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To Find the Comparison of Non-Split Domination Number and the Average Distance of an Interval Graph

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Abstract-

Interval graphs have drawn the attention of many researchers for over 30 years. They are extensively studied and revealed their practical relevance for modeling problems arising in the real world. In an interval graph G = (V,E) the distance between two vertices u, v is defined as the smallest number of edges in a path joining u and v. The aim of this paper is to show that the comparison of non-split domination number and the average distance of an interval graph corresponding to an interval family I.

Keywords - Interval family, interval graph, dominating set, non-split dominating set, distance, average distance.

I. INTRODUCTION

In general an undirected graph $G=(V,E)$ is an interval graph (IG), if the vertex set V can be put into one-to-one correspondence with a set of intervals I on the real line R, such that two vertices are adjacent in G, if and only if their corresponding intervals have non-empty intersection. The set I is called an interval representation of G and G is referred to as the intersection graph I.

Let $I=\{I_1, I_2, I_3, I_4, ...I_n\}$ be any interval family where, each I_i is an interval on the real line and $I_i=[a_i, b_i]$ for $i=1,2,3,4,..........n$. Here a_i is called the left end point labeling and b_i is the right end point labeling of I_i.Without loss of generality we assume that all end points of the intervals in I are distinct numbers between 1 and $2n$. Two intervals i and j are said to be intersect each other if they have non empty intersection. Also we say that the intervals contain both its end points and that no two intervals share a common end point.

The intervals and vertices of an interval graph are one and the same thing. The graph G is connected, and the list of sorted end point is given and the intervals in I are indexed by increasing right end points, that is $b_1 < b_2 < b_3 < .... < b_n$. Let $G=(V,E)$ be a graph. A set $D \subseteq V(G)$ is a dominating set of G if every vertex in V/D is adjacent to some vertex in D. The research of the domination in graphs has been an evergreen of the graph theory. Its basic concept is the dominating set and the domination number. The theory of domination in graphs was introduced by Ore [1] and Berge [2]. A survey on results and applications of dominating sets was presented by E.J.Cockayane and S.T.Hedetniemi [3]. In 1997 Kulli et.al introduced the concept of Non-split domination [4] and studied these parameters for various standard graphs and obtained the bounds for these parameters. The distance between two vertices u and v of a graph is the length of the shortest path. The set of all central vertices of a graph is called the centre of the graph G.

In this section we discuss about the computation of average distance of an interval graph G[9,10]. The average distance μ(G) of a connected interval graph is defined to be the average of all distances in G[11].

$$\mu(G)=\frac{1}{n(n-1)}\sum_{\substack{x,y\in v(G)\\ x\neq y}}\delta(x,y) \quad \text{or} \quad \mu(G)=\frac{1}{2\ {}^{n}c_{2}}\sum_{\substack{x,y\in v(G)\\ x\neq y}}\delta(x,y)$$

Where δ(x,y) denotes the length of shortest path joining the vertices x and y . The average distance can be used as a tool in analytic networks where the performance time is proportions to the distance between any two nodes.

II. MAIN THEOREMS

Theorem.1: Let D be a domination number of the given interval graph G. If I and j are any two intervals in I. Such that $i \in \gamma_{ns}(D)$, $j \neq 1$and j is contained in I and if there is one interval $k \neq i$, to the right of j that intersects j. Then the non-split domination number is greater than or equal to the average distance of G.

Proof: Let I = $\{I_1, I_2, I_3, \ldots\ldots, I_n\}$ be an interval family and let G be an interval graph corresponding to an interval family. Now we have to prove that the non-split dominating set is greater than equal to the average distance of G.

Suppose there is at least one interval $k \neq i$ to the right of j that intersects j. Then it is obvious that j is adjacent to k in v- $\gamma_{ns}(D)$. So that there will not be any disconnection in v- $\gamma_{ns}(D)$. Since there is at least one interval to the left of j that intersect j, there will not be any disconnection in v- $\gamma_{ns}(D)$ to its left. Thus we get the non-split domination number $\gamma_{ns}(D)$.

