International Journal of
Emerging Research in Management &Technology
ISSN: 2278-9359 (Volume-4, Issue-10)

Research Article | **October 2015**

Portfolio Selection with Hyper Power Law with Exponential Cut-Off Utility Function

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Abstract—

We analyse a single period portfolio selection problem for a group of investors who are clients under an agent. The agent here wants to maximize the expected utility for the total wealth available to him. The utility function is hyper power law with exponential cut-off (HPLEC) utility function, which can represent all types of risk averse investors. This utility is used due to the fact that the risk tolerance of each investor under the agent may be different and the agent may not be sure about the risk profile of the clients under him. We show why this utility is the best to be used when the risk profile of each investor under an agent may be different. Illustrations for different models under this scenario are also presented here.

Keywords—Portfolio management, HPLEC, utility function, optimal policy, agent perspective.

I. INTRODUCTION

All financial investments involve complex decision making criteria within themselves. The decision, as to how the given wealth for investment will be distributed among the various available financial instruments, is a very tough one. Every investor wants to make his portfolio an optimum one, taking into consideration the market conditions and his risk preference. Often agents/ financial consultantstake this optimum decision on behalf of their clients. Thus it becomes a difficult task for the agent to manage each and every client. So also the manner in which one can arrive at an optimum decision differs from scenario to scenario. In reality the customers under an agent will not be governing the investment policy. Instead, the agent takes a cue from his interaction with the investors under him and his own past experience. Ultimately the agent then considers a few utility functions, which he thinks represent the clients under him.

Different investors may have different end goals and different preferences towards financial instruments. Their decision also depends on their wealth and time. In order to achieve these goals various methods have been developed. In the pioneering work by [1], minimizing the variance of the returns of the assets, in order to obtain the best portfolio optimization decision, was suggested. However it suffers with a few drawbacks particularly of not taking the investors' time horizon into consideration.[2]-[3] have corrected some of his errors.

Classical method of optimization depends only on mean and variances of returns; however, this does not take into account investors preferences and the amount of wealth to be invested. Optimization of a portfolio based on utility functions of investors will be rather more appropriate and has been considered recently. A utility function is non-decreasing, real valued function defined on real numbers. Since [4] has given importance to the use of utility functions as a basis for investment decisions, their use have increased tremendously. Some of the commonly used utility functions are quadratic utility, power utility, logarithmic utility and exponential utility as highlighted by [5]. A new utility function called the PLEC utility is suggested by [6], which incorporates all most all the risk preference structures of investors as described by [7] and [8].

Many have worked with utility functions in order to obtain an optimal solution to the portfolio problem. Some of them are [9],[2],[10], [11], and [12]. In a survey paper by [13], 208 papers, which show the diversity of different models and approaches used to analyze this problem of optimum portfolio based on utility functions, for both single period and multi-period cases, are reviewed. As highlighted by [14] a huge amount of work is done in this direction by a number of academicians.

The work discussed so far covers only individual investors who may be taking the decision of optimal investment on their own. But usually the optimal decision for investment is taken by an agent/investment consultant on behalf of his/her clients. In many cases, the agent may not be sure of the risk criteria of the client. Based on his past experience the agent may choose different types of utility functions for his clients. He can then assign different proportions to these selected utility functions and hence obtain a prior probability of different combinations of the utility functions.Therefore, one needs to obtain the optimal solution based on a combination of different parameters of a utility function as suggested by [15]. They have studied this scenario using the exponential utility function whose combination leads to the hyperexponential utility functions. This utility however can represent only Constant Absolute Risk Averse (CARA) type of investors, ignoring the Increasing Absolute Risk Averse (IARA) and Decreasing Absolute Risk Averse (DARA) investors.

