International Journal of
Emerging Research in Management &Technology
ISSN: 2278-9359 (Volume-4, Issue-11)

Research Article | November 2015

Crop Production Using Interval-valued Intuitionistic Fuzzy TOPSIS Method

A. Sahaya Sudha*
Assistant Professor,
Nirmala College for Women,
Coimbatore, Tamil Nadu, India

J. Rachel Inba Jeba
Mphil Scholar,
Nirmala College for Women,
Coimbatore, Tamil Nadu, India

Abstract—

The socioeconomic environment becomes more complex, the preference information provided by decision-makers is usually imprecise; that is, there may be hesitation or uncertainty about preferences because a decision should be made under time pressure and lack of knowledge or data, or the decision-makers have limited attention and information processing capacities. In such cases, it is suitable and convenient to express the decisionmakers' preferences by IVIFSs. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. In this paper, an interval-valued intuitionistic fuzzy TOPSIS method based on the improved score functionto solve multi-criteria decision-making problems in which the performance rating values are expressed by IVIFSs.

Keywords—Intuitionistic Fuzzy Set, TOPSIS, Interval-valued Intuitionistic Fuzzy Set, Interval-valued Intuitionistic Fuzzy Number.

I. INTRODUCTION

K. Atanassov [1] generalized the L. A. Zadeh's fuzzy sets [12] and a higher order fuzzy set i.e. Intuitionistic fuzzy set (IFS) and later on K. Atanassov and Gorgav further introduced the interval-valued Intuitionistic fuzzy sets (IVIFS). The characteristics of IVIFS are the values of its membership functions and non-membership functions which are intervals rather than exact numbers. Entropy of fuzzy set describes the fuzziness degree of a fuzzy set and was first mentioned by L. A. Zadeh [12] in 1965. In 1972 A. Deluca and S. Termini [3] presented some axioms to describe the fuzziness degree of fuzzy set, with which fuzzy entropy based on Shannon's function was proposed. Later on in 1975 A. Kaufmann [7] proposed a method for measuring the fuzziness degree of a fuzzy set by a metric distance between its membership function and the membership function of its nearest crisp set.J. H. Park et.al., [9] extended the TOPSIS method to solve multiple attribute group decision making problems under interval-valued intuitionistic fuzzy environment in which all the preference information provided by the decision-makers is presented as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by an interval-valued intuitionistic fuzzy number (IVIFN), and the information about attribute weights is partially known. H. Lai and Chen [8] extended a similarity measure in the technique for order preference based on similarity to the ideal solution (TOPSIS) approach by measuring the similarity of each alternative to positive and negative ideal interval-valued fuzzy numbers (IVFNs) and applied the similarity measure between IVFNs to the decision-making process to increase the ability of the process to account for risks in a variable, complex, and uncertain environment.

II. PRELIMINARIES

A. Definition

Let "X" be the universal set. A fuzzy set $\widetilde{A}$ in X represented by $\widetilde{A}=\{(x,\mu_{\widetilde{A}}(x))/\,x\in X\}$, where the function $\mu_{\widetilde{A}}(x): X\rightarrow[0,1]$ is the membership degree of element x in the fuzzy set X.

B. Definition

An Intuitionistic Fuzzy Set (IFS) $\widetilde{A}^I$ in X is defined as an object of the form $\widetilde{A}^I=\{\langle x,\mu_{\widetilde{A}^I}(x),\nu_{\widetilde{A}^I}(x)\rangle : x\in X\}$ where the functions $\mu_{\widetilde{A}^I}:X\rightarrow[0,1]$ and $\nu_{\widetilde{A}^I}:X\rightarrow[0,1]$ define the degree of the membership and the degree of non-membership of the element $x\in X$ in $\widetilde{A}^I$, $0\le\mu_{\widetilde{A}^I}(x)+\nu_{\widetilde{A}^I}(x)\le 1$ holds.

For every common fuzzy subset $\widetilde{A}$ on X, Intuitionistic Fuzzy Index of x in $\widetilde{A}^I$ is defined as $\pi_{\widetilde{A}^I}(x)=1-\mu_{\widetilde{A}^I}(x)-\nu_{\widetilde{A}^I}(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in $\widetilde{A}^I$. Obviously, for every $x\in X$, $0\le\pi_{\widetilde{A}^I}(x)\le 1$.