Next we will find the average distance of G. First we will discuss the distance of G. For any two vertices I, j in an interval graph G, corresponding to an interval family I. The distance from I to j is denoted by d(i,j) and defined as the length of a shortest i-j path in an interval graph G. The term distance we jest defined satisfies all four of the following axioms.

1. $d(i, j) \geq 0$, for all $i, j \in V(G)$.
2. $d(i, j) = 0$, if and only if $i = j$.
3. $d(i, j) = d(j, i)$, for all $i, j_j \in V(G)$.
4. $d(i, k) \leq d(i, j) + d(j, k)$, for all $i, j, k \in V(G)$.

If an interval an graph G is not a connected and suppose $V(G_1)$and $V(G_2)$ are two components of G, is denoted by W(G).Which are $W(V(G))=W(V(G_1)) \cup W(V(G_2))$ and
$E(W(V(G_1))) \cup E(W(V(G_2)))$And
$W(V(G_1)) \cap W(V(G_2))= \emptyset$
That is $E(W(V(G_1))) \cap E(W(V(G_2)))= \emptyset$ and also $E(W(V(G_1))) \cap V(W(V(G_2)))= \emptyset$
Then $d(i, j)= \infty$ for $i \in V(W(V(G_1)))$ & $j \in V(W(V(G_1)))$
Under this distance function the set V(G) is a metric space. In this fact the interval graph corresponding to interval family $I = \{I_1,I_2,I_3,\ldots\ldots,I_n\}$ where n is a number of vertices of G.

In this section we discuss about the computation of average distance of an interval graph G. The average distance μ(G) of a connected an interval graph is defined to be the average of distance in G.

Where $\mu(G)=\frac{1}{2\ {}^{n}c_2}\sum_{\substack{i,j\in v(G)\\ i\neq j}}\delta(i, j)$. Hence the theorem is hold.

III. EXPERIMENTAL PROBLEM OF THEOREM.1

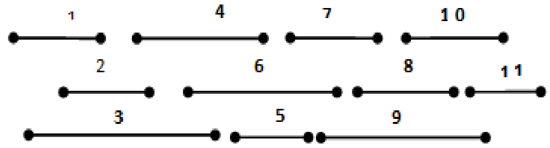

Fig.1: Interval family I

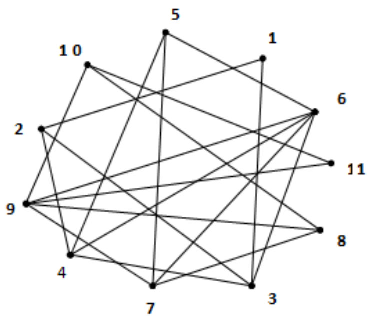

Fig. 2: Interval graph G

Dominating Set = {3,5,9}, $\gamma_{ns}(G)=3$

IV. TO FIND THE DISTANCES FROM G

d(1,1)=0	d(2,1)=1	d(3,1)=1	d(4,1)=2	d(5,1)=3	d(6,1)=2
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=1	d(5,2)=2	d(6,2)=2
d(1,3)=1	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=2	d(6,3)=1
d(1,4)=2	d(2,4)=1	d(3,4)=1	d(4,4)=0	d(5,4)=1	d(6,4)=1
d(1,5)=3	d(2,5)=2	d(3,5)=2	d(4,5)=1	d(5,5)=0	d(6,5)=1
d(1,6)=2	d(2,6)=2	d(3,6)=1	d(4,6)=1	d(5,6)=1	d(6,6)=0
d(1,7)=3	d(2,7)=3	d(3,7)=2	d(4,7)=2	d(5,7)=1	d(6,7)=1
d(1,8)=4	d(2,8)=4	d(3,8)=3	d(4,8)=3	d(5,8)=2	d(6,8)=2
d(1,9)=3	d(2,9)=3	d(3,9)=2	d(4,9)=2	d(5,9)=2	d(6,9)=1
d(1,10)=4	d(2,10)=4	d(3,10)=3	d(4,10)=3	d(5,10)=3	d(6,10)=2
d(1,11)=4	d(2,11)=4	d(3,11)=3	d(4,11)=3	d(5,11)=3	d(6,11)=2

