

# Fixed Point Theorems & Common in Fuzzy 3-Metric Spaces

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## Abstract—

In this paper, we give some new definitions of results extend, generalize and improve the corresponding results given by many authors compatible mappings of types -I & II in fuzzy-3 metric space prove. Some common fixed point theorems for mappings under the condition of compatible mappings of types -I & II in complete fuzzy-3 metric space.

**Keywords—** Fuzzy metric space, Fuzzy 3-metric space, Compatible mappings, Common fixed point.

## I. INTRODUCTION

Impact of fixed point theory in different branches of mathematics and its applications is immense. The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [10] in 1922. In the general setting of complete metric space, this theorem runs as the follows, Theorem 1.1(Banach's contraction principle) Let  $(X, d)$  be a complete metric space,  $c \in (0, 1)$  and  $f: X \rightarrow X$  be a mapping such that for each  $x, y \in X$ ,  $d(fx, fy) \leq c d(x, y)$  Then  $f$  has a unique fixed point  $a \in X$ , such that for each  $x \in X, \lim_{n \rightarrow \infty} f^n x = a$ . After the classical result, R.Kannan [11] gave a subsequently new contractive mapping to prove the fixed point theorem [12,13] & also in common [14] & 2-fuzzy [15]. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. In 2002, A. Branciari [1] analyzed the existence of fixed point for mapping  $f$  defined on a complete metric space  $(X, d)$  satisfying a general contractive condition of integral type.

## II. PRELIMINARY NOTES

**Definition 2.1** A binary operation  $*$  :  $[0,1] \times [0,1] \rightarrow [0,1]$  is a continuous t-norms if  $([0,1], *)$  is an abelian topological monoid with unit 1 such that  $a_1 * b_1 * c_1 \leq a_2 * b_2 * c_2$  whenever  $a_1 \leq a_2, b_1 \leq b_2$  and  $c_1 \leq c_2$  for all  $a_1, a_2, b_1, b_2, c_1, c_2$  are in  $[0,1]$ .

**Definition 2.2** A 3-tuple  $(X, M, *)$  is said to be a fuzzy 2- metric space if  $X$  is an arbitrary set,  $*$  is a continuous t-norm and  $M$  is a fuzzy set on  $X^3 \times (0, \infty)$  satisfying the following conditions: for all  $x, y, z, t \in X$  and  $t_1, t_2, t_3 > 0$ ,

- (1)  $M(x, y, z, t) > 0$ ;
- (2)  $M(x, y, z, t) = 1, t > 0$  when at least two of the three points are equal
- (3)  $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$
- (4)  $M(x, y, z, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3) \leq M(x, y, z, t_1 + t_2 + t_3)$

The function value  $M(x, y, z, t)$  may be interpreted as the probability that the area of triangle is less than  $t$ .

- (5)  $M(x, y, z, \cdot): [0,1] \rightarrow [0,1]$  is left continuous.

**Definition 2.3 [08]** Let  $(X, M, *)$  be a fuzzy- 3 metric space.

- (1) A sequence  $\{x_n\}$  in fuzzy -3 metric space  $X$  is said to be convergent to a point  $x \in X$  (denoted by  $\lim_{n \rightarrow \infty} x_n = x$  or  $x_n \rightarrow x$ )

if for any  $\lambda \in (0,1)$  and  $t > 0$ , there exists  $n_0 \in \mathbb{N}$  such that for all  $n \geq n_0$  and  $a \in X, M(x_n, x, a, t) > 1 - \lambda$ .

That is

$$\lim_{n \rightarrow \infty} M(x_n, x, a, t) = 1 \text{ for all } a \in X \text{ and } t > 0.$$

- (2) A sequence  $\{x_n\}$  in fuzzy- 2 metric space  $X$  is called a Cauchy sequence, if for any  $\lambda \in (0,1)$  and  $t > 0$ , there exists  $n_0 \in \mathbb{N}$  such that for all  $m, n \geq n_0$  and  $a, b \in X, M(x_n, x_m, a, b, t) > 1 - \lambda$ .

- (3) A fuzzy- 3 metric space in which every Cauchy sequence is convergent is said to be complete.

