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Removal of Image Blurriness using Fractional Fourier Transform

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Abstract—

There are several domain transform like Fourier, sine, cosine and wavelet transform to processes an image. But the problem with these transform are inefficient to solve certain problem. Fractional Fourier transform gives extra degree of flexibility. It helps to improve image such as Image sharpness, filter and watermark.

Keywords— Image processing, Fractional Fourier transform, FrFT, Image blurriness.

I. INTRODUCTION

The traditional two-dimensional Discrete Fourier Transform (DFT), a two-dimensional Discrete Cosine Transform (DCT), a two-dimensional Discrete Sine Transform (DST) and their transform ideas are that images are switched between the time-space domain and frequency domain, image information is extracted and image feature is analysed. This paper is divided into three section started with introduction to fundamentals of fractional Fourier Transform section, section III result and analysis of Fractional Fourier Transform(FrFT) and finally conclusion is given in section IV.

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II. FRACTIONAL FOURIER TRANSFORM

Fractional Fourier Transform (FrFT) is a new time-frequency analysis tool which is developed in recently. Fourier Transform is the generalized form of already existing Fourier transform. In essence, the signal makes the representation on the Fractional Fourier domain while this is the integration of signal information in the time domain and frequency domain. It is a relatively new technique not only is closely linked with the Fourier Transform. It is also is very meaningful with other time-frequency analysis tools. Various applications like differential equations, filter design, signal analysis and pattern recognition field [8], fractional Fourier transform[5][7][8][10][12] can be applied. Most applied research of FrFT is focused on the linear FM signal estimation, detection and filtering aspects. FrFT is less in image processing application. Image processing of Fractional Fourier Transform is only limited to the chirp digital watermark detection of image [2],[9],[12]. Therefore, exploration of FrFT in image analysis, Image Compression, watermarking in image have great significance.

A. FrFT Methods

The FRFT can be best describe as rotation of Time-frequency graph with angle a. The representation of the Fractional Fourier domain is formed after the signal does counter clockwise rotation any angle from origin in the time-frequency plane axes and this is a generalized form of Fourier transform.
FRFT of the signal x(t) is defined as[10]:

$$X_\alpha(u)=\{F^\alpha[x(t)]\}(u)=\int_{-\infty}^{\infty}x(t)K_\alpha(t,u)dt \tag{1}$$

where, FRFT transform kernel Kα (t,u).

$$K_\alpha(t,u)=\begin{cases}\sqrt{\dfrac{1-j\cot\alpha}{2\pi}}\exp\left(j\dfrac{t^2+u^2}{2}\cot\alpha-tu\csc\alpha\right), & \alpha\neq n\pi\\ \delta(t-u), & \alpha=2n\pi\\ \delta(t+u), & \alpha=(2n\pm1)\pi\end{cases} \tag{2}$$

The rotation angle is α = pπ/2. For given 2D signal x(s,t), the FRFT for 2D can be expressed as:

$$X_{p1,p2}(u,v)=F_{p2}^{t\to v}\left\{F_{p1}^{s\to u}[x(s,t)]\right\} \tag{3}$$

By the discrete FRFT can also be achieved by using digital methods. The decomposition fast algorithm which is proposed in[6]. The signals can be decomposed into FRFT convolution by the algorithm. Decomposition type FRFT transformation matrix as follows:

$$F_p=DK_pJ \tag{4}$$

where, D and J are, respectively the twice inside difference between the original matrix and the matrix of the extraction operation, Kα is a discrete FRFT transforming matrix, given under:

$$K_p = \frac{A_\alpha}{2\Delta x}\exp\left(\frac{j\pi(\cot\alpha)m^2}{(2\Delta x)^2} - \frac{j2\pi(\csc\alpha)mn}{(2\Delta x)^2} + \frac{j2\pi(\cot\alpha)n^2}{(2\Delta x)^2}\right)|m|,|n| \le N \quad (5)$$

A discrete dimensionless normalized Fractional Fourier transform is defined as:

$$X_p(u) = \sqrt{\frac{1-j\cot(\alpha)}{2\pi}}\exp\left[j\pi\cot(\alpha)u^2\right] \times \int_{-\infty}^{+\infty} x(t)\exp\left[j\pi\cot(\alpha)t^2\right]\exp\left[-j2\pi\csc(\alpha)tu\right]dt \quad (6)$$