Noted that, A is a crisp set if and only if $\forall\, x\in X$, either $\mu_A(x)=0,\nu_A(x)=1$ or $\mu_A(x)=1,\nu_A(x)=0$.

Note: Throughout this paper, μ represents membership values and ν represents non membership values.

C. Definition

A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single valued) numbers. Any fuzzy number can be thought of as a function whose domain is a specified set. Each numerical value in the domain is assigned a specific "grade of membership".

D. Definition

An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is

a. an intuitionistic fuzzy subset of the real line,

b. convex for the membership function $\mu_{\tilde{A}^I}(x)$, that is $\mu_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\geq \min(\mu_{\tilde{A}^I}(x_1),\mu_{\tilde{A}^I}(x_2))$, for every $x_1, x_2 \in R, \lambda \in [0,1]$.

c. concave for the membership function $v_A(x)$, that is, $v_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\leq \max(v_{\tilde{A}^I}(x_1),v_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.

d. normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0)=1, v_{\tilde{A}^I}(x_0)=0$.

E. Definition

Let X be a universe of discourse and $\text{int}(0,1)$ denote all closed subintervals of the interval $[0,1]$. An interval-valued intuitionistic fuzzy set $\tilde{A}^I$ in X is an object having the form

$$\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), v_{\tilde{A}^I}(x)\rangle \mid x \in X\},$$

where $\mu_{\tilde{A}^I} : X \to \text{int}(0,1)$, $v_{\tilde{A}^I} : X \to \text{int}(0,1)$ with the condition $0 \leq \sup(\mu_{\tilde{A}^I}(x)) + \sup(v_{\tilde{A}^I}(x)) \leq 1$.

The intervals $\mu_{\tilde{A}^I}(x)$ and $v_{\tilde{A}^I}(x)$ denote the degree of membership and non-membership of x to $\tilde{A}^I$, respectively. For convenience, let $\mu_{\tilde{A}^I}(x)=[\mu^-_{\tilde{A}^I}(x),\mu^+_{\tilde{A}^I}(x)]$, $v_{\tilde{A}^I}(x)=[v^-_{\tilde{A}^I}(x),v^+_{\tilde{A}^I}(x)]$, so

$$\tilde{A}^I = \{\langle x, [\mu^-_{\tilde{A}^I}(x),\mu^+_{\tilde{A}^I}(x)], [v^-_{\tilde{A}^I}(x),v^+_{\tilde{A}^I}(x)]\rangle \mid x \in X\}$$

with the condition $0 \leq \mu^+_{\tilde{A}^I}(x) + v^+_{\tilde{A}^I}(x) \leq 1$.

We call the interval $[1-\mu^+_{\tilde{A}^I}(x)-v^+_{\tilde{A}^I}(x), 1-\mu^-_{\tilde{A}^I}(x)-v^-_{\tilde{A}^I}(x)]$, abbreviated by $[\pi^-_{\tilde{A}^I}(x),\pi^+_{\tilde{A}^I}(x)]$ or $\pi_{\tilde{A}^I}(x)$, the interval-valued intuitionistic index of x in $\tilde{A}^I$, which is a hesitancy degree of x to $\tilde{A}^I$.

Clearly, if $\mu_{\tilde{A}^I}(x)=\mu^-_{\tilde{A}^I}(x)=\mu^+_{\tilde{A}^I}(x)$ and $v_{\tilde{A}^I}(x)=v^-_{\tilde{A}^I}(x)=v^+_{\tilde{A}^I}(x)$, then the given IVIFS $\tilde{A}^I$ is reduced to an ordinary IFS.

F. Definition

An IVIFS value denoted by $\tilde{A}^I = ([a,b]);([c,d])$ for convenience. In order to make comparisons between two IVIFSs, some metric methods should be introduced by the following score function and accuracy functions.

