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Study and Analysis of Sound Transmission through Multilayered Structures Using Generalized Matrix Method

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Abstract-

In this paper, Generalized Matrix Method (GMM) is used to find out the transmission loss of multilayered structures. Classical solution for sound transmission is available only for two media and three media. Therefore, by the use of GMM, generalized matrix for n-media is obtained and a matlab code that can be used to calculate the sound transmission loss (STL) in decibels, for any number of layers has been developed. Sound transmission loss of multilayered structures with different materials is found out using this method. Double fiberboard walls and EPS sandwich panels are analyzed to obtain sound transmission loss in 1/3rd octave bands between 125 Hz and 4 kHz. Comparison with the Transfer Matrix Method is also carried out. Validation of GMM is done by comparing with the published experimental results measured in third octave bands with the pressure method according to ISO 140-3 standards. The result show good agreement with the experimental results in low frequency range and tends to overestimate the sound transmission loss in high frequency range.

Keywords- GMM, TMM, Transmission loss, Multilayered structure, Plane wave propagation.

I. INTRODUCTION

In aerospace, automobile, and building applications, sufficient airborne and structure borne sound insulation is needed for a comfortable environment. So the use of multilayered structures has increased in these areas. The acoustic performances of these structures are usually specified in terms of transmission loss. Transmission loss is the logarithmic ratio of the sound energy incident on the panel to that transmitted by the panel. Therefore the prediction of sound transmission loss (STL) is of major importance. In this paper, Generalized Matrix Method (GMM)[1] is used to predict the sound transmission loss (STL) of multilayered structures.

Different methods available for the prediction of sound transmission loss are Transfer Matrix Method, Finite Element Method, Wave Based Model, and Statistic Energy Analysis etc. Classical solution for sound transmission is limited to two media and three media only. The analytical solution for sound transmission through multilayered structures is more complex and difficult to achieve if the structure consists of more than three layers of material. Therefore by generalized matrix method, which uses plane wave theory is used for overcoming these limitations and structures which consist of any number of layers can be analyzed without complexity.

II. ACOUSTIC MODELLING

A. Generalized Matrix Method

Generalized Matrix Method (GMM) is a numerical approach based on plane wave theory. Nelson *et al.*[1] proposed this method for the design of single expansion reactive mufflers. Coefficient matrix and complex pressure amplitude matrix is introduced and the matrix inverse technique is used for analysis. The analytical solution for multi-layer sound transmission is more complex and difficult to achieve if the media consists of more than three layers of material. By GMM approach, any number of layers can be analyzed easily. The advantage of GMM is that the mathematical complications are less and acoustical behavior of n- medium can be analyzed without the complexity of the equations.

1) *Sound Transmission through Two Media-Basic formulation*

In sound transmission through two media, the plane wave in medium I is travelling in the positive x-direction and is incident normally at the interface and part of the incident energy is reflected back and part of it is transmitted to the other side.

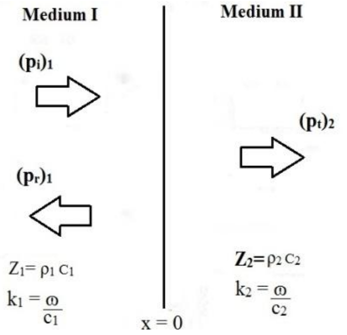

Fig.1. Reflection and transmission of plane waves at a boundary.

The boundary conditions for transmission are -

1. The acoustic pressures on the two sides of the boundary are equal
2. The particle velocities normal to the interface are equal.

Let the incident and reflected wave in medium 1 be-

$$(p_i)_1 = A_1 e^{j(\omega t - k1x)} \qquad (1)$$

$$(p_r)_1 = B_1 e^{j(\omega t + k1x)} \qquad (2)$$

Where,
p_i - Sound pressure of incident wave
p_r - Sound pressure of reflected wave
p_t - Sound pressure of transmitted wave
k – Wave number
c – Velocity of sound wave (m/s)
f – Frequency of sound wave in Hertz
A_1- Complex pressure amplitude of incident wave in the first medium.
B_1- Complex pressure amplitude of reflected wave in the first medium
ω - Angular frequency
Let the transmitted wave into medium II be-

