\therefore \quad \frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, t)} - 1\right)$$

$$Q(\xi v, \xi u, t) = \min\left\{\begin{matrix} M(A\xi v, B\xi z, t), M(B\xi v, A\xi z, t), M(A\xi v, T\xi v, t), \\ \frac{M(B\xi v, A\xi z, t)M(A\xi v, T\xi v, t)}{M(A\xi v, B\xi z, t)}, \frac{M(B\xi v, T\xi v, t)M(B\xi z, T\xi z, t)}{M(A\xi z, T\xi z, t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(\xi v, \xi z, t), M(\xi v, \xi z, t), M(\xi v, \xi v, t), \\ \frac{M(\xi v, \xi z, t)M(\xi v, \xi v, t)}{M(v, z, t)}, \frac{M(\xi v, \xi v, t)M(\xi z, \xi z, t)}{M(z, z, t)} \end{matrix}\right\}$$

$$= \min\{M(\xi v, \xi z, t)M(\xi v, \xi z, t), 1, 1, 1\}$$

$$= M(\xi v, \xi z, t)$$

$$\therefore \quad \frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, t)} - 1\right)$$

This is a contradiction. Hence $\xi v = \xi z$.

Thus A, T and B have a unique fixed point.

Theorem 4.2: Let (X, Ω, M, *) be a complete Random fuzzy 2-metric space (RF-2M) in which fuzzy contractive sequences are Cauchy and T,R and S be mapping from (X, Ω, M, *) into itself $\xi: \Omega \to X$ is a measurable selector and a(ξ)=a > 0 satisfying the following conditions:

$$T(X) \subseteq A(X) and T(X) \subseteq B(X) \qquad (2.3.2.1)$$

$$\frac{1}{M\left(T(\xi x), T(\xi y), \xi a, t\right)} - 1 \le k\left(\frac{1}{Q(\xi x, \xi y, \xi a, t)} - 1\right) \qquad (2.3.2.2)$$

With 0 < k < 1 and

$$Q(\xi x,\xi y,\xi a,t) = \min\left\{\begin{matrix} M(A\xi x, B\xi y, \xi a, t), M(B\xi x, A\xi y, \xi a, t), M(A\xi x, T\xi x, \xi a, t), \\ \frac{M(B\xi x, A\xi y, \xi a, t) M(A\xi x, T\xi x, \xi a, t)}{M(A\xi x, B\xi y, \xi a, t)}, \frac{M(B\xi x, T\xi x, \xi a, t) M(B\xi y, T\xi y, \xi a, t)}{M(A\xi y, T\xi y, \xi a, t)} \end{matrix}\right\}$$

The pairs T, B and T, A are compatible. A, T and B are W-continuous. (2.3.2.3)
Then A, T and B have a unique common fixed point. (2.3.2.4)

Proof: Let $\xi x_0 \in X$ be an arbitrary point. Since $T(x) \subseteq A(x) and T(x) \subseteq B(x)$,we can construct a sequence $\{x_n\}$ in X such that

$$T\xi \chi_{n-1} = A\xi \chi_n = B\xi \chi_n \qquad (2.3.2.5)$$

$$Q(\xi x,\xi y,\xi a,t) = \min\left\{\begin{matrix} M(A\xi x, B\xi y, \xi a, t), M(B\xi x, A\xi y, \xi a, t), M(A\xi x, T\xi x, \xi a, t), \\ \frac{M(B\xi x, A\xi y, \xi a, t) M(A\xi x, T\xi x, \xi a, t)}{M(A\xi x, B\xi y, \xi a, t)}, \frac{M(B\xi x, T\xi x, \xi a, t) M(B\xi y, T\xi y, \xi a, t)}{M(A\xi y, T\xi y, \xi a, t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t), M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t), M(T\xi \chi_{n+1}, T\xi \chi_n, \xi a, t), \\ \frac{M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t) M(T\xi \chi_{n+1}, T\xi \chi_n, \xi a, t)}{M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t)}, \frac{M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t) M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t)}{M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t)} \end{matrix}\right\}$$

$$= \min\left\{M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t), M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t)\right\}$$

We now claim that $M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t) < M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t)$

Otherwise we claim that $M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t) \ge M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t)$

i.e $\quad Q(\xi \chi_n, \xi \chi_{n+1}, \xi a, t) = M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t) \qquad (2.3.2.6)$

$$\therefore \quad \frac{1}{M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t)} - 1 \le k\left(\frac{1}{M(T\xi \chi_n, T\xi \chi_{n+1} \xi a,, t)} - 1\right) \qquad \text{[by2.3.2.2]}$$

This is a contradiction.

