

## Development of Relation Between Normal Curing and Accelerated Curing for Various Grades of Concrete

K. T. Phalak<sup>1</sup>, K. L. Bidkar<sup>2</sup>, S. G. Pande<sup>3</sup>

<sup>1,2</sup> Associate Professor, <sup>3</sup> Assistant Professor

<sup>1,2,3</sup> Department of Civil Engineering, Sandip Institute of Engineering and Management,  
Nasik, Maharashtra, India

### Abstract:

**I**S code has given a combined generalized relation between compressive strength obtained after 28 days and accelerated compressive strength obtained after 24 hours for all the grades of concrete. This project aims at developing the relation between accelerated curing and normal curing for individual grade of concrete. Accelerated curing results, obtained after 24 hours, are used to predict the 28 days compressive strength of concrete. The objective of this project is to develop the relation between compressive strength of concrete after 28 days normal curing and boiling water method of accelerated curing. The model developed in this study is useful for calculating the strength of concrete in around 01 day as against 28 days. This will help in increasing the speed of work and cost saving of the project.

**Keywords:** Concrete mix, Compressive strength, Normal Curing, Accelerated Curing, Regression.

### I. INTRODUCTION

Traditionally, quality of concrete in construction works is calculated in terms of its 28 days compressive strength. This procedure requires 28 days of moist curing before testing, which is too long a period to be of any value for either concrete construction control or applying timely corrective measures. If after 28 days, the quality of concrete is found to be dubious, it would have considerably hardened by that time and also might have been buried by subsequent construction. Thus replacement of the concrete mass of questionable attributes becomes very difficult and often impracticable. On the other hand, if the concrete is found to possess excessive strength than required, it would be too late to prevent wasteful use of cement on uneconomical mix proportioning. Hence, standard 28 days of cube testing of concrete is not feasible for quality control.

What is essentially needed for assessing quality of controlled concrete is an acceptance test which can supply results, while the concrete is still accessible and sufficient green to make its removal practicable, that is, within about 24 hours after casting. With the assistance of reliable test methods employing accelerated curing techniques, it is now possible to test the compressive strength of concrete within a short period and thereby to estimate whether it is likely to reach the specified strength at 28 days or not.

The need for having a reliable and fast method for evaluating controlled concrete in the field using accelerated curing technique was recognised by Cement and Concrete Sectional Committee and as a result, the Committee decided to evolve a standard method of determining compressive strength of test specimens cured by accelerated curing methods.

This standard lays down the method of making; curing and testing in compression concrete specimens cured by two accelerated methods namely warm-water method and boiling-water method. The method laid down in this standard may be used for quality –

control purposes, or for the prediction of normal strength of concrete at later ages, by the use of an appropriate correlation-curve obtained by testing normally-cured and accelerated cured concrete specimens of the mix proportion and materials to be used at the site.

#### 1.1 Objectives

Accelerated curing results, obtained after only 24 hours, are used to predict the 28 days compressive strength of concrete. Various curing methods are available. The designed concrete mix needs 28 days to confirm its strength. This results in delay in the construction activity. To avoid this IS code suggests two procedures of accelerated curing of concrete as follows:

i) Warm Water Method, 23 to 24 hours at  $350C \pm 30C$ .

ii) Boiling Water Method, 23 hours at  $210C \pm 50C$  and 3.5 hours at  $100C$ .

IS code gives a single relation for all grades of concrete to predict its compressive strength based on compressive strength calculated by accelerated curing basis.

Here the attempt is to develop the separate relation for different grades of concrete to predict its compressive strength based on compressive strength calculated by accelerated curing basis.

### II. CONCRETE MIX DESIGN

#### 2.1 Mix Design:-

The concrete mix was designed using I. S code method. The following proportion was used for the laboratory work.

Sr. No	Grade of Concrete	Proportion	Water	Cement	Sand	Aggregates
1	M20	0.48:1:1.4:3.01	188.79	393.31	552.2	1187.2
2	M25	0.43:1:1.32:2.58	188.79	439.05	580.38	1134.56
3	M30	0.38:1:1.05:2.37	186	489.47	514.95	1162.84

### III. STATISTICS

#### 3.1 General

The objective of the statistical analysis is to establish a relationship between compressive strength after 28 days by normal curing and compressive strength by accelerated curing. Compressive strength of concrete after accelerated curing is the independent variable in linear regression model and its contribution to the compressive strength after 28 days i.e. dependent variable can be worked out by writing normal equations and solving them simultaneously.

