International Journal of
Emerging Research in Management &Technology
ISSN: 2278-9359 (Volume-3, Issue-8)

Research Article **August 2014**

Low-complexity Equalization of OFDM systems using Krylov subspace methods over Doubly Selective Channels

Tazeen*
ECED, Thapar University,
India

Ankush Kansal
Assistant Professor, ECED, Thapar University,
India

Abstract— Low complexity equalization with LSRQ and GMRES algorithm is compared with traditional equalization methods. As the next generation technologies like LTE and WiMax have a large number of subcarriers the traditional equalization methods like MF, ZF, MMSE, VBLAST, and SAGE which require matrix inversion become highly complex in time and consume a lot of time to equalize the received signal. An alternative approach that involves solving the system of linear equations using Krylov subspace based methods like LSQR and GMRES is hence employed to equalize the signal in Doubly Selective channels and their performance is compared to the traditional methods. It is found that LSQR algorithm gives performance comparable to MMSE equalization but not as well as VBLAST and SAGE which give a performance gain of about 4.0477 dB and 4.5593 dB at 30dB SNR; trading accuracy for speed.

Keywords— Doubly Selective Channel, OFDM, Krylov Subspace methods, Equalization, VBLAST, SAGE, GMRES, LSQR

I. INTRODUCTION

OFDM has been adopted as a key technology in 3G and 4G systems like Wimax and LTE as it can effectively mitigate frequency selective fading using simple equalization at the receiver. However the next generation communication systems have to support broadband services in high mobility environments where vehicular speeds typically exceed 120 km/hr. In such scenarios where the transmitter and/or receiver are moving with respect to each other the channel becomes doubly selective (i.e. time-selective and frequency-selective). Equalization is the process of undoing the effect of channel on the received signal. In general, there are two designs of equalizers, i.e., coherent equalizers and non-coherent equalizers. Coherent equalizers are assumed to be informed of the channel state parameters explicitly. Non-coherent equalizers, however, perform the task of equalization without explicit state knowledge of the channel. Coherent equalization can be further classified as linear and non-linear equalization. In linear equalization-based detection techniques, an 'equalization matrix' $\widehat{G}$ is used to estimate the transmitted data vector s as $\hat{s} = \widehat{G}Y$. The Matched Filter (MF), Zero Forcing (ZF) and MMSE equalizers all fall in the category of linear equalization techniques. Whereas in non-linear equalization techniques maximization or minimization of certain parameters is carried out (EM methods), for e.g. VBLAST and SAGE.

The traditional equalization methods like MF, ZF, MMSE, VBLAST, and SAGE require matrix inversion, SINR calculation or expectation and maximization steps that are highly complex and consume a lot of time to equalize the received signal. An alternative approach is to consider the received signal ignoring the noise component. The received signal can then be expressed as linear system of equations [1-3] and equalizing it is reduced to finding the solution of the square system. There are many methods to find the solution of linear equations for square systems which can be categorized as direct or iterative. Direct methods include Gaussian Elimination, QR Factorization [4] etc. But they use $O(N^3)$ number of operations where N is the number of subcarriers. Iterative methods start with an initial approximation to the solution and adjust this approximation over iterations to approach the true solution. Krylov subspace based methods like LSQR and GMRES fall under this category.

II. SYSTEM MODEL

Consider the transmission of one OFDM symbol as depicted in Fig. 1 over a doubly selective channel (time and frequency). Let the number of subcarriers be N. An OFDM symbol is constructed by stacking a serial information sequence $s = \{s_0, s_1, s_2, s_3, \dots, s_{N-1}\}$ of length N by serial-to-parallel (S/P) conversion. Subsequently, an N-point IDFT is applied to produce the N dimensional time-domain data, which is parallel-to-serial (P/S) converted. Then a cyclic prefix (CP) of length Ncp is inserted at the beginning of the symbol in order to avoid ISI. The transmitted symbol {x(n)} can be expressed as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k e^{j\frac{2\pi kn}{N}} \quad , \ n \in [-N_{cp}, N-1] \quad (1)$$