d(7,1)=3	d(8,1)=4	d(9,1)=3	d(10,1)=4	d(11,1)=4
d(7,2)=3	d(8,2)=4	d(9,2)=3	d(10,2)=4	d(11,2)=4
d(7,3)=2	d(8,3)=3	d(9,3)=2	d(10,3)=3	d(11,3)=3
d(7,4)=2	d(8,4)=3	d(9,4)=2	d(10,4)=3	d(11,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2	d(10,5)=3	d(11,5)=3
d(7,6)=1	d(8,6)=2	d(9,6)=1	d(10,6)=2	d(11,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1	d(10,7)=2	d(11,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1	d(11,8)=2
d(7,9)=1	d(8,9)=1	d(9,9)=0	d(10,9)=1	d(11,9)=1
d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0	d(11,10)=1
d(7,11)=2	d(8,11)=2	d(9,11)=1	d(10,11)=1	d(11,11)=0

V. TO FIND THE AVERAGE DISTANCE OF G

Table-1

Vertices	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	2	3	2	3	4	3	4	4
2	1	0	1	1	2	2	3	4	3	4	4
3	1	1	0	1	2	1	3	3	2	3	3
4	2	1	1	0	1	1	2	3	2	3	3
5	3	2	2	1	0	1	1	2	2	3	3
6	2	2	1	1	1	0	1	2	1	2	2
7	3	3	2	2	1	1	0	1	1	2	2
8	4	4	3	3	2	2	1	0	1	1	2
9	3	3	2	2	2	1	1	1	0	1	1
10	4	4	3	3	3	2	1	1	1	0	1
11	4	4	3	3	3	2	2	2	1	1	0
Total	27	25	19	19	20	15	18	23	17	24	25

Therefore Average distance

$$\mu(G) = \frac{1}{2\ {}^{n}c_{2}} \sum_{\substack{i,j \in v(G) \\ i \neq j}} \delta(i,j).$$

$$\mu(G) = \frac{1}{11 \times 10} 232$$

$$\mu(G) = 2.109$$

Therefore 3 > 2.109.

Therefore $\gamma_{ns}(G) > \mu(G)$.

Theorem.2: Let $\gamma_{ns}(D)$ be a domination number of the given interval graph G. If i and j are two intervals in I such that $i \in \gamma_{ns}(D)$ and j=1 and j is contained in i and if there is one or more intervals other than i that intersect j, then the non split domination number $\gamma_{ns}(D) > \mu(G)$.

Proof: Let I be an interval family and G is an interval graph corresponding to I. Let j=1 be the interval contained in i where $i \in \gamma_{ns}(D)$. Suppose k is an interval, $k \neq i$ and k intersect j. since $i \in \gamma_{ns}(D)$, $< V - \gamma_{ns}(D) >$ does not contain i. further in $V - \gamma_{ns}(D)$, the vertex j is adjacent to the vertex k and hence there will not be any disconnection in $V - \gamma_{ns}(D)$.

Therefore we get non split domination number in G. We have already proved the average distance $\mu(G)$ in theorem.1.