B. Energy distribution of the image in Fractional Fourier domain:

The discrete FrFT energy distribution of an image reflects the characteristics of the transforming image. The transform purpose is that the energy as much as possible is focused to a small number of several coefficients after transformation. As a result it is found only a few coefficients is non-zero. This show higher compression ratio. The popular transformation DCT energy accumulation is better as compared to other transformation. In this paper, for checking the energy accumulation of the image FRFT domain, a normalized residual error factor ρ is used . ρ for the MxN image, the coefficient of the FRFT is Fα(k,h) is is need to be defined . As other transformation, the FRFT domain energy is also concentrated in the central region, which is given by:

$$\rho = \frac{\sum\limits_{(k,h)\in r}\sum \left|F^\alpha(k,h)\right|^2}{\sum\limits_{k=-\frac{M}{2}}^{\frac{M}{2}-1}\sum\limits_{h=-\frac{N}{2}-1}^{\frac{N}{2}-1}\left|F^\alpha(k,h)\right|^2} \quad (7)$$

where, the r corresponding area is:

$$k = (0, \left|\frac{M}{4}-1\right|) h = (0, \left|\frac{N}{4}-1\right|)$$

C. Fractional Fourier Transform amplitude and phase information:

The phase information is more important than amplitude information and it is the most vulnerable during transmission, the channel delay and Doppler shift. This will affect the phase information, therefore the study the phase and amplitude information and associated characteristics is necessary in context of FrFT. To study phase these characteristics of the image FRFT, the traditional Fourier Transform must be reviewed.

Let F(k, h) is that the two-dimensional Fourier transform of the 2D image f(x, y):

$$F(k,h) = FT_{2D}f(x,y) \quad (8)$$

F(k, h) can be decomposed into the amplitude component and phase components as given below:

$$F(k,h) = \left|F(k,h)\right| \cdot P(k,h) = A(k,h) \bullet P(k,h) \quad (9)$$

Where A(k, h) = |F(k, h)| is amplitude function and P(k, h) = F(k, h)/A(k, h) is phase function. The 2D Fourier inverse transform is to get α(k, h) and p(k, h) of A(k, h) and P(k, h) respectively.

III. RESULT AND ANALYSIS

Here the comparison of single defocused blurred and image restoration is given this section. Figure 1(a) is the original image into focus, Fig. 1(b) is captured by the camera defocus blurred image, Fig. 1(c) and 1(d) are the FRFT recovery results with different order and the number of iterations .

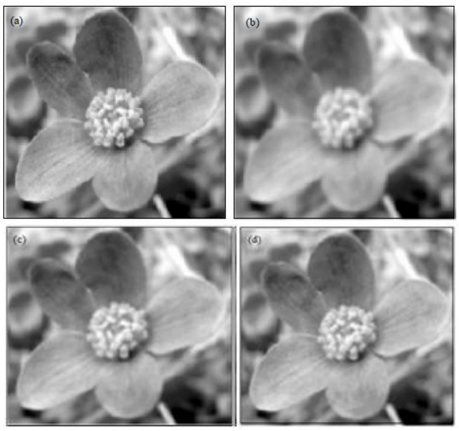

Fig. 1 flower image (a) actual image (b) Blurred image (c) FrFT processed image of image b with a=0.02 and (d) FrFT processed image of image b with 0.05

TABLE 1: Comparison among different type of scheme

S.no.	Recovery type	Mean Square Error
1	Blurred Image	982.5
2	FrFT a=0.02	417.2
3	FrFT a=0.05	510.0
4	Wierner filtering[3]	483.3
5	Gaussian filtering[3]	495.7

Table 1 shows the comparison of the parameter list in the different recovery modes, where in the Minimum Square Error (MSE) represents the mean square error of the original image and the restored image,. If MSE is small means smaller the difference between the two. By comparison, it is found that a = 0.02 is in Fig. d, iterations T = 15, the effect of its minimum MSE value is best in the recovery image and it is better than Wiener filtering and Gaussian filtering.

IV. CONCLUSIONS

It has been concluded that FrFT has greater degree of flexibility it can adapt according to problem. It can be used in different image application. Result shows that FrFT can help to recover the image from blurriness with minimum error. Thus FrFT is given new direction as filter.

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