**Definition 2.4 [08]** Self mappings  $A$  and  $B$  of a fuzzy- 3 metric space  $(X, M, *)$  is said to be compatible, if  $\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, a, b, t) = 1$  for all  $a, b \in X$  and  $t > 0$ ,

Whenever  $\{x_n\}$  is a sequence in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z \text{ for some } z \in X. \text{ Then } \lim_{n \rightarrow \infty} ABx_n = Bz.$$

**Definition 2.5** Let  $(X, M, *)$  is a fuzzy-2 metric space. Then

- (a) A sequence  $\{x_n\}$  in  $X$  is said to converge to  $x$  in  $X$  if for each  $\epsilon > 0$  and each  $t > 0, \exists n_0 \in \mathbb{N}$  such that  $M(x_n, x, t) > 1 - \epsilon$  for all  $n \geq n_0$ .
- (b) a sequence  $\{x_n\}$  in  $X$  is said to Cauchy to if for each  $\epsilon > 0$  and each  $t > 0, \exists n_0 \in \mathbb{N}$  such that  $M(x_n, x_m, t) > 1 - \epsilon$  for all  $n, m \geq n_0$ .

(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

**Definition 2.6** [3] Two self mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  are called compatible if

$$\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) = 1 \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that } \lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = x \text{ For some } x \text{ in } X.$$

**Definition 2.7** [1] Two self mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  are called reciprocally continuous on  $X$  if

$$\lim_{n \rightarrow \infty} fgx_n = fx \text{ and } \lim_{n \rightarrow \infty} gfx_n = gx \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that } \lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = x \text{ for some } x \text{ in } X.$$

**Lemma 2.2.1**[08] Let  $(X, M, *)$  be a fuzzy- 2 metric space. If there exists  $q \in (0, 1)$  such that  $M(x, y, z, qt + 0) \geq M(x, y, z, t)$  for all  $x, y, z \in X$  with  $z \neq x, z \neq y$  and  $t > 0$ , then  $x = y$ ,

**Lemma 2.2.2**[4] Let  $X$  be a set,  $f, g$  ovc self maps of  $X$ . If  $f$  and  $g$  have a unique point of coincidence,  $w = fx = gx$ , then  $w$  is the unique common fixed point of  $f$  and  $g$ .

### III. MAIN RESULTS

**Theorem 3.1** Let  $(X, M, *)$  be a complete fuzzy 3-metric space and let  $P, R, S$  and  $T$  be self-mappings of  $X$ . Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be ovc. If there exists  $q \in (0, 1)$  such that

$$M(Px, Ry, a, b, qt) \geq \min \{ M(Sx, Ty, a, b, t), M(Sx, Px, a, b, t), M(Ry, Ty, a, b, t), M(Px, Ty, a, b, t), M(Ry, Sx, a, b, t), M(Px, Ry, a, b, t), M(Sx, Ty, a, b, t) * M(Ry, Ty, a, b, t) \}$$

$$\dots\dots\dots(1)$$

For all  $x, y \in X$  and for all  $t > 0$ , then there exists a unique point  $w \in X$  such that  $Pw = Sw = w$  and a unique point  $z \in X$  such that  $Rz = Tz = z$ . Moreover  $z = w$  so that there is a unique common fixed point of  $P, R, S$  and  $T$ .

**Proof :** Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be ovc, so there are points  $x, y \in X$  such that  $Px = Sx$  and  $Ry = Ty$ . We claim that  $Px = Ry$ . If not, by inequality (1)

$$\begin{aligned} M(Px, Ry, a, b, qt) &\geq \min \{ M(Sx, Ty, a, b, t), M(Sx, Px, a, b, t), M(Ry, Ty, a, b, t), M(Px, Ty, a, b, t), M(Ry, Sx, a, b, t), M(Px, Ry, a, b, t), \\ &M(Sx, Ty, a, b, t) * M(Ry, Ty, a, b, t) \} \\ &\geq \min \{ M(Px, Ry, a, b, t), M(Px, Px, a, b, t), M(Ty, Ty, a, b, t), M(Px, Ry, a, b, t), M(Ry, Px, a, b, t), \\ &M(Px, Ry, a, b, t), M(Px, Ry, a, b, t) * M(Ty, Ty, a, b, t) \} \\ &\geq \min \{ M(Px, Ry, a, b, t), M(Px, Px, a, b, t), M(Ty, Ty, a, b, t), M(Px, Ry, a, b, t), M(Px, Ry, a, b, t), \\ &M(Px, Ry, a, b, t), M(Px, Ry, a, b, t) \} \\ &= M(Px, Ry, a, b, t). \end{aligned}$$