or in matrix notation

$$x = F^{-1} s \qquad (2)$$

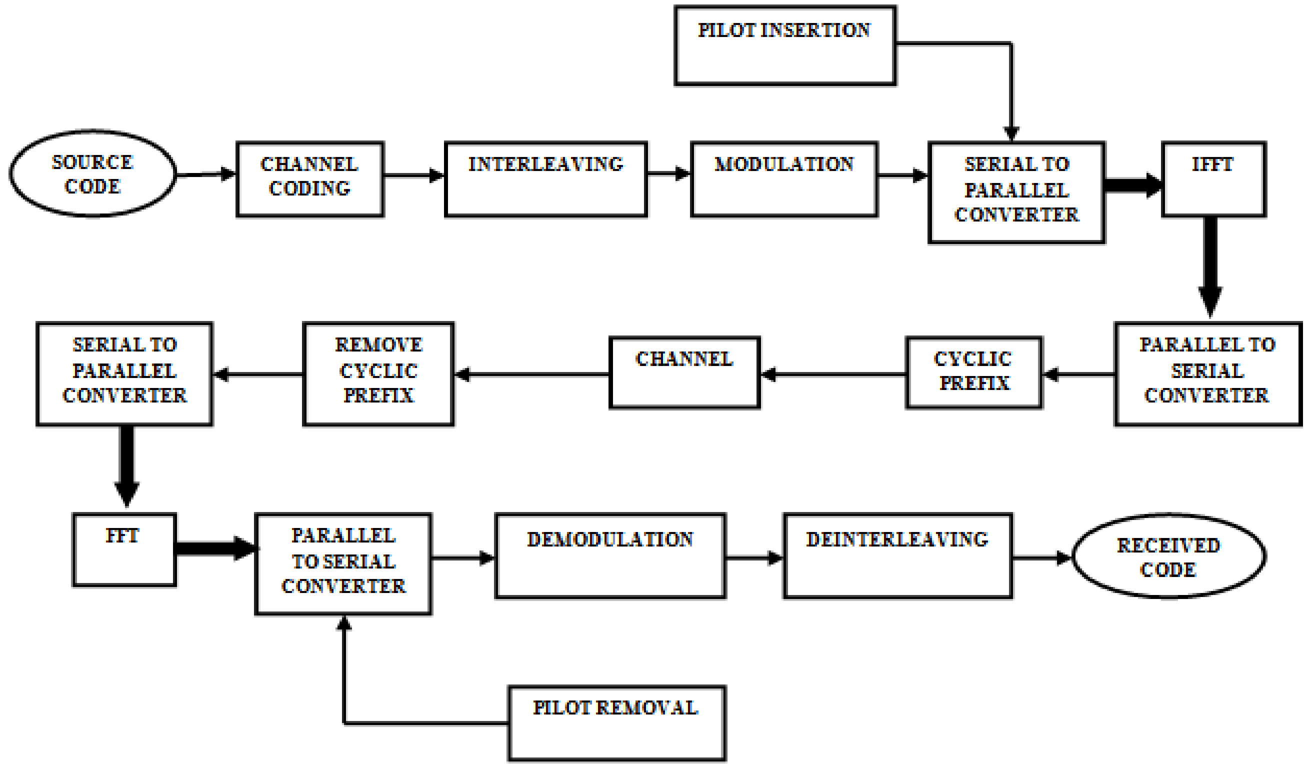

Fig. 1 Block diagram of an OFDM system

where $x = \{x(0), x(1), \ldots, x(N-1)\}$, F is the FFT matrix and consequently F-1 is the IFFT matrix.
If the channel has maximum order or number of taps L such that Ncp ≥ L then the system does not suffer from inter-symbol interference (ISI) between OFDM symbols that are adjacent to each other. In discrete-time baseband equivalent representation the nth received sample can be expressed as [5]

$$y(n) = \sum_{l=0}^{L} h(n;l)x(n-l) + w(n) \qquad (3)$$

where h(n;l) denotes the lth discrete channel tap and w(n) is the zero mean additive white Gaussian noise (AWGN) with variance σ_w^2 at time n. Or in matrix notation

$$\begin{aligned} y &= Hx + w \qquad (4a) \\ &= HF^{-1}s + w \qquad (4b) \end{aligned}$$

where $y = \{y(0), y(1), \ldots, y(N-1)\}$, $w = \{w(0), w(1), \ldots, w(N-1)\}$ and H is the time domain channel matrix defined as

$$H = \begin{bmatrix} h(0;0) & 0 & \cdots & 0 & h(0;L-1) & \cdots & h(0;3) & h(0;2) & h(0;1) \\ h(1;1) & h(1;0) & 0 & \cdots & 0 & h(1;L-1) & \cdots & h(1;3) & h(1;2) \\ \vdots & & & & \ddots & & & & \vdots \\ h(L-1;L-1) & & \cdots & & \cdots & h(L-1;0) & 0 & \cdots & \cdots\ 0 \\ 0 & & h(L;L-1) & & \cdots & & h(L;0) & 0 & \cdots\ \cdots\ 0 \\ \vdots & & & & \ddots & & & & \vdots \\ 0 & \cdots & 0 & h(N-1;L-1) & & h(N-1;L-2) & \cdots & & h(N-1;0) \end{bmatrix} \qquad (5)$$