This paper intends to discuss this variation by suggesting a model that can cover all types of investors in the market. Here we suggest a model where all types of investors of the market (under a particular agent) are represented by a combination of different forms of the PLEC utility function called as the HPLEC (Hyper Power law with exponential cut-off) utility function. The optimal solution to this model is also shown here. The PLEC Utility has been shown to be able to represent all the three risk averse type of investors through changes in the utility parameters by [5]. Here we take a combination of different parameters of this utility function to achieve the desired model. The same model can be used to represent an investor who changes his risk profile with the change in market state.Illustrations showing the sensitivity of the solutions for a constant and mixed risk tolerance models are also presented here.

Section II introduces the main problem and describes our model suggested through the HPLEC utility functions, it is shown that the optimum solution for this model exists and is worked out. Section III explains the various types of investors that can be represented using this utility function. Section IV contains illustrations based on the above models and their corresponding optimal solutions. Section V consists of the conclusions.

II. HPLEC UTILITY FUNCTIONS

We consider an agent who chooses a portfolio among a risk free asset and *n* risky assets. Let the agent based on his past experience choose *k* probable utility functions for all the investors under him. The probability that an investor under him will have thei^{th} utility function will be observed with some probability P_i where$\sum_{i=1}^{k} P_i = 1$.We assume that an investor has the i^{th} utility described by the PLEC utility function

$U(i,x) = K_i - C_i e^{\beta_i x} x^{\alpha_i};\ \alpha_i + \beta_i x < 0\ and\ \hat{\alpha} + x\hat{\beta} < 0$ Eq. 2.1

with some probability P_i. Here, K_i, C_i, α_i and β_i denote the parameters of the i^{th} PLEC utility function. Note that the above two conditions mentioned with the utility are because of the property of the PLEC utility to hold as a valid utility function. Also the structure of the utility function Eq. 2.1 is similar to the one in [5]. Our aim here is to determine the optimal portfolio of risky and risk free assets which maximize the expected utility of the terminal wealth in a single period setting. The behaviour of all the investors under a particular agent can be represented using the utility function

$U(x) = \sum_{i=1}^{k}(P_i K_i - P_i C_i e^{\beta_i x} x^{\alpha_i})\ ;\ \alpha_i + \beta_i x < 0\ and\ \hat{\alpha} + x\hat{\beta} < 0$

Eq.2.2which is a mixture of the PLEC utility functions called as the HPLEC utility function where the notations $\hat{\alpha} = \sum_{i=1}^{k} \alpha_i$ and$\hat{\beta} = \sum_{i=1}^{k} \beta_i$.

We shall refer Eq. 2.2 as the general mixed risk tolerance investors model. If $\alpha_i = \alpha$and $\beta_i = \beta$ for all utilities *i*, then we call it the standard risk tolerance investors model.

In this paper, unless stated otherwise, a vector $\boldsymbol{z}$ is a column vector so that its transpose, denoted by z', is always a row vector. Let $u = [u_1, u_2, \dots, u_n]$denote the amount of wealth that is invested in n risky assets. The random returns of the risky assets are given by the random vector, $R = [R_1, R_2, \dots, R_n]$ andr_f is the return of the risk free asset. Therefore, after one period the wealth becomes

$W = r_f(x - 1'u) + R'u = r_f x + (R^e)'u$ Eq. 2.3

where$1 = (1, \dots, 1)$, $R^e = R - r_f$ is the excess return vector and x is the initial wealth or budget of the investors at the beginning of the period.

Let $g(x,u)$denote the expected utility using the investment policy u when the amount of money available for investment is x. Hence,

$$g(x,u) = \sum_{i=1}^{k} P_i\, E\big[U\big(i, r_f x + R^{e\prime} u\big)\big]$$

$= \sum_{i=1}^{k} P_i K_i - \sum_{i=1}^{k} P_i\, C_i e^{\beta_i x r_f} E\big[e^{\beta_i R^e u}\big(x r_f + R^{e\prime} u\big)^{\alpha_i}\big]$ Eq. 2.4