Therefore  $Px = Ry$ , i.e.  $Px = Sx = Ry = Ty$ . Suppose that there is a another point  $z$  such that  $Pz = Sz$  then by (1) we have  $Pz = Sz = Ry = Ty$ , so  $Px = Pz$  and  $w = Px = Sx$  is the unique point of coincidence of  $P$  and  $S$ . By Lemma 2.8  $w$  is the only common fixed point of  $P$  and  $S$ . Similarly there is a unique point  $z \in X$  such that  $z = Rz = Tz$ .

Assume that  $w \neq z$ . we have

$$\begin{aligned} M(w, z, a, b, qt) &= M(Pw, Rz, a, b, qt) \\ &\geq \min \{ M(Sw, Tz, a, b, t), M(Sw, Pw, a, b, t), M(Rz, Tz, a, b, t), M(Pw, Tz, a, b, t), M(Rz, Sw, a, b, t), \\ &M(Pw, Rz, a, b, t), M(Sw, Tz, a, b, t) * M(Rw, Tz, a, b, t) \} \\ &\geq \min \{ M(w, z, a, b, t), M(w, w, a, b, t), M(z, z, a, b, t), M(w, z, a, b, t), M(z, w, a, b, t), \\ &M(w, z, a, b, t), M(w, z, a, b, t) \} \\ &= M(w, z, a, b, t). \end{aligned}$$

Therefore we have  $z = w$  and  $z$  is a common fixed point of  $P, R, S$  and  $T$ . The uniqueness of the fixed point holds.

**Theorem 3.2** Let  $(X, M, *)$  be a complete fuzzy 2- metric space and let  $P, R, S$  and  $T$  be self-mappings of  $X$ . Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be ovc. If there exists  $q \in (0, 1)$  such that

$$M(Px, Ry, a, b, qt) \geq \emptyset \text{ (min} \{ M(Sx, Ty, a, b, t), M(Sx, Px, a, b, t), M(Ry, Ty, a, b, t), M(Px, Ty, a, b, t), M(Ry, Sx, a, b, t), M(Px, Ry, a, b, t) \})$$

$$\dots\dots\dots(2)$$

For all  $x, y \in X$  and  $\emptyset : [0, 1] \rightarrow [0, 1]$  such that  $\emptyset(t) > t$  for all  $0 < t < 1$ , then there exists a unique common fixed point of  $P, R, S$  and  $T$ .

**Proof :** Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be ovc, so there are points  $x, y \in X$  such that  $Px = Sx$  and  $Ry = Ty$ . We claim that  $Px = Ry$ . If not, by inequality (2)

$$\begin{aligned} M(Px, Ry, a, b, qt) &\geq \emptyset \text{ (min} \{ M(Sx, Ty, a, b, t), M(Sx, Px, a, b, t), M(Ry, Ty, a, b, t), M(Px, Ty, a, b, t), M(Ry, Sx, a, t), \\ &M(Px, Ry, a, b, t) \}) \\ &> \emptyset (M(Px, Ry, a, b, t)). \end{aligned} \quad \text{From Theorem 3.1}$$

Assume that  $w \neq z$ . we have

$$\begin{aligned} M(w, z, a, qt) &= M(Pw, Rz, a, qt) \\ &\geq \emptyset \text{ (min} \{ M(Sw, Tz, a, b, t), M(Sw, Pw, a, b, t), M(Rz, Tz, a, b, t), M(Pw, Tz, a, b, t), M(Rz, Sw, a, b, t), M(Pw, Rz, a, b, t) \}) \\ &= M(w, z, a, b, t). \end{aligned} \quad \text{From Theorem 3.1}$$

Therefore we have  $z = w$  and  $z$  is a common fixed point of  $P, R, S$  and  $T$ . The uniqueness of the fixed point holds.