The channel frequency response at discrete frequency 2πk/N at time n is given as [5]

$$G_k(n) = \sum_{l=0}^{L} h(n;l)\, e^{-j\frac{2\pi kl}{N}} \qquad (6)$$

After removing the CP and applying discrete Fourier transform (DFT) on the received signal we obtain the frequency-domain received signal for $k \in [0, N-1]$ as

$$Y_k = \sum_{n=0}^{N-1} G_{k,n} s_n + W_k \qquad (7)$$

Where

$$G_{k,n} = \frac{1}{N}\sum_{m=0}^{N-1} G_k(m)e^{j\frac{2\pi m(n-k)}{N}} \qquad (8a)$$

$$W_k = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} w(n)e^{-j\frac{2\pi kn}{N}} \qquad (8b)$$

Or in matrix notation

$$Y = Fy = F(Hx+w) = (FHF^{-1})s + Fw$$
$$= Gs + W \qquad (9)$$

where $Y = \{Y_0, Y_1, \dots, Y_{N-1}\}$, $W = \{W_0, W_1, \dots, W_{N-1}\}$.
If the channel is time-invariant, then the channel frequency response Gk(n) and Gk,n are also time invariant and Gk,n is given by Gk,n = Gk(n)δ(n−k), where δ(•) stands for Kronecker's delta and the frequency domain channel matrix G is a diagonal matrix. However, since the transmitter and the receiver are moving with respect to each other the channel becomes time variant and the channel matrix G is no longer diagonal matrix [6]. There are non-zero terms around the diagonal indicating ICI between the various subcarriers [7].

III. LINEAR EQUALIZATION METHODS

For a channel matrix G, the linear methods for detecting s are Matched Filter (MF), Zero Forcing (ZF) and Minimum Mean Square Error (MMSE). The Matched Filter is the simplest linear detector that neglects the presence of ICI. However it exhibits very poor performance in DS channels [8]. The Zero Forcing detector is used to reduce the effects of ICI but leads to noise enhancement [9], i.e., variance of the transformed noise is much larger than that of the noise introduced by channel. To overcome the effect of noise enhancement, MMSE can be used which alleviates both residual interference and noise enhancement [10]. However it's performance in DS channels is also limited. Table I gives a summary of these techniques and the equalization matrix used in each case where $G^{\dagger}$ denotes the pseudoinverse of the channel matrix G.

TABLE I
LINEAR EQUALIZATION METHODS

Method	Equalization Matrix	Solution
Matched Filter (MF)	$\widehat{G} = G^{\dagger}$	$\hat{s} = Q\{\widehat{G}Y\}$
Zero Forcing (ZF)	$\widehat{G}$ = ($G^{\dagger}$G)-1$G^{\dagger}$	$\hat{s} = Q\{\widehat{G}Y\}$
MMSE	$\widehat{G}$ = ($G^{\dagger}$G + σ2I)-1$G^{\dagger}$	$\hat{s} = Q\{\widehat{G}Y\}$

IV. NON-LINEAR EQUALIZATION METHODS

The time-varying nature of the DS channel can be exploited to provide time diversity when the normalized Doppler frequency becomes large. In [11], to fully exploit the time diversity while containing the ICI and noise enhancement, a recursive detection technique based on the decision-feedback principle, namely the MMSE with Successive Detection (MMSE-SD) algorithm or VBLAST, has been proposed. In this algorithm, the data symbols are detected one-by-one and at each detection step, the contribution of the detected symbol to the received vector Y is subtracted from Y. The performance of this algorithm depends vitally on the sequence in which the data vector components are detected [11]. To minimize the propagation of through feedback and to improve the detection of severely faded components, the detection of more reliable data vector components should be carried out first. The algorithm depends on the value of the signal-to-interference plus noise ratio (SINR) calculated after detection based on MMSE detection as a measure of reliability. The SINR needs to be calculated at each iteration, and hence this algorithm is computationally intensive as the calculation of SINR involves many pseudo-inverse operations. Furthermore the complexity of the algorithm grows exponentially with the total number of subcarriers. The MMSE-SD or VBLAST algorithm starts with the MMSE estimate of the data and then calculates the SINR for each component. The most reliable component, i.e., the component with highest SINR is detected first and its contribution is deducted from the received vector ***Y.*** The algorithm [11] can be stated as