It is clear that $g(x,u)$ is strictly concave in u for all x since every function inside the summation in Eq. 2.4 is strictly concave in u provided that (i)$P\big[R^{e\prime} u = 0\big] < 1$ for all $u \neq [0,0,\dots,0]$ and (ii) the above utility function is valid as per the conditions given in Eq. 2.1. We do suppose without loss of generality that the returns R^e of the risky assets and the parameters α_i and $\beta_i \forall\ i = 1, \dots k$ satisfy these conditions. Therefore, to find the optimal portfolio of risky assets, it is enough to set the gradient w.r.t. $u_t \forall\ t = 1, \dots n$ equal to zero so that the optimality condition becomes,

$\nabla_t g(x,u) = \dfrac{\delta g(x,u)}{\delta u_t}$

$= -\sum_{i=1}^{k} P_i C_i\, e^{\beta_i x r_f}\ E\left[R_t^e e^{\beta_i R^{e\prime} u}\big(x r_f + R^{e\prime} u\big)^{\alpha_i - 1}\big[\beta_i(x r_f + R^{e\prime} u) + \alpha_i\big]\right] = 0$ Eq. 2.5

for all *t*. Now define $B^t(x,u) = -P_i C_i e^{\beta_i x r_f}\ E\left[R_t^e e^{\beta_i R^e(i)' u}\big(x r_f + R^{e\prime} u\big)^{\alpha_i - 1}\big[\beta_i(x r_f + R^{e\prime} u) + \alpha_i\big]\right] = 0$ so that the optimality condition Eq. 2.5 can be written for all *n* assets as,

$$B(x,u) = [B^1(x,u), B^2(x,u), \dots, B^n(x,u)] = [0,0,\dots,0]$$

Note that $B(x,u)$ is a continuous function w.r.t. u due to the property of utility functions.

Lemma 2.1:*The function$B^t(x,u)$ is strictly decreasing in u_t and hence $B(x,u)$ is also strictly decreasing in $u_t$for all x, i and $t = 1, \dots, n$.*

This follows by noting that,

$\dfrac{\delta B^t(x,u)}{\delta u_t} = -P_i C_i e^{\beta_i x r_f} E\left[R_t^{e2} e^{\beta_i R^{e\prime} u}\big(x r_f + R^{e\prime} u\big)^{\alpha_i - 2}\left[{\beta_i}^2\big(x r_f + R^{e\prime} u\big)^2 + 2\alpha_i\beta_i\big(x r_f + R^{e\prime} u\big) + \alpha_i(\alpha_i - 1)\right]\right] < 0$

for all x, *i* and t.This follows because of condition on the PLEC utility which holds for any risk-averse investor.

Theorem 2.1: *The optimal policy is the unique solution $u^*(x) = [u_1^*(x), u_2^*(x), \dots, u_n^*(x)]$that satisfies $B\big(x, u^*(x)\big) = 0$ for any x where$B\big(x, u(x)\big)$is a continuously differentiable function w.r.t. u.*

Proof:

For the existence of the optimal policy satisfying the optimality condition Eq. 2.5, consider the Hessian of the objective function g *w.r.t.* v, which is the symmetric matrix,

$$\nabla^2_{t,v} g(x,u) = \frac{\delta \nabla_t g(x,u)}{\delta u_v} = \frac{\delta^2 g(x,u)}{\delta u_v \delta u_t}$$

$$= -\sum_{i=1}^{k} P_i C_i e^{\beta_i x r_f} E\left[R_t^e R_v^e e^{\beta_i R^{e'} u}\left(x r_f + R^{e'} u\right)^{\alpha_i - 2}\left[\beta_i{}^2\left(x r_f + R^{e'} u\right)^2 + 2\alpha_i\beta_i\left(x r_f + R^{e'} u\right) + \alpha_i(\alpha_i - 1)\right]\right] \quad \text{Eq. 2.6}$$

For any vector $z = [z_1, z_2, \dots, z_m]$,

$$z'\left(\nabla^2_{t,v} g(i,x,u)\right) z$$

$$= -\sum_{i=1}^{k} P_i C_i e^{\beta_i x r_f} E\left[\sum_{k=1}^{n} (z_k R_t^e)^2\, e^{\beta_i R^{e'} u}\left(x r_f + R^{e'} u\right)^{\alpha_i - 2}\left[\beta_i{}^2\left(x r_f + R^{e'} u\right)^2 + 2\alpha_i\beta_i\left(x r_f + R^{e'} u\right) + \alpha_i(\alpha_i - 1)\right]\right] < 0$$