Hence, $\frac{1}{M(T\xi \chi_n, T\xi \chi_{n+1}, \xi a, t)} - 1 \le k\left(\frac{1}{M(T\xi \chi_{n-1}, T\xi \chi_n, \xi a, t)} - 1\right) \qquad (2.3.2.7)$

∴ {Tξx_n} is a fuzzy contractive sequence in (X,Ω, M,*)(X,M,*) So {T(X,Ω, M,*)x_n} is a Cauchy sequence in (X,Ω, M,*).

As X is a complete Random fuzzy 2-metric space, {Tξx_{n-1}} is convergent. So, {Tξx_{n-1}} converges to some point ξz in X.

∴ {Tξx_{n-1}},{Aξx_n},{Bξx_n} converges to ξz. By W-continuity of A,B and T, there exists a point ξu in ξX such that $\xi x_n \to \xi u \; as \; n \to \infty$ and so $ln\, A\xi x_n = ln\, B\xi x_n = ln\, T\xi x_{n-1} = \xi z \; implies$

$$A\xi u = B\xi u = T\xi u = \xi z \qquad (2.3.2.8)$$

Also by compatibility of pair T,B and T,A and T ξu =Aξu = Bu = ξ z implies

$$T\xi z = TA\xi u = AT\xi u = A\xi z \text{ and } T\xi z = TB\xi u = BT\xi u = B\xi z$$

Therefore, $T\xi z = A\xi z = B\xi z \qquad (2.3.2.9)$

We now claim that $T\xi z = \xi z.$

If not $\frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, \xi a, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, \xi a, t)} - 1\right)$

$$Q(\xi z, \xi u, \xi a, t) = \min\left\{\begin{array}{l} M(A\xi z, B\xi u, \xi a, t), M(B\xi z, A\xi u, \xi a, t), M(A\xi z, T\xi z, \xi a, t), \\ \frac{M(B\xi z, A\xi u, \xi a, t)M(A\xi z, T\xi z, \xi a, t)}{M(A\xi z, B\xi u, \xi a, t)}, \frac{M(B\xi z, T\xi z, \xi a, t)M(B\xi u, T\xi u, \xi a, t)}{M(A\xi u, T\xi u, \xi a, t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{l} M(T\xi z, \xi z, \xi a, t), M(T\xi z, \xi z, \xi a, t), M(T\xi z, T\xi z, \xi a, t), \\ \frac{M(Tz, z, a, t)M(Tz, Tz, a, t)}{M(T\xi z, \xi z, \xi a, t)}, \frac{M(Tz, Tz, a, t)M(z, z, a, t)}{M(\xi z, \xi z, \xi a, t)} \end{array}\right\} \quad \therefore$$

$$= \min\{M(T\xi z, \xi z, \xi a, t)M(T\xi z, \xi z, \xi a, t), 1, 1, 1\}$$

$$= M(T\xi z, \xi z, \xi a, t)$$

$$\frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, \xi a, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, \xi a, t)} - 1\right)$$

This is a contradiction.
Hence $T\xi z = \xi z$
So ξz is a common fixed point of A, T and B.
Now suppose $\xi v \neq \xi z$ be another fixed point of A,T and B,T.

$$\therefore \quad \frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, \xi a, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, \xi a, t)} - 1\right)$$

$$Q(\xi v, \xi u, \xi a, t) = \min\left\{\begin{array}{l} M(A\xi v, B\xi z, \xi a, t), M(B\xi v, A\xi z, \xi a, t), M(A\xi v, T\xi v, \xi a, t), \\ \frac{M(B\xi v, A\xi z, \xi a, t)M(A\xi v, T\xi v, \xi a, t)}{M(A\xi v, B\xi z, \xi a, t)}, \frac{M(B\xi v, T\xi v, \xi a, t)M(B\xi z, T\xi z, \xi a, t)}{M(A\xi z, T\xi z, \xi a, t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{l} M(\xi v, \xi z, \xi a, t), M(\xi v, \xi z, \xi a, t), M(\xi v, \xi v, \xi a, t), \\ \frac{M(\xi v, \xi z, \xi a, t)M(\xi v, \xi v, \xi a, t)}{M(v, z, a, t)}, \frac{M(\xi v, \xi v, \xi a, t)M(\xi z, \xi z, \xi a, t)}{M(\xi z, \xi z, \xi a, t)} \end{array}\right\} \quad \therefore$$

$$= \min\{M(\xi v, \xi z, \xi a, t)M(\xi v, \xi z, \xi a, t), 1, 1, 1\}$$

$$= M(\xi v, \xi z, \xi a, t)$$

$$\frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, \xi a, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, \xi a, t)} - 1\right)$$

This is a contradiction. Hence $\xi v = \xi z$.
Thus R, T and S have unique common fixed point. This completes our proof.