#### 3.2 Curve Fitting

Let  $(X_i, Y_i)$ ,  $i = 1$  to  $N$  be a given set of  $N$  pairs of values,  $X$  being independent variable and  $Y$  being dependent variable. The general problem in curve fitting is to find an analytic expression of the form of  $y = \Phi(x)$  for the functional relationship suggested by the given data. Curve fitting is of considerable importance theoretically as well as practically. Theoretically it is useful in the study of correlation and regression. Practically it enables us to represent the relationship between two variables by simple algebraic expressions i.e. polynomials, exponential or logarithmic functions. It may also be used to estimate the values of one variable which would correspond to the specified values of the other variable. Correlation studies are used to measure the strength of the linear relationship between two random variables  $x$  and  $y$ . Regression studies are designed to describe the relationship between the mean of a random variable  $y$  and one or more other variables. The primary purpose of regression is the prediction.

#### 3.3 Statistical Analysis Tool

##### 3.3.1 Simple Linear Regression Analysis: SLRA

In many problems there are two or more variables that are related, and it is important to model and explore this relationship, e.g. compressive strength of concrete based on number of days of curing of concrete. The mathematical relation between these two is developed based on bivariate data on these variables. The relation is given by  $Y = a + b \cdot X$ , in which “ $a$ ” and “ $b$ ” are called regression coefficients. This relation enables to estimate quite accurately, on an average, the value of one variable on the basis of the value of the other variable. It is known as regression equation of one variable on the other. Out of these two variables the variable for which the equation is developed is referred as dependent or response variable, and the variable based on the observed values of which the equation is developed is referred as independent or regressor variable.

##### 3.3.2 Method of Least Squares

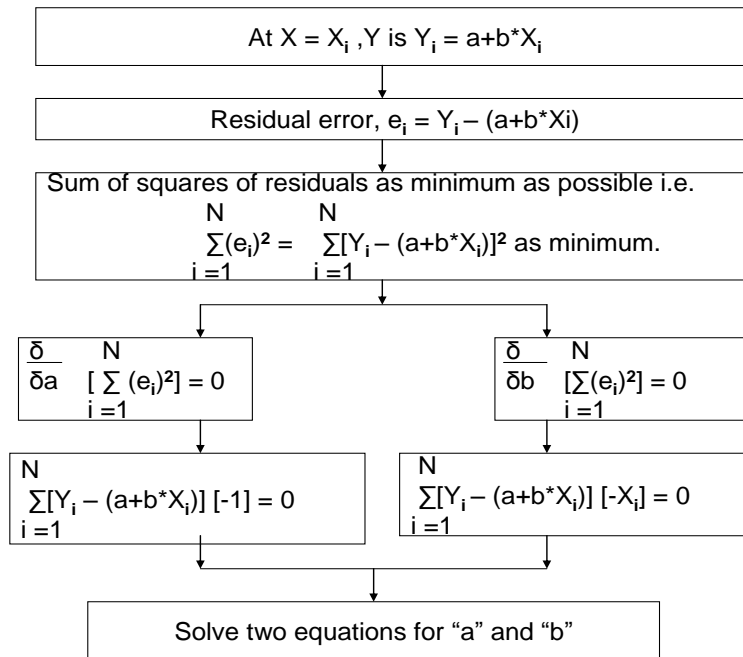
The regression equation between the two random variables is  $Y = a + b \cdot X$ . This is equivalent to finding the equation of the straight line that fits best to a given or observed set on data points. The procedure of finding the equation of the line which best fits a given set of paired data, is called the method of least squares. This tool is used for the evaluation of “ $a$ ” and “ $b$ ” by the use of partial differentiation. The reasoning behind the method of least squares is simple. If the values of  $X$  are plotted along  $X$  axis and the values of  $Y$  along  $Y$  axis then through these points many straight lines can be drawn. This diagram is known as scatter gram or scatter plot. From among the many straight lines one line provides best fit that is closest to the observations. To determine how close the  $i^{\text{th}}$  data point is to the estimated line of regression, the vertical distance from this data point to the line is measured. This distance is called the  $i^{\text{th}}$  residual or the  $i^{\text{th}}$  error, denoted by  $e_i$  and given by,  $e_i = Y_i - (a + b \cdot X_i)$ . Now the values of “ $a$ ” and “ $b$ ” should be such that  $e_i$ ’s are as small as possible. But it is impossible to minimize each of “ $e_i$ ” individually, it is better to make their sum

$$\sum_{i=1}^N (e_i) = 0 \quad \dots (3.1)$$

But residual error will be positive for the points which are above the line, negative for the points which are below the line and zero for the points which are on the line. The sum of their residuals is always tend to zero. Hence in the method of least squares the sum of the squares of residuals instead of the sum of the residuals. The technique selects the line through the data that is best in the sense that it minimizes the sum of squares of the residuals. Mathematically, it may be expressed as

$$\sum_{i=1}^N (e_i)^2 = \sum_{i=1}^N [Y_i - (a + b \cdot X_i)]^2 \text{ as minimum. } \dots (3.2)$$