$$(p_t)_2 = A_2\, e^{j(\omega t - k2x)} \qquad (3)$$

Where, A_2- Complex pressure amplitude of transmitted wave in the second medium

There is no reflected wave in medium II since it is assumed that medium II is extended to infinity.
At the interface, i.e. x = 0, continuity of pressure leads to

$$(p_i)_1 + (p_r)_1 = (p_t)_2 \qquad (4)$$

$$\text{i.e,} A_1 e^{j(\omega t)} + B_1 e^{j(\omega t)} = A_2\, e^{j(\omega t)} \qquad (5)$$

so that,

$$A_1 + B_1 = A_2 \qquad (6)$$

The particle velocities associated with these three waves can be represented by,
$u_i = \frac{p_i}{\rho_1 c_1}$, $u_r = \frac{p_r}{-\rho_1 c_1}$, $u_t = \frac{p_t}{\rho_2 c_2}$
Applying the second boundary condition, the continuity of velocity gives

$$u_i + u_r = u_t \qquad (7)$$

Substituting the respective expressions for the particle velocities at x=0 in the above equation gives,

$$\rho_2 c_2 \; (A_1 - B_1) = \rho_1 c_1\, A_2 (8)$$

Equations (6) and (8) can be combined to eliminate A_2

$$\frac{B_1}{A_1} = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2} (9)$$

Equations (6) and (10) can be combined to eliminate B_1

$$\frac{A_2}{A_1} = \frac{2\rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2} \qquad (10)$$

Intensity of a plane wave is given by,

$$I = \frac{p^2}{2\rho c}$$

Where,
ρ – Density of the medium.
The sound power reflection coefficient is given by,

$$\alpha_r = \frac{I_r}{I_i} = \frac{{B_1}^2}{{A_1}^2}$$

$$\alpha_r = \left(\frac{\rho_2 c_2 - \rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2}\right)^2$$

The sound power transmission coefficient is given by,

$$\alpha_t = \frac{I_t}{I_i}$$

$$\propto_t = \frac{4\, \rho_2 c_2 \rho_1 c_1}{(\rho_1 c_1 + \rho_2 c_2)^2} (11)$$

Transmission loss = $10 1og_{10}\left[\frac{1}{\alpha_t}\right]$
Assuming anechoic termination, the complex pressure amplitude in second medium,

$$A_2 = 1 \qquad (12)$$

In matrix form, first, second and third row is written using equations (6), (9), and (12) respectively.

$$\begin{pmatrix} 1 & 1 & -1 \\ \rho_2 c_2 & -\rho_2 c_2 & -\rho_1 c_1 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} A_1 \\ B_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The above matrix is in the form,

$$[M]_2 \times [A]_2 = [R]_2 \qquad (13)$$

where,
$[M]_2$ = co-efficient matrix for the two media
$[A]_2$ = complex pressure amplitude matrix for the two media
$[R]_2$ = right hand side matrix
Then the pressure amplitude matrix is given by,

$$[A]_2 = [M_2]^{-1} \times [R]_2$$

$$\text{Transmission loss} = 10 \log_{10} (A_1^2 / A_2^2) \qquad (14)$$

2) *Sound Transmission through Three Media-Basic formulation*

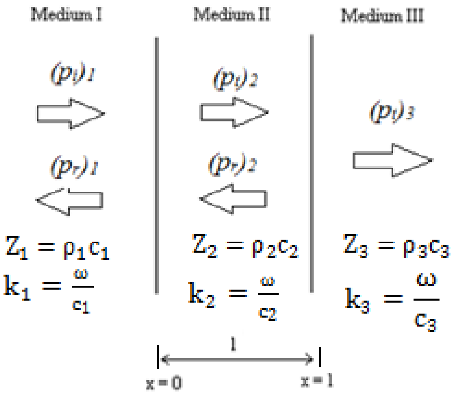

Fig.2. Transmission of plane waves across two boundaries.