Theorem 4.3: Let (X, Ω, M, *) be a complete Random fuzzy 3-metric space (RF-3M) in which fuzzy contractive sequences are Cauchy and T,R and S be mapping from (X, Ω, M, *) into itself $\xi : \Omega \to X$ is a measurable selector and ξa. ξb > 0 satisfying the following conditions:

$$T(X) \subseteq A(X) and T(X) \subseteq B(X) \qquad (2.3.3.1)$$

$$\frac{1}{M(T(\xi x), T(\xi y), \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{Q(\xi x, \xi y, \xi a, \xi b, t)} - 1\right) \qquad (2.3.3.2)$$

With 0 < k < 1 and

$$Q(\xi x, \xi y, \xi a, \xi b, t) =$$

$$\min\left\{\begin{array}{l} M(A\xi x, B\xi y, \xi a, \xi b, t), M(B\xi x, A\xi y, \xi a, \xi b, t), M(A\xi x, T\xi x, \xi a, \xi b, t), \\ \frac{M(B\xi x, A\xi y, \xi a, \xi b, t)M(A\xi x, T\xi x, \xi a, \xi b, t)}{M(A\xi x, B\xi y, \xi a, \xi b, t)}, \frac{M(B\xi x, T\xi x, \xi a, \xi b, t)M(B\xi y, T\xi y, \xi a, \xi b, t)}{M(A\xi y, T\xi y, \xi a, \xi b, t)} \end{array}\right\}$$

The pairs T, B and T, A are compatible. A, T and B are W-continuous. (2.3.3.3)
Then R, T and S have a unique common fixed point. (2.3.3.4)

Proof: Let $\xi x_0 \in X$ be an arbitrary point of X . Since $T(x) \subseteq A(x) and T(x) \subseteq B(x)$,we can construct a sequence $\{x_n\}$ in X such that

$$T\xi x_{n-1} = A\xi x_n = B\xi x_n \tag{2.3.3.5}$$

$$Q(\xi x, \xi y, \xi a, \xi b, t) =$$

$$\min\left\{\begin{matrix} M(A\xi x, B\xi y, \xi a, \xi b, t), M(B\xi x, A\xi y, \xi a, \xi b, t), M(A\xi x, T\xi x, \xi a, \xi b, t), \\ \dfrac{M(B\xi x, A\xi y, \xi a, \xi b, t) M(A\xi x, T\xi x, \xi a, \xi b, t)}{M(A\xi x, B\xi y, \xi a, \xi b, t)}, \dfrac{M(B\xi x, T\xi x, \xi a, \xi b, t) M(B\xi y, T\xi y, \xi a, \xi b, t)}{M(A\xi y, T\xi y, \xi a, \xi b, t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t), M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t), M(T\xi x_{n+1}, T\xi x_n, \xi a, \xi b, t), \\ \dfrac{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) M(T\xi x_{n+1}, T\xi x_n, \xi a, \xi b, t)}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)}, \dfrac{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} \end{matrix}\right\}$$

$$= \min\{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t), M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)\}$$

We now claim that $M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) < M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)$

Otherwise we claim that $M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) \geq M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)$

i.e $Q(\xi x_n, \xi x_{n+1}, \xi a, \xi b, t) = M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)$ (2.3.3.6)

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_n, T\xi x_{n+1} \xi a, \xi b, , t)} - 1\right) \quad \text{[by(2.3.3.2)]}$$

This is a contradiction.
Hence,

$$\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right) \tag{2.3.3.7}$$

$\therefore$ {Tξx_n} is a fuzzy contractive sequence in (X,Ω, M,*)(X,M,*) So {T(X,Ω, M,*)x_n} is a Cauchy sequence in (X,Ω, M,*).

As X is a complete Random fuzzy 3-metric space, {Tξx_{n-1}} is convergent. So, {Tξx_{n-1}} converges to some point ξz in X.