To determine the values of “ $a$ ” and “ $b$ ” the differential calculus is used. A necessary condition for a relative minimum is the vanishing of the partial derivative with respect to “ $a$ ” and “ $b$ ” as shown in the flow chart below:



Flow Chart No 3.1: Procedure for Least Squares Method

Using method of least squares the simple linear equation becomes

$$X_1 = K_1 + K_2 * X_2$$

In this equation,  $K_1$  is constant and  $K_2$  is "Regression Coefficient".

Regression Coefficients indicate the contribution of an independent variable to a dependent variable.

$X_1$  is compressive strength of concrete after 28 days of normal curing.

$X_2$  is compressive strength of concrete using boiling water method of accelerated curing.

### 3.3.3 Normal Equations by the Least Squares

The principle of least squares is used to compute regression coefficients of a set of  $N$  data points  $X_1, X_2$ . The normal equations for determining  $K_1$ , and  $K_2$  are shown in table 4.1:

Table 3.1: Normal equations:

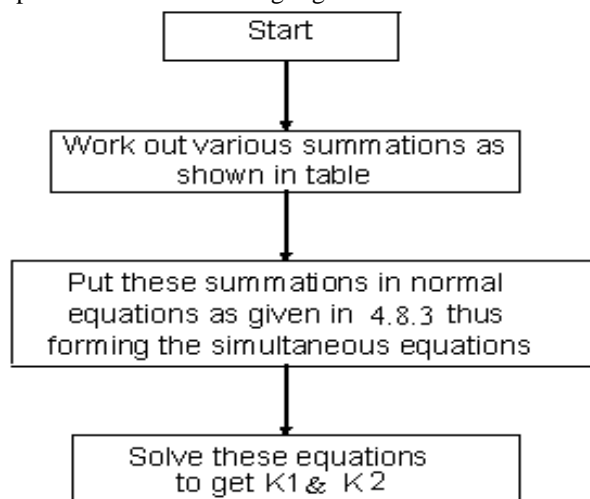
$\sum X_1 = K_1 * N + K_2 * \sum X_2$
$\sum X_1 * X_2 = K_1 * \sum X_2 + K_2 * \sum X_2 * X_2$

In these equations "N" = number of cubes under study.

## IV. DEVELOPMENT OF MODEL

The normal equations for the calculation of regression coefficients using the principle of least squares regression plane are as given in 4.8.3.

The flow chart below shows the procedure for calculating regression coefficients.



Flow chart No.4.1 : Calculation of constant and Regression Coefficient

**4.1 Coefficient of Determination**

The coefficient of multiple determination  $r^2$  is an indicator of goodness of fit of the model. It represents the percent of the variance in the dependent variable explained by the independent variables. It is a measure of relationship between the predicted variable and the predictors in a model. A model with high value of coefficient of determination is said to fit best to determine the dependent variable.

Coefficient of determination is more useful and comprehensible measure for indicating the percentage variation in the dependent variable due to variation in independent variables. For example the value of  $r = 0.6$  doesn't show that 60% of the variation in the dependent variable is due to variation in independent variables. But the coefficient of determination is  $r^2 = 0.36$  which implies that only 36% of the variation in the dependent variable has been explained by independent variables and the remaining 64% variation is due to other factors or variables. The model for which coefficient of determination is close to 1 is said to fit best.

