Belief Propagation Based Combined Decoding Scheme for LDPC-coded OFDM with PTS as PAPR Reduction

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Abstract—

This paper proposes belief propagation based combined decoding scheme for low-density parity-check (LDPC)-coded orthogonal frequency-division multiplexing (OFDM) system with a peak-to-average power ratio (PAPR) reduction using the partial transmit sequence (PTS), which does not transmit PTS side information about the phase factors.

Keywords— Belief Propagation (BP), Partial Transmit Sequence (PTS), Peak to Average Power (PAPR), Low Density Parity Check codes (LDPC), OFDM

I. INTRODUCTION

It is well known that orthogonal frequency-division multiplexing (OFDM) suffers from a high peak-to-average power ratio (PAPR). Among the techniques that have been proposed to reduce the PAPR, the partial transmit sequence (PTS), has attracted a lot of attention [2] because it introduces no distortion in the transmitted signal and achieves significant PAPR reduction. However, one of the critical challenges of PTS schemes is that the phase factor information is required to be transmitted to the receiver as side information, which decreases transmission efficiency and increases system complexity. Recently, some works [5]–[9] have been presented for phase factor recovery with the help of error-correcting codes, which do not require the transmission of phase factor information.

This paper proposes a scheme that does not have to perform phase factor estimation before decoding. We propose combined decoding of the LDPC code and phase factors, which simply uses belief propagation (BP) algorithms [10] and could enable a simple system design. In this paper, We investigate an LDPC-coded OFDM system with the PTS PAPR reduction, which does not transmit PTS side information about the phase factors. We view the PTS processing as a stage of coding and call the resulted code of LDPC coding and PTS processing a concatenated LDPC-PTS code. Then, We derive the parity-check matrix of the concatenated LDPC-PTS code. With the parity-check matrix, the LDPC code and phase factors can be combinedly decoded using Belief Propagation algorithm.

II. LITERATURE REVIEW

A. Basic OFDM System

Orthogonal Frequency-Division Multiplexing (OFDM) is a technique to nullify the effects of ISI [2]. The idea is to divide the signal bandwidth into a number of subcarriers where the bandwidth of each of these subchannels is simply the ratio of the total bandwidth to the number of subcarriers. This is done by using an Inverse Fourier Transform (IDFT/IFFT) block at the transmitter and a corresponding DFT/FFT block at the receiver. To make the channel perform circular convolution, a cyclic prefix (CP) of length greater than or equal to the channel length can be added before the symbols are sent at the transmitter side. Thus after estimating the channel coefficients (in the frequency domain) at the receiver, the symbols can be decoded without any ISI. OFDM works quite well with channel codes and together they are called coded OFDM (COFDM). Channel codes increase the error detecting and correcting capability and thus higher order constellations like 16 QAM and 64 QAM could also be used to achieve higher spectral efficiency. Some of the common coding techniques for COFDM include concatenating inner convolution code with an outer Reed-Solomon code, turbo codes and LDPC codes. However the implementation of COFDM with LDPC codes is more complex than some of the other codes.

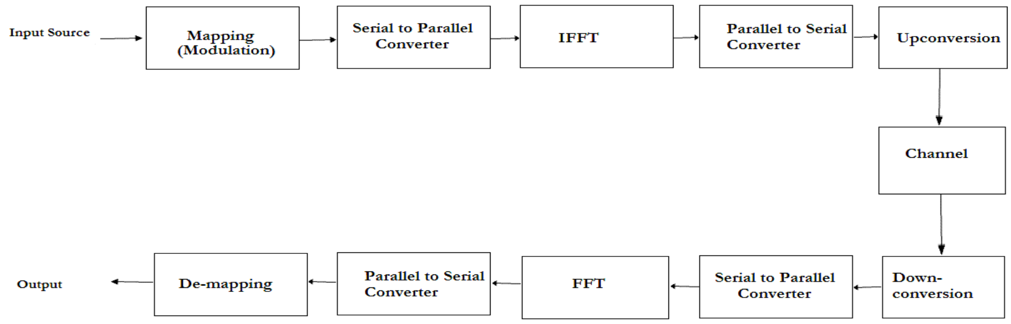

Fig. 1 Basic OFDM System block diagram

B. LDPC Codes

Low-density parity-check (LDPC) codes are a class of linear block codes. The name comes from the characteristic of their parity-check matrix which contains only a few 1's in comparison to the amount of 0's. Their main advantage is that they provide a performance which is very close to the capacity for a lot of different channels and linear time complex algorithms for decoding. Furthermore they are suited for implementations that make heavy use of parallelism. They were first invented by Gallager in the 1963 but it was a forgotten code until this past decade due to its implementation complexity [3].