A score functions to measure the degree of suitability of a IVIFN $\tilde{A}^I$ as follows:

$$s(\tilde{A}^I)=\frac{1}{2}(a-c+b-d)$$

where $s(\tilde{A}^I)\in[-1,1]$. The larger the value of $s(\tilde{A}^I)$, the higher the IVIFN $\tilde{A}^I$. Especially, if $s(\tilde{A}^I)=1$, then $\tilde{A}^I=\langle[1,1],[0,0]\rangle$, which is the larger IVIFN ; if $s(\tilde{A}^I)=-1$ then $\tilde{A}^I=\langle[1,1],[0,0]\rangle$, which is the smallest IVIFN.

An accuracy function h to evaluate the accuracy degree of a IVIFN as follows:

$$h(\tilde{A}^I)=\frac{1}{2}(a+b+c+d),$$

where $h(\tilde{A}^I)\in[0,1]$. The larger value of $h(\tilde{A}^I)$, the higher the degree of accuracy of the IVIFN $\tilde{A}^I$.

Novel accuracy function:

$$M(A)=\frac{a-(1-a-c)+b-(1-b-d)}{2}, \qquad M(A)\in[-1,+1].$$

Another accuracy function:

$$L(A)=\frac{a+b-d(1-b)-c((1-a)}{2},\ L(A)\in[-1,+1].$$

G. Definition Operational laws of IVIFNs

$Let\ \tilde{a}_1=\langle [a_1,b_1],[c_1,c_2]\rangle, \tilde{a}_2=\langle [a_2,b_2],[c_2,d_2]\rangle\ and\ \tilde{a}=\{[a,b],[c,d]\}$ be three IVIFNs; then

i) $\tilde{a}_1\otimes\tilde{a}_2=\langle [a_1a_2,b_1b_2],[c\ \ +c_2-c_1c_2,d_1+d_2-d_1d_2]\rangle$;

ii) $\tilde{a}^{\lambda}=\langle [a^{\lambda},b^{\lambda}],[1-(1-c)^{\lambda},1-(1-d)^{\lambda}]\rangle, \lambda>0$

iii) $\lambda\tilde{a}=\langle [1-(1-a)^{\lambda},1-(1-b)^{\lambda}],[c^{\lambda},d^{\lambda}]\rangle, \lambda>0$

H. Definition

For two IFSs $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in X\}$ and $B=\{\langle x,\nu_B(x),\nu_B(x)\rangle | x\in X\}$, their relations are defined as follows;

1. $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x), \nu_A(x)\ge\nu_B(x)$ for each $x\in X$
2. $A=B$ if and only if $A\subseteq B$ and $B\subseteq A$

III. PROPOSED METHOD

Step 1:

In a multi-criteria decision-making problem, suppose that there exists a set of alternatives $A=\{A_1,A_2,....,A_m\}$. Each alternatives is assessed on n criteria, which are denoted by $C=\{C_1,C_2,...,C_n\}$. The characteristics of an alternative A_i with respect to a criterion C_j can be represented by an IVIFS value $x_{ij}=([a_{ij},b_{ij}],[c_{ij},d_{ij}])\ (i=1,2,...,m;\ j=1,2,...,n)$, which can represent the membership degree and non-membership degree of the alternative $A_i\in A$ with respect to the criterion $C_j\in C$. The interval-valued intuitionistic fuzzy decision matrix $\tilde{D}$ is defined as the following form;

$$\tilde{D}=\begin{pmatrix} [a_{11},b_{11}],[c_{11},d_{11}] & [a_{12},b_{12}],[c_{12},d_{12}] & [a_{13},b_{13}],[c_{13},d_{13}]\ldots\ldots & [a_{1n},b_{1n}],[c_{1n},d_{1n}] \\ [a_{21},b_{21}],[c_{21},d_{21}] & [a_{22},b_{22}],[c_{22},d_{22}] & [a_{23},b_{23}],[c_{23},d_{23}]\ldots\ldots & [a_{2n},b_{2n}],[c_{2n},d_{2n}] \\ . & . & . \quad . & . \\ . & . & . \quad . & . \\ . & . & . \quad . & . \\ [a_{m1},b_{m1}],[c_{m1},d_{m1}] & [a_{m2},b_{m2}],[c_{m2},d_{m2}] & [a_{m3},b_{m3}],[c_{m3},d_{m3}]\ldots\ldots & [a_{mn},b_{mn}],[c_{mn},d_{mn}] \end{pmatrix}$$