**4.1: Various calculations required for development of model i.e regression equation  
For M<sub>20</sub> Grade**

cube no.	X <sub>1</sub>	X <sub>2</sub>	X <sub>1</sub> *X <sub>2</sub>	X <sub>2</sub> *X <sub>2</sub>	X <sub>1</sub> by relation	X <sub>1</sub> by IS code
1	29.4432	14.49548	426.7933	210.1189	31.11167112	31.86259
2	28.20588	15.75988	444.5213	248.3738	31.48340472	33.9362
3	29.73824	16.27748	484.0636	264.9564	31.63557912	34.78507
4	30.18535	14.70904	443.9975	216.3559	31.17445776	32.21283
5	30.05436	14.06968	422.8552	197.9559	30.98648592	31.16428
6	32.68728	15.45652	505.2316	238.904	31.39421688	33.43869
7	33.21564	14.79408	491.3948	218.8648	31.19945952	32.35229
8	33.02472	16.7388	552.7942	280.1874	31.7712072	35.54163
9	33.83724	16.96968	574.2071	287.97	31.83908592	35.92028
10	31.01384	15.57352	482.9947	242.5345	31.42861488	33.63057
11	32.48748	13.04472	423.7901	170.1647	30.68514768	29.48334
12	31.56884	16.7388	528.4245	280.1874	31.7712072	35.54163
13	32.7148	14.6854	480.4299	215.661	31.1675076	32.17406
14	33.81504	15.25184	515.7416	232.6186	31.33404096	33.10302
15	31.82592	14.18568	451.4723	201.2335	31.02058992	31.35452
16	31.27536	13.4008	419.1148	179.5814	30.7898352	30.06731
17	31.9458	15.68196	500.9728	245.9239	31.46049624	33.80841
18	34.17912	16.54732	565.5728	273.8138	31.71491208	35.2276
19	30.8802	15.424	476.2962	237.8998	31.384656	33.38536
20	27.19944	15.9568	434.016	254.6195	31.5412992	34.25915
21	31.28868	16.46296	515.1043	271.0291	31.69011024	35.08925
22	32.86932	14.14772	465.0259	200.158	31.00942968	31.29226
23	30.34452	15.40968	467.5993	237.4582	31.38044592	33.36188
24	27.55908	13.34752	367.8454	178.1563	30.77417088	29.97993
25	31.68828	14.32532	453.9448	205.2148	31.06164408	31.58352
	783.0476	379.4547	11894.2	5789.942	782.8096759	824.5557

The least square equations are

$$\sum X_1 = K_1 * N + K_2 * \sum X_2 \tag{4.1}$$

$$\sum X_1 * X_2 = K_1 \sum X_2 + K_2 \sum X_2 * X_2$$

Where N= Number of cubes

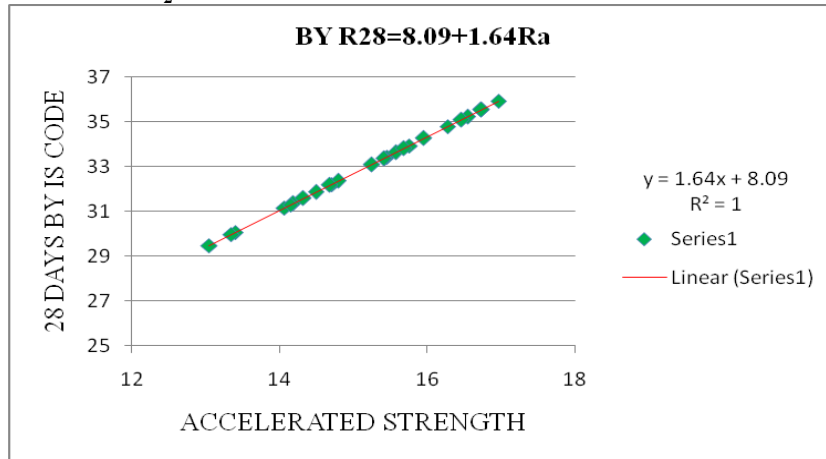
X<sub>1</sub> = Compressive strength by normal curing i.e. R<sub>28</sub>

X<sub>2</sub> = Compressive strength by accelerated curing i.e. R<sub>a</sub>

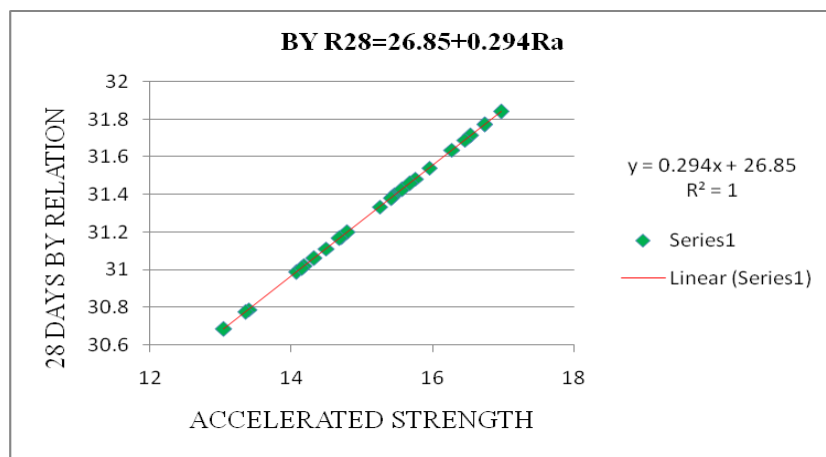
Solving above equations;