VI. EXPERIMENTAL PROBLEM OF THEOREM.2

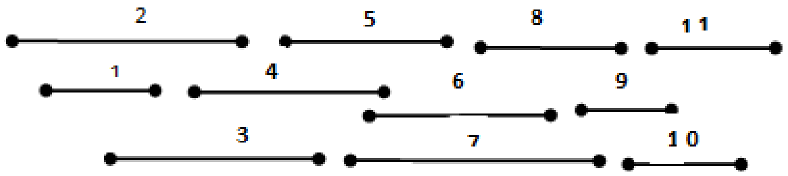

Fig. 3: Interval family I

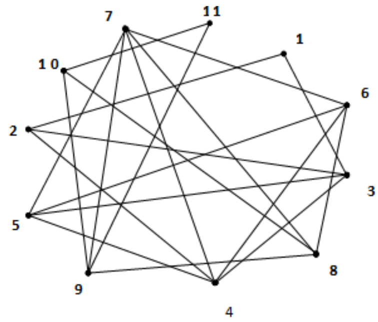

Fig. 2.2: Interval graph G
Dominating Set = {3,8,11}, $\gamma_{ns}(G)=3$

VII. TO FIND THE DISTANCES FROM G

d(1,1)=0	d(2,1)=1	d(3,1)=1	d(4,1)=2	d(5,1)=2	d(6,1)=3
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=1	d(5,2)=2	d(6,2)=2
d(1,3)=1	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=1	d(6,3)=2
d(1,4)=2	d(2,4)=1	d(3,4)=1	d(4,4)=0	d(5,4)=1	d(6,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1	d(4,5)=1	d(5,5)=0	d(6,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2	d(4,6)=1	d(5,6)=1	d(6,6)=0
d(1,7)=3	d(2,7)=2	d(3,7)=2	d(4,7)=1	d(5,7)=1	d(6,7)=1
d(1,8)=4	d(2,8)=3	d(3,8)=3	d(4,8)=2	d(5,8)=2	d(6,8)=1
d(1,9)=4	d(2,9)=3	d(3,9)=3	d(4,9)=2	d(5,9)=2	d(6,9)=2
d(1,10)=5	d(2,10)=4	d(3,10)=4	d(4,10)=3	d(5,10)=3	d(6,10)=2
d(1,11)=5	d(2,11)=4	d(3.11)=4	d(4.11)=3	d(5,11)=3	d(6,11)=3

d(7,1)=3	d(8,1)=4	d(9,1)=4	d(10,1)=5	d(11,1)=5
d(7,2)=2	d(8,2)=3	d(9,2)=3	d(10,2)=4	d(11,2)=4
d(7,3)=2	d(8,3)=3	d(9,3)=3	d(10,3)=4	d(11,3)=4
d(7,4)=1	d(8,4)=2	d(9,4)=2	d(10,4)=3	d(11,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2	d(10,5)=3	d(11,5)=3
d(7,6)=1	d(8,6)=1	d(9,6)=2	d(10,6)=2	d(11,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=1	d(10,7)=2	d(11,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1	d(11,8)=2
d(7,9)=1	d(8,9)=1	d(9,9)=0	d(10,9)=1	d(11,9)=1
d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0	d(11,10)=1
d(7,11)=2	d(8,11)=2	d(9,11)=1	d(10,11)=1	d(11,11)=0

VIII. TO FIND THE AVERAGE DISTANCE OF G

Table-2

Vertices	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	2	2	3	3	4	4	5	5
2	1	0	1	1	2	2	2	3	3	4	4
3	1	1	0	1	1	2	2	3	3	4	4
4	2	1	1	0	1	1	1	2	2	3	3
5	2	2	1	1	0	1	1	2	2	3	3
6	3	2	2	1	1	0	1	1	2	2	3
7	3	2	2	1	1	1	0	1	1	2	2
8	4	3	3	2	2	1	1	0	1	1	2
9	4	3	3	2	2	2	1	1	0	1	1

10	**5**	**4**	**4**	**3**	**3**	**2**	**2**	**1**	**1**	**0**	**1**
11	**5**	**4**	**4**	**3**	**3**	**3**	**2**	**2**	**1**	**1**	**0**
Total	**30**	**23**	**22**	**17**	**18**	**18**	**16**	**20**	**20**	**25**	**28**

Therefore Average distance

$$\mu(G)=\frac{1}{2\ {}^{n}c_2}\sum_{\substack{i,j\in v(G)\\ i\neq j}}\delta(i,j).$$

$$\mu(G)=\frac{1}{11\times 10}237$$

$$\mu(G)=2.1545$$

Therefore $\gamma_{ns}(D) > \mu(G)$.

Theorem.3: Let D be a dominating set of the given interval graph G. If i, j, k are three conjugate intervals such that i < j < k, and $j\in \gamma_{ns}(D)$, i intersect j, j intersect k and i intersect k. Then non-split domination number $\gamma_{ns}(D)$ equal to $\mu(G)$.

Proof: Let I = {i, j, k,….., n}intervals and let G be an interval graph I. Let i, j, k be three conjugative intervals satisfying the hypothesis. Now i and k intersect implies that i and k are adjacent in v- $\gamma_{ns}(D)$. So that there will not be any disconnection in the induced sub graph < v- $\gamma_{ns}(D)$ > is connected. $\mu(G)$ is the average distance of G, we have already proved in theorem.1.