1. *j = 1*
2. $\widehat{\boldsymbol{G}} = (\boldsymbol{G}^{\dagger}\boldsymbol{G} + \sigma_{\omega}^{2}\boldsymbol{I_N})^{-1}\boldsymbol{G}^{\dagger}$
3. $i_1 = \arg\max_k \left\{ SINR_k = \frac{|\langle \widehat{\boldsymbol{g}}_{\boldsymbol{k}}, \boldsymbol{g}_{\boldsymbol{k}}\rangle|^2}{\sum_{m=1, m\neq k}|\langle \widehat{\boldsymbol{g}}_{\boldsymbol{k}}, \boldsymbol{g}_{\boldsymbol{m}}\rangle|^2 + \sigma_{\omega}^2\|\widehat{\boldsymbol{g}}_{\boldsymbol{k}}\|^2} \right\}$
4. *loop*
5. $\hat{s}_{i_{(j)}} = \mathbb{Q}(\widehat{\boldsymbol{g}}_{i_{(j)}}^{\dagger}\boldsymbol{Y})$
6. $\boldsymbol{Y} = \boldsymbol{Y} - \boldsymbol{g}_{i_{(j)}}\hat{s}_{i_{(j)}}$
7. $\boldsymbol{G} = [\boldsymbol{g_0} \dots \boldsymbol{g}_{\boldsymbol{i}_{(j-1)}}\ 0\ \boldsymbol{g}_{\boldsymbol{i}_{(j+1)}} \cdots \boldsymbol{g}_{\boldsymbol{N-1}}]$
8. $\widehat{\boldsymbol{G}} = (\boldsymbol{G}^{\dagger}\boldsymbol{G} + \sigma_{\omega}^{2}\boldsymbol{I_N})^{-1}\boldsymbol{G}^{\dagger}$

9. $i_{j+1} = \arg\max_{k\in\{i_1,\ldots,j\}} \left\{ SINR_k = \frac{|\langle \hat{\boldsymbol{g}}_k, \boldsymbol{g}_k\rangle|^2}{\sum_{\substack{m=1 \\ m\in\{i_1,\ldots,j\} \\ m\neq k}} |\langle \hat{\boldsymbol{g}}_k, \boldsymbol{g}_m\rangle|^2 + \sigma_\omega^2 \|\hat{\boldsymbol{g}}_k\|^2} \right\}$

10. $j = j + 1$

Space-Alternating Generalised Expectation-maximisation (SAGE) algorithm is a modification of the expectation maximisation (EM) algorithm [12]. It is principally well suited for OFDM signals and can be easily extended to MIMO-OFDM systems. For every iteration, this technique comprises of ICI cancellation along with a soft-input/hard-output serial data detector. In the SAGE algorithm out of the parameters to be estimated only a subset is updated while the complement set is held fixed. This allows the system to operate at high vehicle speeds as the data sequence is updated in series leading to a receiver structure that also incorporates ICI cancellation. The SAGE algorithm consists of two steps: the Expectation step and the Maximization step. The algorithm can be described as follows: The received vector Y is decomposed in terms of the column vectors of G, as

$$\boldsymbol{Y} = \boldsymbol{g_0} s_0 + \boldsymbol{g_1} s_1 + \ldots + \boldsymbol{g_{N-1}} s_{N-1} + \boldsymbol{W} \qquad (10)$$

Where $\boldsymbol{g_n} = [G(0,n)\ \ G(1,n)\ \ G(2,n) \ldots G(N-1,n)]^T$

The data vector s = $[s_0, s_1, s_2, s_3, \ldots, s_{N-1}]^T$ needs to be estimated. At the i^{th} iteration, only one element of s is updated: s_n with n = imodN+1. Hence, the complement set is the vector $s_{\bar{n}}$ obtained by omitting the component s_n in s. Thus Y can be expressed as

$$\boldsymbol{Y} = \boldsymbol{u_n} + \sum_{\substack{m=0 \\ m\neq n}}^{N-1} \boldsymbol{g_m} s_m \qquad (11)$$

Where $\boldsymbol{u_n} = \boldsymbol{g_n} s_n + \boldsymbol{W}$

We choose $\boldsymbol{u_n}$, n = 0, 1, ... ,N-1 as hidden data when estimating s_n. The conditional log-likelihood function for the hidden data $\boldsymbol{u_n}$ be denoted by $\ell(\boldsymbol{u_n}|s_n) \triangleq \log p(\boldsymbol{u_n}|s_n)$. At the i^{th} iteration step, the expectation step (E-Step) of the SAGE algorithm computes the $Q(.|.)$ function as

$$Q\left(s_n \middle| \boldsymbol{s}^{(i)}\right) = E\left\{\ell\left(\boldsymbol{u_n}\middle| s_n, s_{\bar{n}}{}^{(i)}\right) | \boldsymbol{Y}, \boldsymbol{s}^{(i)}\right\} \qquad (12)$$