Step 2:

Let $\tilde{A}^I=([a,b],[c,d])$ be an IVIFS value, its improved score function based on the unknown degree is proposed by,

$$I(\tilde{A}^I)=\frac{a+a(1-a-c)+b+b(1-b-d)}{2} \tag{1}$$

where $I(\tilde{A}^I)\in[0,1]$. Especially, when $a=b$ and $c=d$, an IVIFS is degenerated to an IFS, and then the improved score function of IVIFS is degenerated to the score function of IFS.

Based on the improved score function, the interval-valued intuitionistic fuzzy decision matrix $\tilde{D}$ is converted into the following score matrix $\tilde{R}^I(I_{ij}(x_{ij}))$

$$\tilde{R}^I(I_{ij}(x_{ij}))=\ .\ \begin{pmatrix} I_{11}(x_{11}) & I_{12}(x_{12}) & \ldots & I_{1n}(x_{1n}) \\ I_{21}(x_{21}) & I_{22}(x_{22}) & \ldots & I_{2n}(x_{2n}) \\ . & . & . & \\ . & . & . & . \\ . & . & . & . \\ I_{m1}(x_{m1}) & I_{m2}(x_{m2}) & \ldots & I_{mn}(x_{mn}) \end{pmatrix}$$

Assuming that the weight of the criterion $C_j\ (j=1,2,....,n)$, entered by the decision-maker, is $w_j, w_j\in[0,1]$ and $\sum_{j=1}^{n}x_j=1$.

Then the positive ideal solution for the alternatives is denoted by $A^{+} = \left\{ \left\langle C_j, [1,1], [0,0] \right\rangle \mid C_j \in C \right\}$, $j = 1,2,\ldots.,n$, and the negative ideal solution for the alternatives is denoted by $A^{-} = \left\{ \left\langle C_j, [0,0], [1,1] \right\rangle \mid C_j \in C \right\}$ and $j = 1,2,\ldots.,n$.

Step 3:

The score function-based on separation measures $d_i^{+}\left(A^{+}, A_i\right)$ and $d_i^{-}\left(A^{-}, A_i\right)$ of each alternatives from the positive ideal and negative ideal solutions, respectively, are derived by the following forms:

$$d_i^{+}\left(A^{+}, A_i\right) = \sqrt{\sum_{j=1}^{n} \left[w_j \left(1 - I_{ij}\left(x_{ij}\right)\right) \right]^2}\ , \qquad (2)$$

$$d_i^{-}\left(A^{-}, A_i\right) = \sqrt{\sum_{j=1}^{n} \left[w_j \left(I_{ij}\left(x_{ij}\right)\right) \right]^2} \qquad (3)$$

Hence, the relative closeness of an alternatives A_i with respect to the positive ideal solution A^{+} is defined as the following formula:

$$C_i\left(A_i\right) = \frac{d_i^{-}\left(A^{-}, A_i\right)}{d_i^{+}\left(A^{+}, A_i\right) + d_i^{-}\left(A^{-}, A_i\right)} \qquad (4)$$

where $C_i\left(A_i\right)\ \left(i = 1,2,\ldots.,m\right)$ is the relative closeness coefficient of A_i with respect to the positive ideal solution A^{+} and $0 \le C_j\left(A_j\right) \le 1$. Therefore, the alternatives can be ranked according to the descending order of $C_i\left(A_i\right)$. The alternative with the highest value of $C_i\left(A_i\right)$ will be the best choice.

IV. NUMERICAL EXAMPLE

An agricultural cost of cultivation problem is considered where five different types of crops such as Paddy, Cholam, Maize, Blackgram and groundnut are considered as alternatives as A_1,A_2,A_3.A_4.A_5 and analysed with the criteria Ploughing and Land cost, Fertilizer and Manures, Irrigation charges, Cost of labour and Income from the crop as C_1,C_2,C_3,C_4 and C_5 respectively.