IX. EXPERIMENTAL PROBLEM OF THEOREM.3

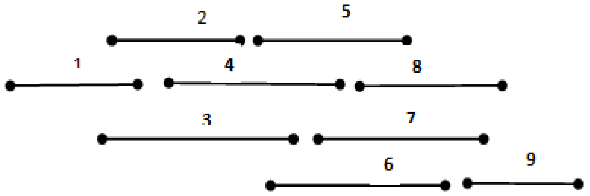

Fig. 2.1: Interval family I

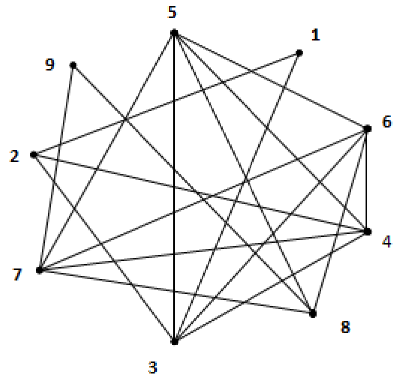

Fig. 2.2: Interval graph G

Dominating Set = {3,8}, $\gamma_{ns}(G)=2$

X. TO FIND THE DISTANCES FROM G

d(1,1)=0	d(2,1)=1	d(3,1)=1	d(4,1)=2	d(5,1)=2
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=1	d(5,2)=2
d(1,3)=1	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=1
d(1,4)=2	d(2,4)=1	d(3,4)=1	d(4,4)=0	d(5,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1	d(4,5)=1	d(5,5)=0
d(1,6)=2	d(2,6)=2	d(3,6)=1	d(4,6)=1	d(5,6)=1
d(1,7)=3	d(2,7)=2	d(3,7)=2	d(4,7)=1	d(5,7)=1
d(1,8)=3	d(2,8)=3	d(3,8)=2	d(4,8)=2	d(5,8)=1
d(1,9)=4	d(2,9)=3	d(3,9)=3	d(4,9)=2	d(5,9)=2

d(6,1)=2	d(7,1)=3	d(8,1)=3	d(9,1)=4
d(6,2)=2	d(7,2)=2	d(8,2)=3	d(9,2)=3
d(6,3)=1	d(7,3)=2	d(8,3)=2	d(9,3)=3
d(6,4)=1	d(7,4)=1	d(8,4)=2	d(9,4)=2
d(6,5)=1	d(7,5)=1	d(8,5)=1	d(9,5)=2
d(6,6)=0	d(7,6)=1	d(8,6)=1	d(9,6)=2
d(6,7)=1	d(7,7)=0	d(8,7)=1	d(9,7)=1
d(6,8)=1	d(7,8)=1	d(8,8)=0	d(9,8)=1
d(6,9)=2	d(7,9)=1	d(8,9)=1	d(9,9)=0

XI. TO FIND THE AVERAGE DISTANCE OF G

Table-2

Vertices	1	2	3	4	5	6	7	8	9
1	0	1	1	2	2	2	3	3	4
2	1	0	1	1	2	2	2	3	3
3	1	1	0	1	1	1	2	2	3
4	2	1	1	0	1	1	1	2	2
5	2	2	1	1	0	1	1	1	2
6	2	2	1	1	1	0	1	1	2
7	3	2	2	1	1	1	0	1	1
8	3	3	2	2	1	1	1	0	1
9	4	3	3	2	2	2	1	1	0
Total	**18**	**15**	**12**	**11**	**11**	**11**	**12**	**14**	**18**

Therefore Average distance

$$\mu(G)=\frac{1}{2\ {}^{n}C_{2}}\sum_{\substack{i,j\in v(G)\\ i\neq j}}\delta(i,j).$$

$$\mu(G)=\frac{1}{9\times 8}122$$

$$\mu(G)=1.69.$$

Therefore $\gamma_{ns}(D) > \mu(G)$.

XII. CONCLUSIONS

The interval graphs are rich in combinatorial structure and have found applications in several disciplines such a Traffic control and computer science and particularly useful in real line scheduling and computer storage allocation problems in this paper we individualized an interval graphs as various graphs. We then extended the results to trace out a specific type of an interval graphs having every pair of vertices as to find the comparison of non-split domination number and the average distance of an interval graph.

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