In the maximisation step (M-Step), only s_n is updated, that is

$$s_n{}^{(i+1)} = \arg\max_{s_n} Q\left(s_n \middle| \boldsymbol{s}^{(i)}\right) \qquad (13)$$

$$s_{\bar{n}}{}^{(i+1)} = s_{\bar{n}}{}^{(i)} \qquad (14)$$

The initial value of the data can be obtained by the MMSE estimate expressed as

$$\boldsymbol{s}^{(0)} \equiv \hat{\boldsymbol{s}}_{MMSE} = (\boldsymbol{G}^\dagger \boldsymbol{G} + \sigma_\omega^2 \boldsymbol{I_N})^{-1} \boldsymbol{G}^\dagger \boldsymbol{Y} \qquad (15)$$

V. KRYLOV SUBSPACE METHODS

Consider the time domain received vector *y* in equation (4a). After ignoring the noise term *w* we can express *y* as

$$y \approx Hx$$

The Cayley-Hamilton theorem states that the inverse of a matrix can be found in a linear combination of its powers. The Krylov subspace generated by a $N \times N$ matrix H and a vector *y* of dimension *N* is the linear subspace spanned by the images of *y* under the first i-1 powers of *H* starting from $H^0 = I$, i.e.,

$$\mathcal{K}_i(H, y) = span\{y, Hy, H^2 y, \ldots, H^{i-1} y\}$$

where i is the order of the Krylov subspace. For large matrices the elements of Krylov subspace are found by first multiplying the vector *y* with the matrix *H* and then multiplying the resulting vector with the matrix *H* and so on instead of matrix-matrix multiplication to obtain powers of *H* that then need to be multiplied with the vector *y*. The two most common Krylov subspace methods are LSQR [13] and GMRES [14].

A. GMRES Algorithm

Generalized Minimum Residual Algorithm constructs an approximation of the solution in the Krylov subspace defined by

$$\mathcal{K}_i(H, y) = span\{y, Hy, H^2 y, \ldots, H^{i-1} y\} \qquad (16)$$

The approximation is given by the vector $s_i \in \mathcal{K}_i(H, y)$ that minimizes norm $\|r_i\|_2$ of the residual r_i given by

$$r_i = Hx_i - y \qquad (17)$$

The vectors in the Krylov subspace may not be orthogonal. So Arnoldi iteration is used to find orthonormal basis vectors $q_1, q_2, \ldots, q_i$ for $\mathcal{K}_i(H, y)$. The approximation vector x_i is written in terms of the orthonormal basis vectors as $x_i = Q_i b_i$ where Q_i is the $N \times i$ matrix formed by $q_1, q_2, \ldots, q_i$ and $b_i \in \mathbb{C}^i$. The Arnoldi process produces a Hessenberg matrix $\mathcal{H}_i$ of dimension $(i+1) \times i$ satisfying the equation

$$HQ_i = Q_{i+1} \mathcal{H}_i \qquad (18)$$

As Q_i has orthogonal columns we can write

$$\|Hx_i - y\|_2 = \|\mathcal{H}_i b_i - \beta e_1\|_2 \qquad (19)$$

Where $e_1 = (1,0,0, \dots ,0)$ and $\beta = \|y\|_2$

Hence x_i can be found by minimizing the norm of the residual given by

$$r_i = \beta e_1 - \mathcal{H}_i b_i \qquad (20)$$

This least squares problem of size i is solved by QR factorization. Hence the GMRES algorithm can be stated as

For every iteration follow the steps:

- Perform one step of Arnoldi method
- Find b_i that minimizes $\|r_i\|_2$ using QR factorization at the cost of $O(i^2)$ flops
- Compute $x_i = Q_i b_i$
- Repeat if the residual is not small enough

B. LSQR Algorithm

LSQR Algorithm constructs an approximation of the solution in the Krylov subspace defined by

$$\mathcal{K}_i(H^H H, H^H y) = span\{H^H y, (H^H H)H^H y, (H^H H)^2 H^H y, \dots, (H^H H)^{i-1} H^H y\} \quad (21)$$

It is equivalent to conjugate gradient method for the normal equations $H^H Hx = H^H y$. At the i[th] iteration the approximation to the solution is given by the vector $x_i \in \mathcal{K}_i(H^H H, H^H y)$ that minimizes norm $\|r_i\|_2$ of the residual r_i given by

$$r_i = Hx_i - y \qquad (22)$$

LSQR consists of two steps: Golub-Kahan bidiagonalization and solution to bidiagonal least squares problem. These two steps are repeated until a solution is reached.