**Theorem 3.3** Let  $(X, M, *)$  be a complete fuzzy 3- metric space and let  $P, R, S$  and  $T$  be self-mappings of  $X$ . Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be ovc. If there exists  $q \in (0, 1)$  such that

$$M(Px, Ry, a, b, qt) \geq \emptyset ( M(Sx, Ty, a, b, t), M(Sx, Px, a, b, t), M(Ry, Ty, a, b, t), M(Px, Ty, a, b, t), M(Ry, Sx, a, b, t), M(Px, Ry, a, b, t))$$

.....(3)

For all  $x, y \in X$  and  $\emptyset: [0, 1]^7 \rightarrow [0, 1]$  such that  $\emptyset(t, 1, 1, t, t, 1) > t$  for all  $0 < t < 1$ , then there exists a unique common fixed point of  $P, R, S$  and  $T$ .

**Proof:** Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be owc, so there are points  $x, y \in X$  such that  $Px = Sx$  and  $Ry = Ty$ . We claim that  $Px = Ry$ . If not, by inequality (3)

$$\begin{aligned} M(Px, Ry, a, b, qt) &\geq \emptyset ( M(Sx, Ty, a, b, t), M(Sx, Px, a, b, t), M(Ry, Ty, a, b, t), M(Px, Ty, a, b, t), M(Ry, Sx, a, b, t), M(Px, Ry, a, b, t)) \\ &\geq \emptyset(M(Px, Ry, a, b, t), M(Px, Px, a, b, t), M(Ty, Ty, a, b, t), M(Px, Ry, a, b, t), M(Ry, Px, a, b, t), \\ &\quad M(Px, Ry, a, b, t)) \\ &\geq \emptyset( M(Px, Ry, a, b, t), M(Px, Px, a, b, t), M(Ty, Ty, a, b, t), M(Px, Ry, a, b, t), M(Px, Ry, a, b, t), M(Px, Ry, a, b, t)) \\ &= \emptyset(M(Px, Ry, a, b, t), 1, 1, M(Px, Ry, a, b, t), M(Px, Ry, a, b, t), M(Px, Ry, a, b, t)) \\ &= M(Px, Ry, a, b, t). \end{aligned}$$

A contradiction, therefore  $Px = Ry$ , i.e.  $Px = Sx = Ry = Ty$ . Suppose that there is a another point  $z$  such that  $Pz = Sz$  then by (3) we have  $Pz = Sz = Ry = Ty$ , so  $Px = Pz$  and  $w = Px = Sx$  is the unique point of coincidence of  $P$  and  $S$ . By Lemma 2.8  $w$  is the only common fixed point of  $P$  and  $S$ . Similarly there is a unique point  $z \in X$  such that  $z = Rz = Tz$ . Thus  $z$  is a common fixed point of  $P, R, S$  and  $T$ . The uniqueness of the fixed point holds from (3).

**Theorem 3.4** Let  $(X, M, *)$  be a complete fuzzy 3- metric space and let  $P, R, S$  and  $T$  be self-mappings of  $X$ . Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be owc. If there exists  $q \in (0, 1)$  for all  $x, y \in X$  and  $t > 0$

$$M(Px, Ry, a, b, qt) \geq M(Sx, Ty, a, b, t) * M(Sx, Px, a, b, t) * M(Ry, Ty, a, b, t) * M(Px, Ty, a, b, t) * M(Ry, Sx, a, b, t) * M(Px, Ry, a, b, t)$$

..... (4)

Then there exists a unique common fixed point of  $P, R, S$  and  $T$ .

**Proof:** Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be owc, so there are points  $x, y \in X$  such that  $Px = Sx$  and  $Ry = Ty$ . We claim that  $Px = Ry$ . If not, by inequality (4) We have

$$\begin{aligned} M(Px, Ry, a, b, qt) &\geq M(Sx, Ty, a, b, t) * M(Sx, Px, a, b, t) * M(Ry, Ty, a, b, t) * M(Px, Ty, a, b, t) * M(Ry, Sx, a, b, t) * M(Px, Ry, a, b, t) \\ &= M(Px, Ry, a, b, t) * M(Px, Px, a, b, t) * M(Ty, Ty, a, b, t) * M(Px, Ry, a, b, t) * M(Ry, Px, a, b, t) * M(Px, Ry, a, b, t) \\ &= M(Px, Ry, a, b, t) * 1 * 1 * M(Px, Ry, a, b, t) * M(Ry, Px, a, b, t) * M(Px, Ry, a, b, t) \\ &> M(Px, Ry, a, b, t). \end{aligned}$$

Thus we have  $Px = Ry$ , i.e.  $Px = Sx = Ry = Ty$ . Suppose that there is a another point  $z$  such that  $Pz = Sz$  then by (4) we have  $Pz = Sz = Ry = Ty$ , so  $Px = Pz$  and  $w = Px = Sx$  is the unique point of coincidence of  $P$  and  $S$ . Similarly there is a unique point  $z \in X$  such that  $z = Rz = Tz$ . Thus  $w$  is a common fixed point of  $P, R, S$  and  $T$ .