Table I Collected data

Alternatives	Criteria				
	Ploughing and Land cost	**Fertilizers and Manures**	**Irrigation charges**	**Cost of Production**	**Income from the Crop**
Paddy	9093	7934	332	1424	182000
Cholam	2932	2404	0	1138	31500
Maize	7345	11032	332	1287	108000
Blackgram	2513	2093	0	4314	32600
Groundnut	3523	7344	1838	2962	115000

Step 1:

$$\tilde{D} = \begin{pmatrix} \text{[0.5, 0.6], [0.1 , 0.2][0.5, 0.6] ,[0.15, 0.2][0.05, 0.1] , [0.45, 0.75] [0.2 , 0.3] , [0.4 , 0.5] [0.8, 0.9] , [0.5 . 0.1]} \\ \text{[0.3, 0.4] , [0.25 , 0.35][0.3 , 0.4] , 0.35 , 0.4] [0.0 , 0.1] , [0.05 , 0.09] [0.1 , 0.2] , [0.5 , 0.8] [0.6 , 0.7] [0.15 , 0.2]} \\ \text{[0.4 , 0.3] , [0.1 , 0.2][0.5 , 0.7] , [0.1 , 0.2] [0.05, 0.1] , [0.45 , 0.75][0.1 , 0.2] , [0.5 , 0.6] [0.7 , 0.8] ,[0.1 , 0.2]} \\ \text{[0.3 , 0.4] , [0.3 , 0.4][0.3, 0.4] , [0.4 , 0.5][0.0 , 0.1] , [0.05 , 0.09][0.4 , 0.5] , [0.3 , 0.4] [0.6 , 0.7 , [0.1 , 0.2]} \\ \text{[0.4 , 0.5] , [0.4 , 0.5] [0.4 , 0.5], [0.15 , 0.2][0.2 , 0.3] , [0.4 , 0.6] [0.3 , 0.4] , [0.2 , 0.3] [0.7, 0.8], [0.05, 0.1]} \end{pmatrix}$$

Step 2:

By using(1), convert the interval-valued intuitionistic fuzzy decision matrix $\tilde{D}$ into the following score matrix $\tilde{R}^{I}\left(I_{ij}\left(x_{ij}\right)\right)$:

$$\tilde{R}^{I}\left(I_{ij}\left(x_{ij}\right)\right) = \begin{pmatrix} 0.71 & 0.6975 & 0.095 & 0.32 & 0.91 \\ 0.4675 & 0.4425 & 0.0905 & 0.17 & 0.76 \\ 0.625 & 0.735 & 0.095 & 0.19 & 0.82 \\ 0.45 & 0.415 & 0.0905 & 0.535 & 0.775 \\ 0.79 & 0.615 & 0.305 & 0.485 & 0.8775 \end{pmatrix}$$

Step 3:

$$d_i^+\left(A^+, A_i\right) = \sqrt{\sum_{j=1}^{n}\left[w_j\left(1 - I_{ij}\left(x_{ij}\right)\right)\right]^2}$$

where the weights are consider as

$W = (0.1738, 0.0957, 0.1941, 0.0316, 0.5048)$

$$d_1^+ = \sqrt{[(0.1738(1-0.71))^2 + (0.0957(1-0.6975))^2 + (0.1941(1-0.095))^2 + (0.0316(1-0.32))^2 + (0.05048(1-0.91))^2}$$

$$d_1^+ = \sqrt{0..3676083271}$$

$$= 0.1917$$

$$d_i^-\left(A^-, A_i\right) = \sqrt{\sum_{j=1}^{n}\left[w_j\left(I_{ij}\left(x_{ij}\right)\right)\right]^2}$$

$$d_1^- = \sqrt{(0.1738 \times 0.71)^2 + (0.6975 \times 0.957)^2 + (0.1941 \times 0.095)^2 + (0.32 \times 0.036)^2 + (0.91 \times 0.5048)^2}$$

$$d_1^- = \sqrt{0.2311439562}$$

$$= 0.4808$$

Table II Ranking alternatives

$d_i^+\left(A^+, A_i\right)$	$d_i^-\left(A^-, A_i\right)$	$c_i\left(A_i\right)$	**Rank**
0.1917	0.4808	0.7149	1
0.2407	0.3949	0.6213	5
0.2113	0.4341	0.6726	3
0.2378	0.4017	0.6366	4
0.1775	0.4590	0.7211	2

The ranking order of the five alternatives is A_5, A_1, A_3, A_4, A_2. Obviously, amongst them, A_5 (Groundnut) is 0.7211.