Golub-Kahan bidiagonalization constructs vectors u_i , v_i and positive constants α_i, β_i for i = 1,2,... as follows:

- Set $\beta_1 = \|y\|_2, u_1 = y/\beta_1 , \alpha_1 = \|H^H y\|_2, v_1 = H^H y/\alpha_1$
- For i =1,2,... set

$$\beta_{i+1} = \|Hv_i - \alpha_i u_i\|_2, \qquad u_{i+1} = Hv_i - \alpha_i u_i/\beta_{i+1}$$
$$\alpha_{i+1} = \|H^H u_i - \beta_i v_i\|_2, \qquad v_{i+1} = H^H u_i - \beta_i v_i/\alpha_{i+1}$$

The process is terminated if $\alpha_{i+1} = 0$ or $\beta_{i+1} = 0$.

The vectors u_i , v_i are orthonormal. Hence the approximation can be reduced to the following least squares problem

$$\min_{w_i} \|B_i w_i - [\beta_1, 0,0, \dots ,0]^T\|_2 \qquad (23)$$

Where B_i is the $(i + 1) \times i$ lower bidiagonal matrix with $\alpha_1, \dots, \alpha_i$ on the main diagonal and $\beta_2, \dots, \beta_{i+1}$ on the first sub-diagonal. This is solved by QR factorization of the bidiagonal matrix B_i. The i[th] approximate solution is computed as

$$x_i = \sum_{j=1}^{i} w_i(j) v_j \qquad (24)$$

VI. COMPARISON OF GMRES AND LSQR

At each iteration of GMRES or LSQR certain operations need to be performed. These are Matrix-vector products, vector additions, scalar multiplications and finding norms of vectors. Out of these the most computationally expensive part is the matrix-vector product. Since GMRES requires matrix-vector product of the form $H^{i-1}y$ and LSQR requires matrix-vector product of the form $(H^H H)^{i-1} H^H y$, LSQR has twice the complexity of GMRES. Table II presents some more comparisons of GMRES and LSQR.

TABLE III
COMPARISON OF GMRES AND LSQR

Property	**GMRES**	**LSQR**
Krylov Subspace	$\mathcal{K}_i(H, y)$	$\mathcal{K}_i(H^H H, H^H y)$
Storage	i+1 vectors of length N	4 vectors of length N
Work per iteration	One application of H and other linear operations	One application of H; one application of H^H and other linear operations
Memory Requirement	O(N)	O(N)
Operations per iteration	$O(N \log N)$	$O(N \log N)$

VII. COMPLEXITY ANALYSIS

The complexity analysis of VBLAST, SAGE, GMRES and LSQR in Table III reveals that the Krylov methods have substantially lower complexity when compared to non-linear equalization methods. Here $G^H G$ is a banded matrix with 4Q bandwidth and i is the number of iterations for Krylov methods. Between GMRES and LSQR the former offers lower complexity as it requires half the number of complex operations.

TABLE IIIII

COMPLEXITY ANALYSIS OF EQUALIZATION TECHNIQUES

Method	Number of Complex Operations	Computational Complexity
VBLAST	$2N^3 + (8Q^2 + 22Q + 6)N^2 + 2N$	$O(N^3) + O(Q^2N^2) + O(QN^2)$
SAGE	$(8Q^2 + 38Q + 8)N + N\log N$	$O(Q^2N) + O(N\log N)$
LSQR	$(i + 1)N\log N$	$O(N\log N)$
GMRES	$(2i + 1)N\log N$	$O(N\log N)$

VIII. SIMULATION RESULTS

The performance of MF, ZF and MMSE equalizers is simulated with N = 128 subcarriers and cyclic prefix length, NCP = 16. The OFDM symbol period is Ts = 144 micro seconds. The bandwidth of the OFDM system is 5MHz and the carrier frequency is fc. = 2.4 GHz. The normalised Doppler frequency is fd = 0.0384 for a mobile terminal travelling at a velocity of 120 km/hr. The modulation technique used is BPSK. The channel is a doubly selective Rayleigh channel having 4 complex zero mean Gaussian taps with power profile as given σ2 = [1 0.448 0.321 0.230]. The channel taps fade according to Jakes model. Fig. 2 shows the results of the simulation carried out. It is found that MMSE gives the better performance compared to MF and ZF.