Similarly in sound transmission through three media, the interface is having a thickness 'l' .The sound wave is incident at right angles to the first boundary and part of incident wave gets reflected and the transmitted sound wave into the second media is again subjected to reflection and absorption and the remaining part is transmitted to the third medium. The final equilibrium equations for the three media after introducing two boundary conditions are

$$A_1 + B_1 = A_2 + B_2 \qquad (15)$$

$$\rho_2 c_2 (A_1 - B_1) = \rho_1 c_1 (A_2 - B_2) \qquad (16)$$

$$(A_2 e^{-j(k2l)} + B_2 e^{j(k2l)}) = A_3 \qquad (17)$$

$$\rho_3 c_3 (A_2 e^{-j(k2l)} - B_2 e^{j(k2l)}) = \rho_2 c_2 A_3 \qquad (18)$$

$$A_3 = 1 \qquad (19)$$

The matrix formulation for sound transmission through three media is shown below.

$$\begin{pmatrix} 1 & 1 & -1 & -1 & 0 \\ Z_2 & -Z_2 & -Z_1 & Z_1 & 0 \\ 0 & 0 & e^{-jk_2 1} & e^{jk_2 1} & -1 \\ 0 & 0 & Z_3 e^{-jk_2 1} & -Z_3 e^{-jk_2 1} & -Z_2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The generalized matrix for any number of layers is generated by analyzing the matrix size of both two media and three media matrices. The co-efficient matrix for n-layers is of the order of (2n-1) x (2n-1) and the complex pressure amplitude matrix and right hand matrix is of the order of (2n-1) x1.The generalized coefficient matrix is given as M_n

$$\begin{pmatrix} 1 & 1 & -1 & -1 & 0 & 0 & - & 0 & 0 & 0 \\ Z_2 & -Z_2 & -Z_1 & Z_1 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & e^{-jk_2 1} & e^{jk_2 1} & -1 & -1 & - & 0 & - & 0 \\ 0 & 0 & Z_3 e^{-jk_2 1} & -Z_3 e^{jk_2 1} & -Z_2 & -Z_2 & - & 0 & - & 0 \\ 0 & - & - & - & - & - & - & - & - & - \\ 0 & - & - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & - & - & e^{-jk_n 1_{n-2}} & e^{jk_n 1_{n-2}} & -1 \\ 0 & 0 & 0 & 0 & - & - & - & Z_n e^{-jk_{n-1} 1_{n-2}} & -Z_n e^{-jk_{n-1} 1_{n-2}} & Z_n \\ 0 & 0 & 0 & 0 & 0 & 0 & - & 0 & 0 & 1 \end{pmatrix}$$

Similarly, complex pressure amplitude matrix and right hand matrix is given by
$\{A\}_n^T = [A_1\ B_1\ A_2\ B_2\ A_3\ B_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots A_n]$
$\{R\}_n^T = [0\ 0\ 0\ 0\ 0\ 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1]$

The above matrix is in the form,

$$[M]_n \times [A]_n = [R]_n \qquad (20)$$

$$[A]_n = [M]_n^{-1} \times [R]_n$$

$$\text{Transmission loss} = 10 \log_{10} (A_1^2 / A_n^2) \quad (21)$$

Where A_n is the complex pressure amplitude of the transmitted wave in the n^{th}media.Based on this generalized matrix, a MATLAB code that can be used to calculate the transmission loss for any number of layers has been developed.

B. Transfer Matrix Method

The TMM is a general method for modeling acoustic fields in layered media. The method assumes infinite layers and represents the plane wave propagation in different media in terms of transfer matrices. The global transfer matrix of the stratified material is the product of all elementary transfer matrices $[T_i]$ of each layer.Xin Sun *et al*[3] have presented the transfer matrices for the characterization of sound transmission loss of laminated glass.

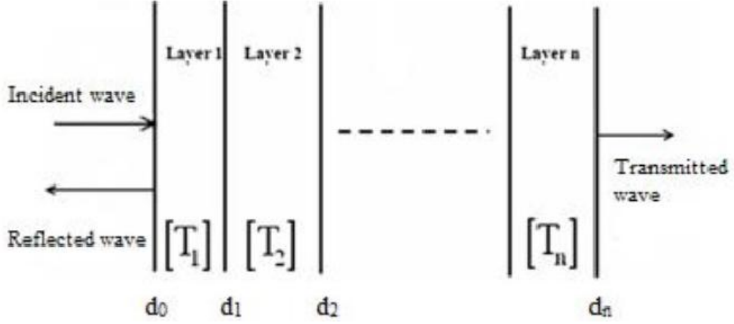

Fig.3. Wave propagation through multi-layer structure.