Solving above equation: K<sub>1</sub> = 26.85 K<sub>2</sub> = 0.294

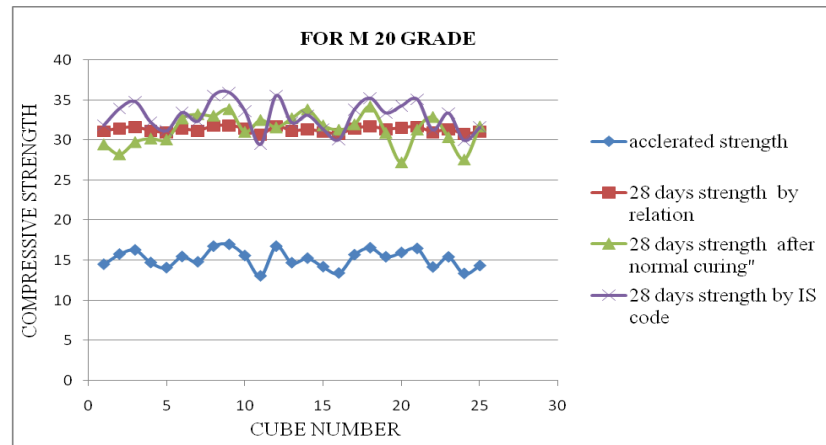
The relation is  $X_1 = 26.85 + 0.294 * X_2$



Graph no. 4. 1: Accelerated strength v/s 28 days strength by IS code for all grades by I.S code



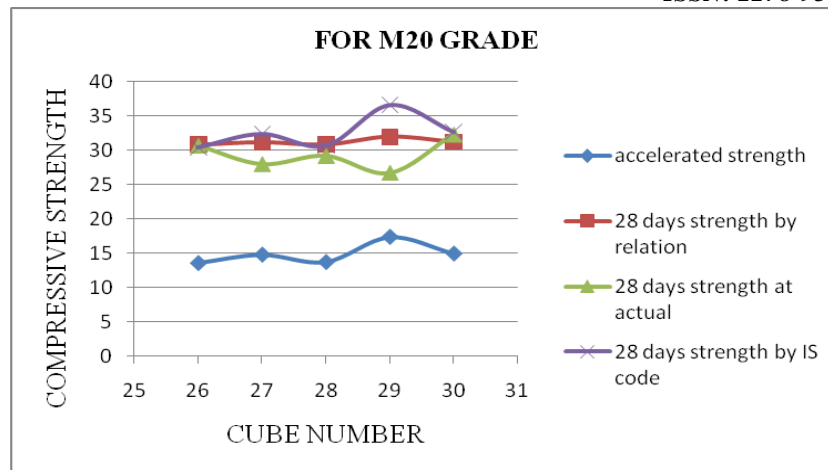
Graph no. 4. 2: Accelerated strength v/s 28 days strength by relation for M<sub>20</sub> grade



Graph no. 4. 3: Cube number v/s compressive strength by IS code, by relation, by normal curing, by accelerated curing for M<sub>20</sub> grade

Verification of equation:

Cube no	X2	X1 by relation	X1 at actual	X1 by IS Code Eqn.
26	13.61592	30.85308	30.7026	30.42011
27	14.8216	31.20755	27.9764	32.39742
28	13.76588	30.89717	29.193	30.66604
29	17.38548	31.96133	26.697	36.60219
30	14.98388	31.25526	32.327	32.66356



Graph no. 4. 4: Cube number v/s compressive strength by IS code, by relation, by normal curing, by accelerated curing for M<sub>20</sub> grade for verification

#### 4. 2 Developed Models

Putting the values of constant K<sub>1</sub> and regression coefficient K<sub>2</sub> in equation 4.7, following models are developed.

For M<sub>20</sub> concrete –  $X_1 = 26.85 + 0.294 * X_2$

### V. RESULTS AND CONCLUSION

#### 5.1 General

The equations for various grades of concrete are as below:

For M20 concrete –  $X_1 = 26.85 + 0.294 * X_2$

The coefficient of determination work out to be 1 for all equations.

#### 5.2 Future scope of study

Present work gives simple linear equation for various grades of concrete. Multiple linear equations can be developed by considering density of cubes and non-destructive strength as additional variables.

#### 5.3 Strengths & Weaknesses of developed models

##### Strengths:

The model developed in this study is useful for calculating the strength of concrete in around 01 day as against 28 days. This will help in increasing the speed of work and cost saving of the project.

The results of the model are matching with the results given by the single equation mentioned by I.S code.

##### Weakness:

The models so developed give result with average error of +/- 10%

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