$\therefore$ {Tξx_{n-1}},{Aξx_n},{Bξx_n} converges to ξz. By W-continuity of A,B and T, there exists a point ξu in ξX such that $\xi x_n \to \xi u \text{ as } n \to \infty$ and so $\ln A\xi x_n = \ln B\xi x_n = \ln T\xi x_{n-1} = \xi z$ implies

$$A\xi u = B\xi u = T\xi u = \xi z \tag{2.3.3.8}$$

Also by compatibility of pair T,S and T,R and T ξu =Rξu = Su = ξ z implies

$$T\xi z = TA\xi u = AT\xi u = A\xi z \text{ and } T\xi z = TB\xi u = BT\xi u = B\xi z$$

Therefore, $T\xi z = A\xi z = B\xi z$ (2.3.3.9)

We now claim that $T\xi z = \xi z$.

If not $\quad \dfrac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \leq k\left(\dfrac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$

$$Q(\xi z, \xi u, \xi a, \xi b, t) =$$

$$\min\left\{\begin{matrix} M(A\xi z, B\xi u, \xi a, \xi b, t), M(B\xi z, A\xi u, \xi a, \xi b, t), M(A\xi z, T\xi z, \xi a, \xi b, t), \\ \dfrac{M(B\xi z, A\xi u, \xi a, \xi b, t) M(A\xi z, T\xi z, \xi a, \xi b, t)}{M(A\xi z, B\xi u, \xi a, \xi b, t)}, \dfrac{M(B\xi z, T\xi z, \xi a, \xi b, t) M(B\xi u, T\xi u, \xi a, \xi b, t)}{M(A\xi u, T\xi u, \xi a, \xi b, t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, T\xi z, \xi a, \xi b, t), \\ \dfrac{M(Tz, z, \xi a, \xi b, t) M(Tz, Tz, \xi a, \xi b, t)}{M(T\xi z, \xi z, \xi a, \xi b, t)}, \dfrac{M(Tz, Tz, \xi a, \xi b, t) M(\xi z, \xi z, \xi a, \xi b, t)}{M(\xi z, \xi z, \xi a, \xi b, t)} \end{matrix}\right\} \quad \therefore$$

$$= \min\{M(T\xi z, \xi z, \xi a, \xi b, t) M(T\xi z, \xi z, \xi a, \xi b, t), 1, 1, 1\}$$

$$= M(T\xi z, \xi z, \xi a, \xi b, t)$$

$$\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$$

This is a contradiction.
Hence $T\xi z = \xi z$
So ξz is a common fixed point of R, T and S.
Now suppose $\xi v \neq \xi z$ be another fixed point of R,T and

$$\therefore \quad \frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, \xi a, \xi b, t)} - 1\right)$$

$$Q(\xi v, \xi u, \xi a, \xi b, t)$$

$$= \min\left\{\begin{array}{l} M(A\xi v, B\xi z, \xi a, \xi b, t), M(B\xi v, A\xi z, \xi a, \xi b, t), M(A\xi v, T\xi v, \xi a, \xi b, t), \\ \frac{M(B\xi v, A\xi z, \xi a, \xi b, t) M(A\xi v, T\xi v, \xi a, \xi b, t)}{M(A\xi v, B\xi z, \xi a, \xi b, t)}, \frac{M(B\xi v, T\xi v, \xi a, \xi b, t) M(B\xi z, T\xi z, \xi a, \xi b, t)}{M(A\xi z, T\xi z, \xi a, \xi b, t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{l} M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi v, \xi a, \xi b, t), \\ \frac{M(\xi v, \xi z, \xi a, \xi b, t) M(\xi v, \xi v, \xi a, \xi b, t)}{M(v, z, \xi a, \xi b, t)}, \frac{M(\xi v, \xi v, \xi a, \xi b, t) M(\xi z, \xi z, \xi a, \xi b, t)}{M(\xi z, \xi z, \xi a, \xi b, t)} \end{array}\right\} \therefore$$

$$= \min\{M(\xi v, \xi z, \xi a, \xi b, t) M(\xi v, \xi z, \xi a, \xi b, t), 1, 1, 1\}$$

$$= M(\xi v, \xi z, \xi a, \xi b, t)$$

$$\frac{1}{M(T\xi\chi_n, T\xi\chi_{n+1}, \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi\chi_{n-1}, T\xi\chi_n, \xi a, \xi b, t)} - 1\right)$$

This is a contradiction. Hence $\xi v = \xi z$.
Thus A,T and B have unique common fixed point. This completes our proof.

V. CONCLUSIONS

Application Of This Theorem Are In Random Fuzzy Metric Space & Expend Random Fuzzy2-Metric Space, Random Fuzzy3-Metric Space Etc. Also In Rational Expression in any type of theorem & convert in integral type theorem

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