For normal incidence of plane wave, the transmission coefficient is given by

$$T = \frac{m_{11}m_{22} - m_{12}m_{21}}{m_{22}}$$

where,

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = T_n T_{n-1} \ldots T_0$$

$$T_n = \frac{1}{2} \begin{bmatrix} \left[1+\frac{Z_n}{Z_{n+1}}\right] e^{i(k_n-k_{n+1})d_n} & \left[1-\frac{Z_n}{Z_{n+1}}\right] e^{-i(k_n+k_{n+1})d_n} \\ \left[1-\frac{Z_n}{Z_{n+1}}\right] e^{i(k_n+k_{n+1})d_n} & \left[1+\frac{Z_n}{Z_{n+1}}\right] e^{-i(k_n-k_{n+1})d_n} \end{bmatrix}$$

Where $z_n=\rho_n c_n$ is the impedance and $k_n = \frac{\omega}{c_n} = \frac{2\pi f}{c}$, is the wave number of the nth layer. The position of the interface is represented by d_n.

III. RESULTS AND DISCUSSIONS

A. Double Fibreboard Walls

Multilayered structures like double fibreboard walls and EPS (Expanded Polystyrene) sandwich panels are often used in buildings, aerospace and automotive applications for sound attenuation. Here fibreboard with smooth surface is analysed. An air cavity of 6 mm is provided in between the two fibreboards of thickness 9.5mm each. The GMM results obtained are compared with TMM (normal incidence) results and available TMM (oblique incidence) and experimental results. The variation of transmission loss with frequency is shown in figure.5.

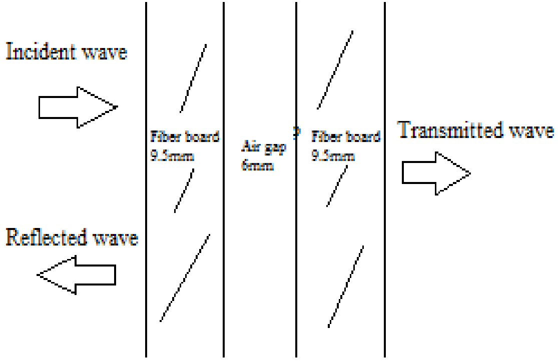

Fig.4. Double fibreboard wall

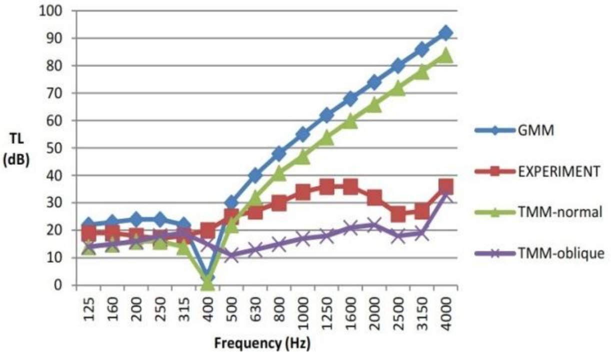

Fig.5.TL versus frequency plot for double fibreboard panel

B. Expanded Polystyrene Sandwich Panels

Expanded polystyrene is a lightweight cellular material consisting of fine spherical shaped particles which are comprised of 98% air and 2% polystyrene. EPS has the advantage of being light-weight and effective in thickness as low as 1/4 inch, replacing thicker, heavier materials and effectively reduce the transmission of airborne sound in automotive and building applications. Two types of sandwich panels with a core of expanded polystyrene (EPS) were analyzed. In the first configuration, the panel consists of a core of EPS with a 4 mm fiberboard plate glued on each side with an air cavity in between the first fiber board plate and the EPS core. In the second configuration, the sandwich panel consists of a core of EPS with a 3 mm fiberboard plate glued on each side with an air cavity in middle of the EPS core.

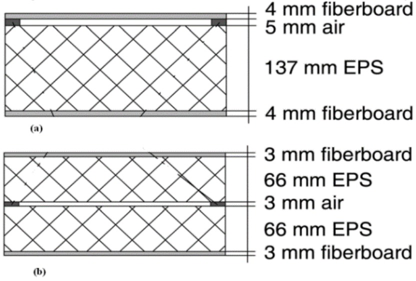

Fig.6. Type 1 and Type 2 sandwich structures

The agreement between measurement and GMM results is good over the low frequency range as shown in figure 7 and figure 8. The material properties used are shown in table 1.