**Corollary 3.5** Let  $(X, M, *)$  be a complete fuzzy 2- metric space and let  $P, R, S$  and  $T$  be self-mappings of  $X$ . Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be owc. If there exists  $q \in (0, 1)$  for all  $x, y \in X$  and  $t > 0$

$$M(Px, Ry, a, b, qt) \geq M(Sx, Ty, a, b, t) * M(Sx, Px, a, b, t) * M(Ry, Ty, a, b, t) * M(Px, Ty, a, b, t) * M(Ry, Sx, a, b, t) * M(Px, Ry, a, b, t)$$

.....(5)

Then there exists a unique common fixed point of  $P, R, S$  and  $T$ .

**Proof:** We have

$$\begin{aligned} M(Px, Ry, a, b, qt) &\geq M(Sx, Ty, a, b, t) * M(Sx, Px, a, b, t) * M(Ry, Ty, a, b, t) * M(Px, Ty, a, b, t) * M(Ry, Sx, a, b, t) * M(Px, Ry, a, b, t) \\ &\geq M(Sx, Ty, a, b, t) * M(Sx, Px, a, b, t) * M(Ry, Ty, a, b, t) * M(Px, Ty, a, b, t) * M(Sx, Ty, a, b, t) * M(Ty, Ry, a, b, t) * \\ &\quad M(Px, Ry, a, b, t) \\ &\geq M(Sx, Ty, a, b, t) * M(Sx, Px, a, b, t) * M(Ry, Ty, a, b, t) * M(Px, Ty, a, b, t) * M(Px, Ry, a, b, t) \\ &= M(Px, Ry, a, b, t) * M(Px, Px, a, b, t) * M(Ty, Ty, a, b, t) * M(Px, Ry, a, b, t) * M(Ry, Px, a, b, t) * M(Px, Ry, a, b, t) \\ &= M(Px, Ry, a, b, t) * 1 * 1 * M(Px, Ry, a, b, t) * M(Ry, Px, a, b, t) * M(Px, Ry, a, b, t) \\ &> M(Px, Ry, a, t). \end{aligned}$$

And therefore from theorem 3.4,  $P, R, S$  and  $T$  have a common fixed point.

**Corollary 3.6** Let  $(X, M, *)$  be a complete fuzzy 3-metric space and let  $P, R, S$  and  $T$  be self-mappings of  $X$ . Let the pairs  $\{P, S\}$  and  $\{R, T\}$  be owc. If there exists  $q \in (0, 1)$  for all  $x, y \in X$  and  $t > 0$

$$M(Px, Ry, a, b, qt) \geq M(Sx, Ty, a, b, t)$$

.....(6)

Then there exists a unique common fixed point of  $P, R, S$  and  $T$ .

**Proof:** The Proof follows from Corollary 3.5

**Theorem 3.7** Let  $(X, M, *)$  be a complete fuzzy 2- metric space. Then continuous self-mappings  $S$  and  $T$  of  $X$  have a common fixed point in  $X$  if and only if there exists a self mapping  $P$  of  $X$  such that the following conditions are satisfied

- (i)  $PX \subset TX \cap SX$
  - (ii) The pairs  $\{P, S\}$  and  $\{P, T\}$  are weakly compatible,
  - (iii) There exists a point  $q \in (0, 1)$  such that for all  $x, y \in X$  and  $t > 0$
- $$M(Px, Py, a, b, qt) \geq M(Sx, Ty, a, b, t) * M(Sx, Px, a, b, t) * M(Py, Ty, a, b, t) * M(Px, Ty, a, b, t) * M(Py, Sx, a, b, t) * M(Px, Sy, a, b, t)$$

.....(7)

Then P, S and T have a unique common fixed point.