Implying that the Hessian of $g(x,.)$ is negative definite for all x. Every negative definite matrix has an inverse which is also negative definite. The implicit function theorem can be applied to the optimality condition Eq. 2.5 which can be rewritten as

$$\nabla g(i,x,u) = [\nabla_1 g(i,x,u), \nabla_2 g(i,x,u), \dots, \nabla_n g(i,x,u)] = [0,0,\dots,0].$$

Note that $\nabla g(i,x,u)$is a continuously differentiable function. The Hessian in Eq. 2.6 gives the matrix of the first order derivatives of $\nabla g(x,u)$with respect to$u = [u_1, u_2, \dots, u_n]$. This is clearly invertible at any fixed point (x,u), since it is negative definite. The proof is now immediate through the implicit function theorem for$\nabla g(x,u) = B(x,u) = 0$.

Therefore the existence and uniqueness of the optimal policy or portfolio $u^*$is established, where $u_t^*(x)$is the amount of money invested in asset *t* if the wealth of the investor is *x*.

III. GENERAL DISTRIBUTION MODEL

In this setting, the distributions of the risky asset returns are assumed to be arbitrary. Our analysis in this section is presented in two parts. In the first one, we assume that the risk tolerance is same for all investors under the agent or in other words the risk tolerance is constant. In the second one, the risk tolerance varies with the investors under the agent or in other words there is a mixed risk profile of investors under the agent. In this part we analyse the case with *1* and *n* risky asset.

A. *Standard risk tolerance investor's model*

If the risk tolerance of all the investors under the agent is the same so that $\beta_i = \beta, \alpha_i = \alpha, K_i = K$ and $C_i = C\ \forall\ i$ then the expected utility Eq. 2.4 becomes,

$$g(x,u) = \sum_{i=1}^{k} P_i K_i - e^{\beta x r_f} E\left[e^{\beta R^e u}\left(x r_f + R^{e\prime} u\right)^{\alpha}\right] \sum_{i=1}^{k} P_i\, C_i \quad \text{Eq. 3.1}$$

And the optimality condition Eq. 2.5 simplifies to

$$\nabla_t g(x,u) = e^{\beta x r_f}\ E\left[R_t^e e^{\beta R^{e'} u}\left(x r_f + R^{e'} u\right)^{\alpha - 1}\left[\beta\left(x r_f + R^{e\prime} u\right) + \alpha\right]\right] C \sum_{i=1}^{k} P_i = 0$$

It is clear that $\nabla_t g(x,u)$ is equal to zero if and only if $E\left[R_t^e e^{\beta R^{e'} u}\left(x r_f + R^{e'} u\right)^{\alpha - 1}\left[\beta\left(x r_f + R^{e\prime} u\right) + \alpha\right]\right]$ is zero. Therefore, in this setting, the optimal decision $u^*(x)$ is the vector that satisfies

$$E\left[R_t^e e^{\beta R^{e'} u}\left(x r_f + R^{e'} u\right)^{\alpha - 1}\left[\beta\left(x r_f + R^{e\prime} u\right) + \alpha\right]\right] = 0 \quad \text{Eq. 3.2}$$

for all *t* and *x*. Observe that in this case it does not matter if the number of investors under the agent is 1, 10, 100, 1000 or more.

B. *Mixed risk tolerance investor's model*

If the risk tolerance of all the investors under the agent is not the same so that $\beta_i \neq \beta$, $\alpha_i \neq \alpha$, $K_i \neq K$and $C_i \neq C\ \forall\ i$, then the expected utility and the optimality condition is given by Eq. 2.4 and Eq. 2.5 respectively.