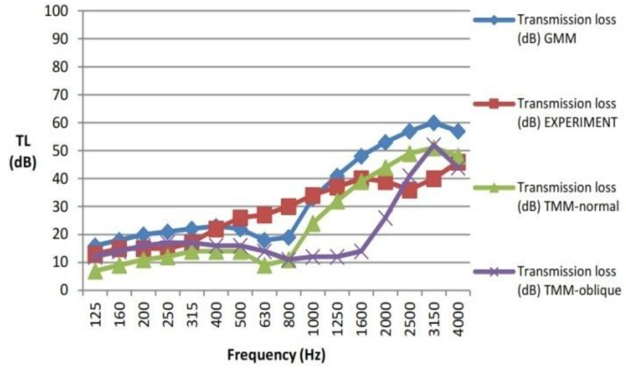

Fig.7. TL versus frequency plot for type 1 sandwich structure

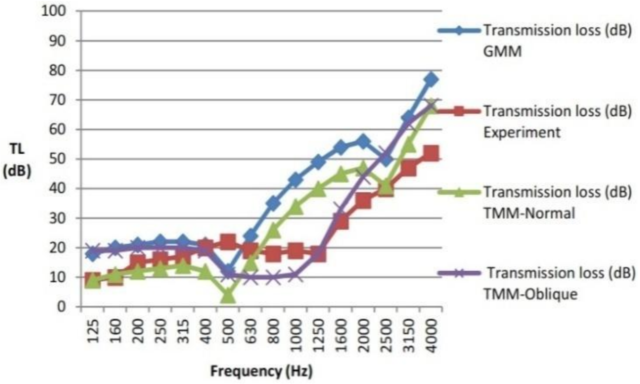

Fig.8. TL versus frequency plot for type 2 sandwich structure

C. Steel-Polyethylene Sandwich Structure

Figure (9) shows the comparison of GMM, TMM (normal incidence) and available numerical results for a two layer panel made of 5mm polyethylene and 2mm steel.

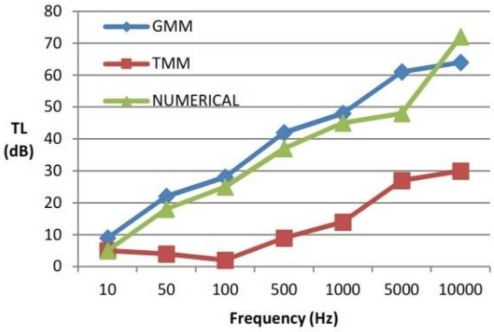

Fig.9. TL versus frequency plot for steel-polyethylene sandwich structure

GMM results match with the numerical results closely while TMM results show much deviation.

D. Gypsum Board-Mineral Wool Structure

Gypsum board and mineral wool are commonly used in aeronautical and automotive applications for sound attenuation. The sandwich panel is made of two gypsum boards of 12 mm thickness each attached to a mineral wool core of 180mm thickness. An air gap of 60mm thickness is provided in the middle of mineral wool core. This sandwich panel represents plane wave transmission through seven media and the analysis is shown in figure 10. GMM and experimental results match closely except for a dip in transmission loss at 800 Hz.

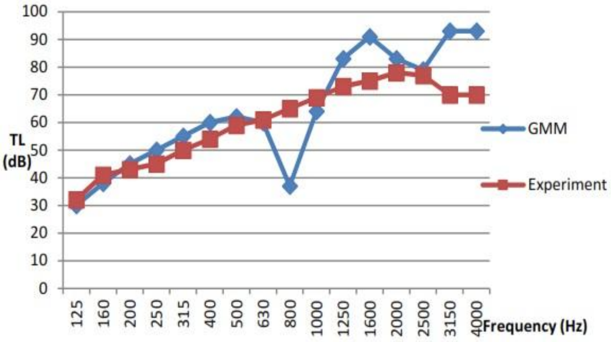

Fig.10. TL versus frequency plot for gypsum board-mineral wool sandwich structure

E. Concrete wall with an air column

Sound transmission through a concrete wall with an air column in between is analyzed for its acoustic behavior at a frequency of 1000Hz.Thus it comprises of five medium. The total thickness is kept same that of a normal concrete wall (20cm) and the thickness of air column is varied from 1cm to 5cm. It is observed that the sound transmission loss increases with increasing thickness of air column as shown in figure 11.