**Proof:** Since compatible implies ows, the result follows from Theorem 3.4

**Theorem 3.8** Let  $(X, M, *)$  be a complete fuzzy 2- metric space and let P and R be self-mappings of X. Let the P and R are owc. If there exists  $q \in (0, 1)$  for all  $x, y \in X$  and  $t > 0$

$$M(Sx, Sy, a, b, qt) \geq \alpha M(Px, Py, a, b, t) + \beta \min\{M(Px, Py, a, b, t), M(Sx, Px, a, b, t), M(Sx, Py, a, b, t)\}$$

.....(8)

For all  $x, y \in X$  where  $\alpha, \beta > 0, \alpha + \beta > 1$ . Then P and S have a unique common fixed point.

**Proof:** Let the pairs  $\{P, S\}$  be owc, so there are points  $x \in X$  such that  $Px = Sx$ . Suppose that exist another point  $y \in X$  for which  $Py = Sy$ . We claim that  $Sx = Sy$ . By inequality (8) We have

$$M(Sx, Sy, a, b, qt) \geq \alpha M(Px, Py, a, b, t) + \beta \min\{M(Px, Py, a, b, t), M(Sx, Px, a, b, t), M(Sx, Py, a, b, t)\}$$

$$= \alpha M(Sx, Sy, a, b, t) + \beta \min\{M(Sx, Sy, a, b, t), M(Sx, Sx, a, b, t), M(Sx, Sy, a, b, t)\}$$

$$= (\alpha + \beta)M(Sx, Sy, a, b, t)$$

A contradiction, since  $(\alpha + \beta) > 1$ . Therefore  $Sx = Sy$ . Therefore  $Px = Py$  and Px is unique.

From lemma 2.2.2, P and S have a unique fixed point.

## REFERENCES

- [1] A. Branciari. A fixed point theorem for mappings satisfying a general contractive condition of integral type. Int. J. Math. Sci. 29(2002), no.9, 531 - 536.
- [2] A. George, P. Veeramani, "On some results in fuzzy metric spaces", Fuzzy Sets and Systems, 64 (1994), 395-399.
- [3] Badard R. Fixed point theorems for fuzzy numbers. Fuzzy Set Syst 1984; 13:291-302.
- [4] Bose BK, Sabani D. Fuzzy mappings and fixed point theorems. Fuzzy Set Syst 1987; 21-53.
- [5] O. Kramosil and J. Michalek, "Fuzzy metric and statistical metric spaces", Kybernetika, 11 (1975), 326-334.
- [6] B. Schweizer and A. Sklar, "Statistical metric spaces", Pacific J. Math. 10 (1960), 313-334.
- [7] L.A. Zadeh, Fuzzy sets, Inform and Control 8 (1965), 338-353.
- [8] Jinkyu Han. Common fixed point theorem on fuzzy-2 metric spaces. Int J Chung cheong Math Soci. 2010:302-729.
- [9] M.S. Chauhan, Bharat Singh. Fixed point in intuitionistic fuzzy-3 metric space. Int. J. Engg and Techo. Vol.3 (2), 2011, 144-150.
- [10] S. Banach, Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales, Fund. Math. 3, (1922) 133-181 (French).
- [11] R. Kannan, Some results on fixed points, Bull. Calcutta Math. Soc., 60(1968), 71-76.
- [12] Jitendra Singhi, Ramakant Bhardwaj, Sarvesh Agrawal and Rajesh Shrivastava, "Fixed Point Theorem in Fuzzy Metric Spaces." International Mathematical Forum, 5, 2010, no. 30, 1473 - 1480.
- [13] K.S. Dersanambika, Aswathy M.R., "Fixed Point Theorems in Fuzzy Metric Spaces." Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 22, 1079 - 1089.
- [14] Namdeo Kailash, Shrivastav Rajesh, Solanki Manoj and Sharma Manoj, "Common Fixed Point Theorem for four Mappings in Fuzzy Metric Space." Research Journal of Mathematical and Statistical Sciences ISSN 2320-6047 Vol. 1(9), 11-14, October (2013).
- [15] R Krishnakumar and P Parakath Nisha Bagam, Fixed Point Theorems In Fuzzy 2-Metric Space With Implicit Map." Int. J. Engg. Res. & Sci. & Tech. , IS