Consider the case where there is *n=1* risky asset. For this case, since the distribution of the risky asset is general, we have to consider the generalised optimality condition Eq. 2.5. We know that B*(x,u)* is decreasing in *u* from Section 2. Note that,

$$B(x,0) = \sum_{i=1}^{k} B_i(x,0) = -\sum_{i=1}^{k} P_i C_i e^{\beta_i x r_f}\left[\beta_i x r_f + \alpha_i\right]\left(x r_f\right)^{\alpha_i - 1} E[R^e] = -\left(\sum_{i=1}^{k} P_i C_i e^{i x r_f}\left[\beta_i x r_f + \alpha_i\right]\left(x r_f\right)^{\alpha_i - 1}\right)\mu$$

where$\mu = \bar{r} - r_f$ is the mean of the excess return. Therefore, the sign of $B(x,0)$ depends on the sign of μ.If $\mu > 0$, then $B(x,0) > 0$ so that $u^*(x) > 0$ since $B(x,0)$is strictly decreasing in *u* and the optimal decision satisfying $B\left(x, u^*(x)\right) = 0$ is greater than zero for all *x*. Moreover, when $\mu < 0$, then$u^*(x) < 0$ for all *x* by a similar argument.

IV. ILLUSTRATIONS

In this section, computational issues are addressed and it is demonstrated how the results obtained can be put to work, through a numerical illustration, for the HPLEC utility under the various cases. Consider a market with four risky assets and one riskless asset where the returns of the risky assets follow an arbitrary multivariate distribution The illustration is based on data obtained during September 2011 to September 2014 from daily return information of four assets (Tata chemicals, Bata India, Reliance communications and Syndicate Bank) traded in National stock exchange; and the daily respective treasury rate of Reserve Bank of India.

Table I The return from the riskless asset and the expected return of each risky asset

r_f	μ_1	μ_2	μ_3	μ_4
1.0002	1.0003	1.0009	1.0022	1.0013

and the covariance matrixis

$$\Sigma = \begin{pmatrix} 0.228 & 0.039 & 0.148 & 0.147 \\ 0.039 & 0.298 & 0.052 & 0.035 \\ 0.148 & 0.052 & 1.034 & 0.407 \\ 0.147 & 0.035 & 0.407 & 0.715 \end{pmatrix}$$

Note that these values are obtained by multiplying the actual numbers by 1,000 for simplification. We consider the problem of investors with initial wealth $x = Rs.1000$ who want to maximize the expected utility of terminal wealth.

It is difficult to calculate optimal $\mathrm{u}(x)$values numerically for an arbitrary distribution using Eq. 2.5. The approach here is similar to [14] and [16] i.e., is to use Taylor series expansion of the utility function around the expected value $\bar{W} = E[W]$ of the terminal wealth $W = X_T$.[17]gives a detailed discussion on the benefits, advantages and disadvantages of using Taylor series expansion in optimal portfolio allocation. In particular, they give a convincing argument for using the first 4 moments in the approximation. From the data it is recognized that the return distributions have very small skew-ness and kurtosis, so it was decided to use the first four moments. Taylor series expansion is

$$U(W) = \sum_{j=0}^{+\infty} U^{(j)}(\bar{W}) \frac{(W - \bar{W})^j}{j!}$$

where$U^{(j)}(\bar{W})$ is the j^{th} derivative of the utility function at$\bar{W}$. Taking expectations the above equation can be written as,

$$E[U(W)] = U(\bar{W}) + \frac{1}{2!}U^2(\bar{W})\mu_p^2 + \frac{1}{3!}U^3(\bar{W})\mu_p^3 + \frac{1}{4!}U^4(\bar{W})\mu_p^4 + E[R_4(W,\bar{W})]$$

where$R_4(W,\bar{W})$ is the remainder for the first four moments and μ_p^n is the nth moment of the portfolio defined as$\mu_p^2 = E[(W - \bar{W})^n]$