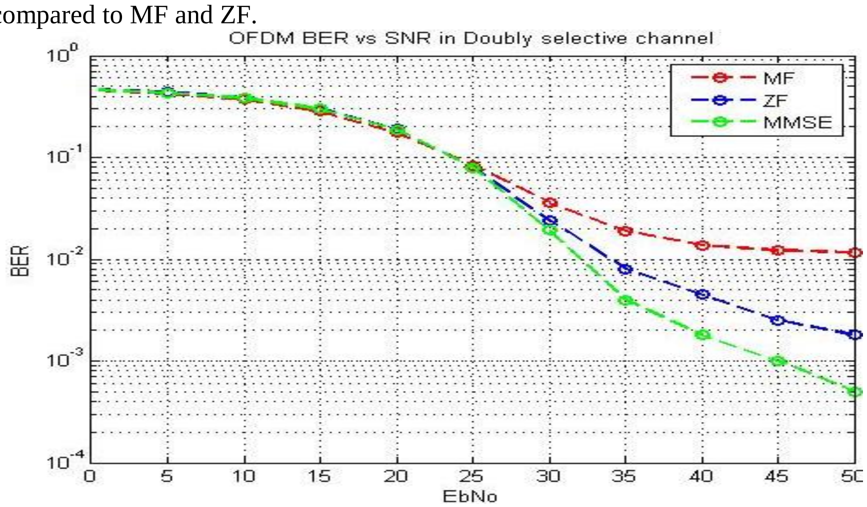

Fig. 2 Comparison of Linear Equalization Methods

In Fig. 3 the performance of SAGE and VBLAST in the same scenario is compared to the linear detection techniques. It is found that both outperform the linear detection methods significantly. In particular, it is seen that the SAGE algorithm exhibits a detection gain of about 4 dB over MMSE detection for BPSK modulation at SER = 10-2, respectively. Moreover, it is also seen that the detection gain of SAGE over MMSE increases for higher SNR values. The performance of VBLAST in the same scenario is compared to the linear detection techniques. It is found that the VBLAST algorithm exhibits a detection gain of about 4 dB over MMSE detection for BPSK modulation at SER = 10-2, respectively. The Krylov subspace methods LSQR and GMRES are also employed for the same channel. It is found that LSQR algorithm gives performance comparable to MMSE equalization but not as well as VBLAST and SAGE which give a performance gain of about 4.0477 dB and 4.5593 dB at 30dB SNR; trading accuracy for speed. The GMRES method does not converge and the BER remains constant at 0.5.

The plot of residual error versus number of iterations is shown in Fig. 4 for both LSQR and GMRES algorithms. It is seen that LSQR converges to a solution much faster than GMRES. The relative residual for GMRES is very high at nearly 1.