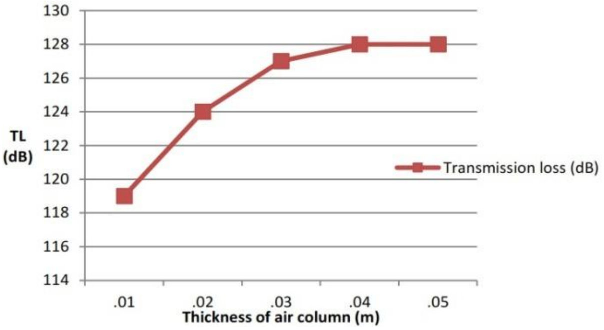

Fig.11. TL versus frequency plot for Concrete wall with an air column

Table.1.Material properties used for GMM and TMM analysis.

Material	Density (kg/m^3)	Speed of sound (m/s)
EPS	15	1179
Mineral wool	50	180
Fiber board	750	2629
Air	1.21	343
Water	1000	1481.4
Concrete	2300	3200
Gypsum board	960	6800

IV. COMPARISON BETWEEN GMM AND TMM

The generalized matrix method and transfer matrix method is used to describe the plane wave propagation through infinite multilayered structures and analyzes the acoustic behaviour of multi-layered structures for normal incidence of plane wave. The analysis of the sandwich structure is shown in figure 12 and 13 and it is seen that the transmission loss curve is similar for both GMM and TMM, but TMM greatly underestimates the transmission loss while GMM shows good agreement with the experimental results.

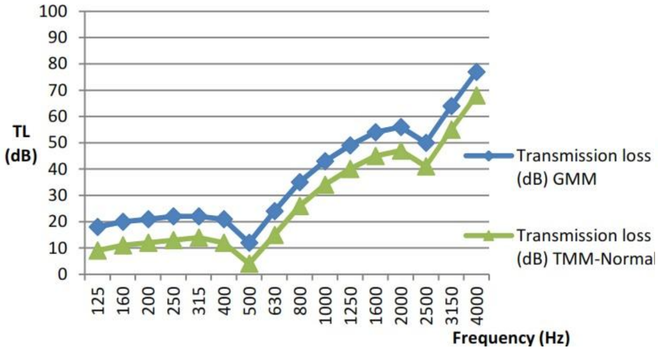

Fig.12. GMM and TMM results for type 1 sandwich structure.

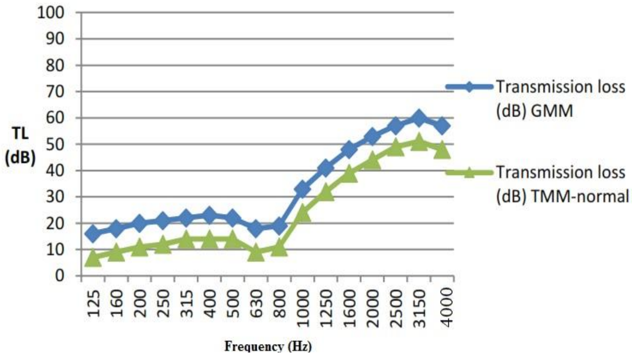

Fig.13. GMM and TMM results for type 2 sandwich structure.

V. CONCLUSIONS

The Generalized Matrix Method is used for analyzing the transmission phenomena of multilayered structures. Generalized matrix for sound transmission through n- number of layers is obtained and software is developed based on this approach. Conventional Transfer matrix method is also modeled for comparison. The sound transmission loss of double fiberboard walls and two types of sandwich panels with a core of Expanded Polystyrene (EPS) is found out using

Generalized Matrix Method and the results are validated with the available experimental results.Comparison with TMM results are also carried out. It is seen that the GMM results show better agreement with experimental results than the results obtained by TMM. GMM results overestimate the sound transmission losses in the high frequency range while it shows good agreement in the low frequency range. The discrepancy with the GMM results may be due to the reactance component in the impedance value of the solid material which is not accounted in this work

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