V. CONCLUSIONS

In this paper, investigate the decision making problem under interval-valued intuitionistic fuzzy environment, and extended TOPSIS method to handling the situation where the attribute values are characterized by IVIFSs and the information about criteria weight is partially known. The proposed approaches first usually interval-valued intuitionistic fuzzy decision matrices into improved score function. The proposed score function to calculate the separation measures of each alternative from the positive and negative ideal solution to determine the relative closeness coefficients. According to the values of the closeness coefficients, the alternatives can be ranked and the most desirable one can be selected in the decision-making process. The highest value of A_5 (Groundnut) is the best choice and viable represents the criteriaof ploughing and land cost, fertilizers and manures, irrigation charges, cost of production and income from the crop.

REFERENCES

[1] Attanassov. K., Intuitionistic Fuzzy Set, *Fuzzy set and systems*, vol 30, no.1, 87 – 96, 1986.

[2] Cuiping Wei and Yuzhong Zhang, Entropy Measures for Interval – Valued Intuitionistic Fuzzy Sets and Decision – Making, *Hindwai Publishing corporation.*, 13, 2015.

[3] De Luca. A. and S. Termini, A Definition of a Non tic Entropy in the Setting of Fuzzy Sets theory, *Information and control*, vol 20, 301 – 302, 1972.

[4] Dong Gun Park , Young Chel Kwun, Jin Han Park and II Young Park , Correlation Coefficient of Interval – Valued Intuitionistic Fuzzy Sets its Application to Multiple Attribute Group Decision Making Problems , *Mathematical and Computer Modelling*,50, 1279 – 1293, 2009.

[5] Henry Amirtharaj. E. C., John Robinson. P., Attributes weight determination for Interval Valued Intuitionistic Fuzzy Multiple Attribute Group Decision Making Problems, *International Journal Of computing Algorithm*, vol 2, 339 – 353, 2013.

[6] Jin Han Park , II Young Park, Young Chel Kwun and Xuegong Tan, Extension of the Topsis Method for Decision Making Problems Under Interval – Valued Intuitionistic fuzzy environment,*Applied Mathematical Modelling*, 35,pp.2544 – 2556, 2011.

[7] Kaufmann, A .,Introduction to the Theory of Fuzzy Subsets vol 1 , Fundamental Theoretical Elements, *Accadamic Press,New York*, 1975.

[8] Lai H. and Tchen, A Fuzzy risk-assessment method using a TOPSIS approach based on interval-valued Fuzzy numbers, *Journal of the Chinese Institute of Industrial Engineers*, Vol 28(6), 467-484, 2011.
[9] Park J. H., Park I .Y., Kwun Y. C . and Tan X., Extensionof the Topsis method for Decision making problems under interval – valued Intuitionistic Fuzzy Environment, *Applied Mathematical Modelling*,Vol.35, no 5 pp 2544 – 2556, 2011.
[10] Rajeev Kaushik and Rakesh K Bajaj , On Exponential Interval Valued Intuitionistic Fuzzy Entropy Of order α and β and its Applications in Decision Making, *International Journal of Computer Applications*, vol.95, 0975 – 8887, 2014.
[11] Sahaya Sudha. A and Rachel Inba Jeba .J, An agricultural Economic Problem Using Intuitionistic Fuzzy, *International Journal of Multidisciplinary and Current Research*, vol 3, 2013.
[12] Zadeh. L. A., Fuzzy sets information and control ,8, 338 – 353.
[13] Zhi – Young Bai , An Interval – Valued Intuitionistic Fuzzy Topsis Method Based on an Improved Score Function , *Hindwai Publishing Corporation* ,2013.