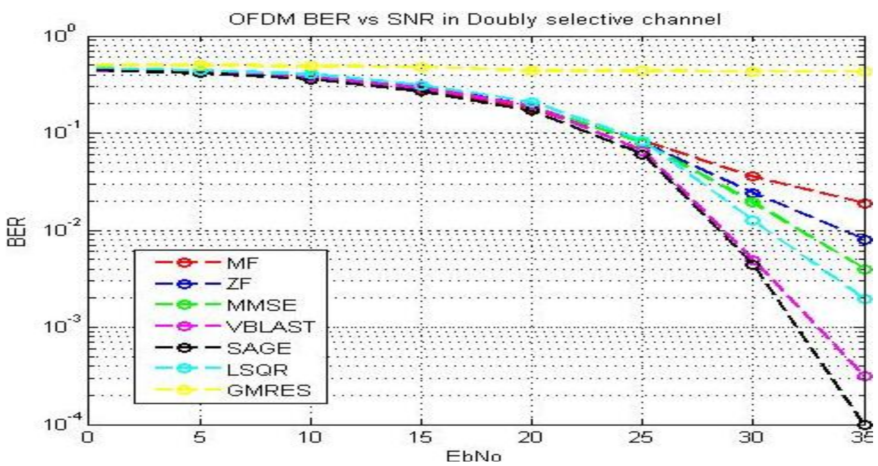

Fig. 3 Comparison of LSQR and GMRES with traditional Equalization techniques

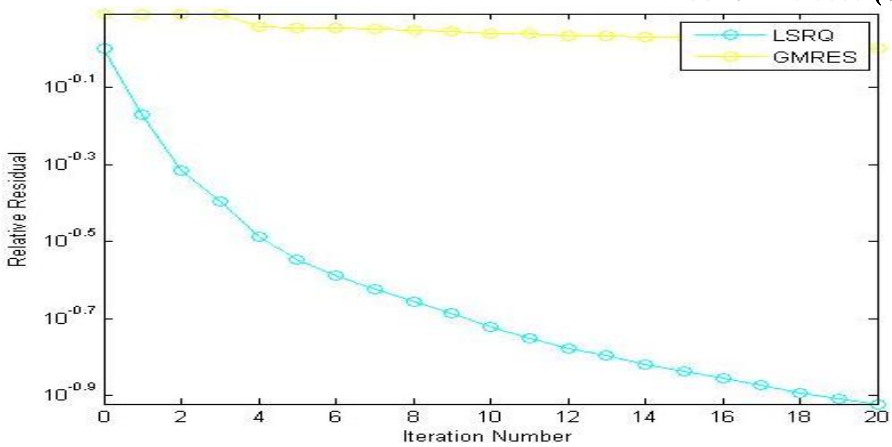

Fig. **Error! No text of specified style in document.** Relative Residual of GMRES and LSQR

VIII. CONCLUSIONS

Between LSQR and GMRES the former gives acceptable results. GMRES is not suitable for Doubly Selective channels as the algorithm fails to converge and the residual error is high. LSQR method on the other hand gives results comparable to MMSE but not as well as VBLAST and SAGE which give a performance gain of about 4.0477 dB and 4.5593 dB at 30dB SNR. But LSQR offers very little complexity as compared to both VBLAST and SAGE. In 4th generation communication systems the number of subcarriers is more than 1024. In such scenarios LSQR method is more appropriate than VBLAST or SAGE even at the cost of performance.

REFERENCES

[1] James W. Demmel, *Applied Numerical Linear Algebra*, Society for Industrial and Applied Mathematics, 1997.
[2] Gene Golub and Charles F. Van Loan. *Matrix Computations*, The John Hopkins University Press, 3rd Edition, 1996.
[3] Lloyd N. Trefethen and David III Bau. *Numerical Linear Algebra*, Society for Industrial and Applied Mathematics 1997.
[4] Hussein Hijazi and Laurent Ros, “Joint Data QR Detection and Kalman Estimation for OFDM Time-Varying Rayleigh Channel Complex Gains”, *IEEE Transactions on Communications*, vol. 58, no. 1, pp. 170-178, Jan. 2010.
[5] A. Molisch, “Orthogonal Frequency Division Multiplexing” *Wireless Communications*, pp. 417-443, 2011.
[6] Shuichi Ohno and Kok Ann Donny Teo, “Approximate BER expression of ML Equalizer for OFDM over Doubly Selective Channels”, *IEEE International Conference on Audio, Speech and Signal Processing*, pp. 3049-3052, 2008.
[7] A. F. Molisch, M. Toeltsch, S. Vermani, "Iterative Methods for Cancellation of Intercarrier Interference in OFDM Systems", *IEEE Transactions on Vehicular Technology*, vol. 56, no. 4, pp. 2158 – 2167, July 2007.
[8] J. Guerreiro, R. Dinis and P. Montezuma, “ML-based receivers for underwater networks using OFDM signals with strong non linear distortion effects,” *Military Communication Conference*, MILCOM-2012, pp. 1-7, 2012.
[9] Li Yabo, I. Ngebani and Xia Xiang-Gen, “On performance of V-OFDM with Zero Forcing Receiver” *IEEE International Conference on Signal Processing*, vol. 2, pp. 1506-1511, Oct. 2012.
[10] Y.-S. Choi, P. J. Voltz, and F. A. Cassara, “On channel estimation and detection for multicarrier signals in fast and selective Rayleigh fading channels,” *IEEE Trans. Comms*., vol. 49, no. 8, pp. 1375–1387, Aug. 2001.
[11] P. Schniter, “Low-complexity equalization of OFDM in doubly selective channels,” *IEEE Trans. Signal Processing*, vol. 52, no. 4, pp. 1002–1011, Apr. 2004.
[12] J. Fessler and A. Hero, “Space-alternating generalized expectation-maximization algorithm”, *IEEE Trans. Signal Process*., vol. 42, no. 10, pp. 2664–2677, 1994.
[13] C. C. Paige and M. A. Saunders, “LSQR: An Algorithm for Sparse Linear Equations and Sparse Least Squares Problems”, *ACM Transactions on Math Soft.*, vol. 8, pp. 43-71, 1982.
[14] Y. Saad and M. Schultz, “GMRES: A Generalized Minimum Residual Algorithm for Solving Non-Symmetric Linear Systems”, *SIAM J, Scientific and Stat. Comp.*, vol. 7, pp. 856-869, 1986.