C. Partial Transmit Sequence (PTS)

To overcome the problem of high Peak to Average Power Ratio(PAPR), the partial transmit sequences (PTS) method for PAPR reduction in single antenna systems as well as for multiantenna OFDM systems is used. The PTS approach is a distortion less technique based on combining signal subblocks which are phase-shifted by constant phase factors. The technique significantly reduces the PAPR. The functional block diagram of partial transmit sequence based PAPR reduction algorithm is shown in Fig. 2. It is based on dividing the original OFDM sequence into several sub-sequences; and for each sub-sequence, multiplying by different weights until an optimum value is achieved.

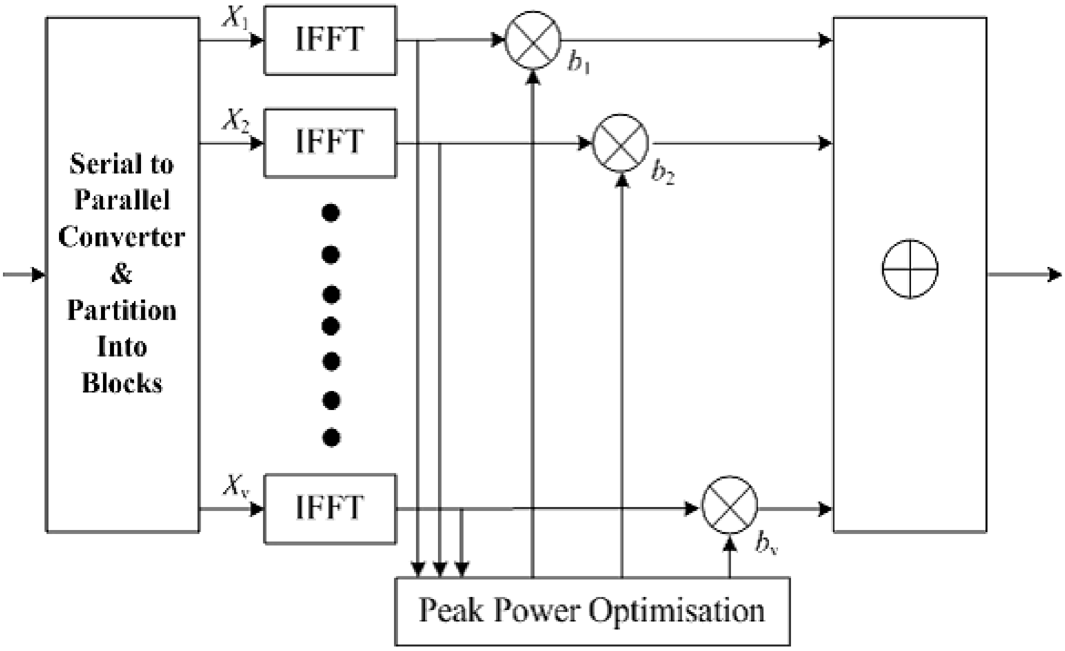

Fig. 2 The functional block diagram of partial transmit sequence based PAPR reduction algorithm

III. PROPOSED SYSTEM

A. Block Diagram

In the LDPC-coded OFDM system shown in Fig. 3, an LDPC [9] codeword is mapped onto OFDM subcarriers with phase-shift keying (PSK) modulation/quadrature-amplitude modulation (QAM). The LDPC codeword, which is denoted by row vector A, is mapped onto one OFDM data block of N_c subcarriers after a random interleaver to obtain data block vector X= [X(1), . . . , X(N_c)], where X(k) represents the PSK/QAM symbol on the kth subcarrier.