Using the definitions in [17] for any market state, the second moment can be expressed as$\mu_p^2 = \gamma' M_2 \gamma$where $M_2 = \boldsymbol{\Sigma}(.)$ is the covariance matrix. Similarly $\mu_p^3 = \gamma' M_3(\gamma \otimes \gamma)$where $\otimes$is the Kronecker product, and M_3 is the 4 x 16 co-skewness matrix with elements$s_{abc} = E[(R_a - \mu_a)(R_b - \mu_b)(R_c - \mu_c)]$for *a,b,c=1,2,3,4.* Finally,$\mu_p^4 = \gamma' M_4(\gamma \otimes \gamma \otimes \gamma)$where M_4 is the 4 x 64 co-kurtosis matrix with elements$k_{abcd} = E[(R_a - \mu_a)(R_b - \mu_b)(R_c - \mu_c)(R_d - \mu_d)]$for *a,b,c,d=1,2,3*,4

Consider 2 agents, one having a set of investors who have the same or constant risk preferences and another having a set of investors with different or mixed risk preferences.

Illustration 1:

Consider an agent having all his clients having the same risk preference which can be represented by the HPLEC utility with parameters$(\alpha = -4, \beta = -0.0003)$. Taking$C_j = 10^{15}\ \forall\, j$.These parameters represent a DARA investor. Substituting these parameter values in the Taylor series expansion of Eq. 3.1 and taking the gradient with respect to $u(x)$ and setting it to equal to zero, we obtain the first order condition to the problem. From this first order conditionthe optimal $u(x)$values are determined numerically using an R program. The computational time was less than a half a minute on a laptop with 2.1 GHz processor.The corresponding optimal investment amount is given as,

Table II Optimal amounts to be invested in each of the assets

Asset	**Tata Chemicals**	**Bata India**	**Reliance Com.**	**Syndicate Bank**	**Risk free**
Amount	-78.15	94.22	74.01	24.79	885.13

Since the investors under this agent are of the DARA type they prefer to invest more in the less risky stocks than the risky ones i. e., they prefer the ones with lower variance.

Illustration 2:

Consider an agent having not all his clients the same risk preference. They may be having any one of the risk profiles from the set of parameters of the HPLEC function $\{(\alpha = -4, \beta = -0.0003), (\alpha = -4, \beta = -0.0002)\}$. Taking$C_j = 10^{15}\ \forall\, j$and $P_j = 0.5\ \forall\, j$. These parameters represent two different DARA investor preferences. We will assume here that there is equal probability of having investors in this set who prefer each type of risk profile.Substituting these parameter values in the Taylor series expansion of Eq. 2.5 and taking the gradient with respect to $u(x)$ and setting it to equal to zero, we obtain the first order condition to the problem. From this first order conditionthe optimal $u(x)$values are determined numerically using an R program. The computational time was less than a half a minute on a laptop with 2.1 GHz processor. The corresponding optimal investment amount is given as,

Table III Optimal amounts to be invested in each of the assets

Asset	**Tata Chemicals**	**Bata India**	**Reliance Com.**	**Syndicate Bank**	**Risk free**
Amount	-55.15	72.77	58.17	21.33	902.88

Again since the investors under this agent are of different DARA types they prefer to invest more in the less risky stocks than the risky ones i. e., they prefer the ones with lower variance.

We observe that the optimal amounts to be invested obtained from these models is different for each type of investor. This holds if one studies a standard risk model or a combination of different risk models through the analysis method presented here. Figure 6.1 illustrates this clearly. Here the two sets of parameters mentioned for the mixed risk model is individually studied as constant risk model and we call it as model A and B respectively in the figure. The same sets of parameters are studied together in the mixed risk model and we call it as model C in Figure 1.

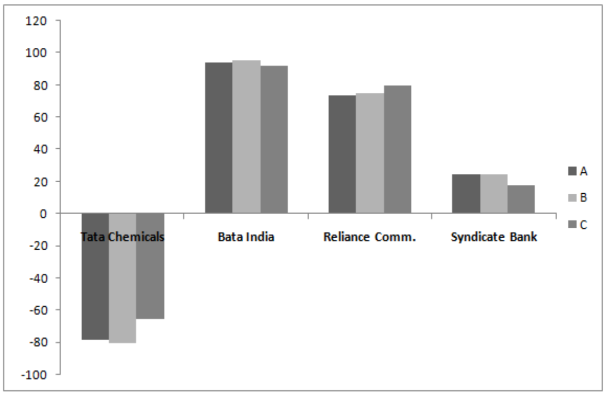

Fig 1.Optimal investment in each asset under different models.

As it can be observed clearly that each model yields a different optimal solution for each of the assets hence one should be careful while selecting the appropriate model before finding the optimal solution for it. It also highlights the fact that an agent may have different types of investors for whom he may want to obtain an optimal solution. It is very difficult for the agent to individually attend each and every one of the investors under him. The models presented here help the agent save time and money in doing so and at the same time the optimal solution for each of his investors is achieved.

Illustration 3:

Consider an agent having not all of his clients with the same risk preference as described in Illustration 2 here. Let the same set of parameters be chosen by the investor. Suppose based on the past experience the agent arrives at the probabilities of choosing each utility as $P_1 = 0.25$ and $P_2 = 0.75$. Here we assume that based on the past experience the agent arrives at the above mentioned probabilities. Let us call this model as Model D. Proceeding in the manner described in illustration 2 we obtain the solution to the portfolio problem here as,

Table IV Optimal amounts to be invested in each of the assets

Asset	**Tata Chemicals**	**Bata India**	**Reliance Com.**	**Syndicate Bank**	**Risk free**
Amount	-54.87833	72.73459	58.01825	21.56133	902.56416

Hence we observe from Table 4 that the solution is clearly different from the one obtained in Illustration 2. However the behaviour of the investors under the agent is similar due to the DARA utilities chosen.

Importance of using the appropriate model:

In the solutions obtained in illustration 1, illustration 2 and illustration 3, we notice that the expected utility values based on each of the three solutions are 739.2848, 778.3441 and 797.7951 respectively. Hence if the agent chooses model A to represent all the investors, when in fact he may have customers from two different DARA risk profiles as in Model C or D, he may get only an apparent optimal solution. Although all the three solutions are optimal for the model under consideration, they may not be appropriate for the actual situation of investors under the agent.

V. CONCLUSIONS

Portfolio decision making has been an important line of research where in expected utility maximization method is widely used. The availability of information on the risk profile of the investors is a major challenge in the application of this highly useful method. This difficulty is majorly faced by investment consultants/ agents who handle investment of a large number of clients under them. Based on the past trend and interaction with his customers, the agent decides about the utility functions to be considered for arriving at an optimal solution for his clients. Therefore, for modelling the behaviours of such investors under an agent it is reasonable to use a combination of utility functions which can represent each and every investor in the group.

Reference [15] has worked in a similar direction where the investors considered were only of CARA type. But in reality there could be investors who are of IARA and DARA type as well. We have therefore suggested a model based on the HPLEC utility function. This utility is a combination of different forms of the PLEC utility function. The PLEC utility function is more generalised utility and can represent all the three types of risk averse investors based on different parameter values.

We show that the expected utility function here is a concave function of the decision variables and therefore the optimal solution is obtained by setting the gradient equal to zero. If there is a single asset with an arbitrary distribution, then the optimal policy is positively increasing in the wealth level if the mean excess return is positive, and it is negatively decreasing otherwise.

Three illustrations showing the applicability of the model for this situation is also presented here. The optimal solutions follow the nature of the risk profile chosen for the analysis. The solution suggested here may not be optimal for the clients individually but is optimal for all the investors taken together by the agent. This however is true in case of most funds, especially the like of mutual funds, where the investment policy is not directly governed by the individual clients. We also show that choosing the right model based on the investors under the agent is a necessary task and one may achieve different optimal solutions for each of the assets based on the selected parameters of the model. Analysis also suggests that different proportions selected for different utilities of the agents yield varying solutions.

ACKNOWLEDGEMENT

The first author wishes to gratefully acknowledge the Department of Science and Technology, Ministry of Science and Technology, New Delhi for providing financial assistance through the INSPIRE fellowship programme.

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