With the PTS PAPR reduction, X is partitioned into W blocks, i.e., $X^{(1)}, \ldots, X^{(W)}$, where $X^{(w)} = [X^{(w)}(1), \ldots, X^{(w)}(Nc)]$, for $w = 1, 2, \ldots, W$. The partition satisfies that $X^{(w)}(k) = 0$ or X(k) for $k = 1, 2, \ldots, N_c$ and $w = 1, 2, \ldots, W$, and $\sum X(w) = X$.

Then, the candidate data block of PTS, which is denoted by X_{PTS}, can be obtained by independently rotating X(w) with the phase factor p_w ($w = 1, 2, \ldots, W$), i.e.,

$$\mathbf{X}_{\mathrm{PTS}} = \sum_{w=1}^{W} p_w \mathbf{X}^{(w)}.$$

In this paper, we assume that $p_w \in \{1, -1\}$ for all w. We also assume that the PSK modulation/QAM scheme employed in the system satisfies the following:

Multiplying the PSK/QAM symbols by a phase factor of −1 flips several bits of the symbols and does not change the other bits, and the indexes of bits that are flipped when the symbols are multiplied by −1 are common to all symbols and known to the receiver.

Obviously, the widely adopted PSK modulation/QAM with Gray mapping satisfies this assumption.

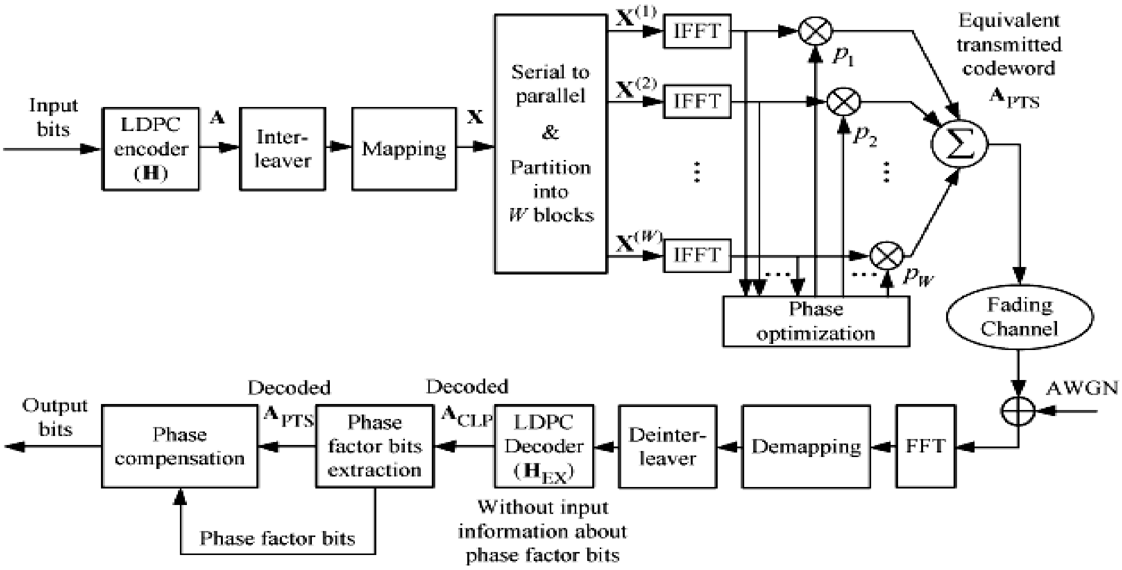

Fig. 3 The functional block diagram of proposed system

Let A_w (w = 1, 2, . . . , W) denote the row vector of the bits that are flipped when p_w is turned from 1 to −1, and let A_0 denote the row vector of all other bits that are never affected by the PTS processing. This way, the original LDPC codeword A is divided into W + 1 vectors, i.e., $A_0, A_1, \ldots, A_W$. Denote the length of vector A_w by N_w for $w = 0, 1, \ldots, W$.

The corresponding vectors after the PTS processing are denoted by $(A_{PTS})_0$, $(A_{PTS})_1$, . . . , $(A_{PTS})_W$, respectively. Apparently, $(A_{PTS})_0 = A_0$. Due to the PTS processing and the aforementioned assumptions, we have

$$(\mathbf{A}_{\mathrm{PTS}})_w = \mathbf{A}_w \oplus [\underbrace{b_w, \ldots, b_w}_{N_w}], \quad \text{for } w = 1, 2, \ldots, W$$

where ⊕ represents modulo-2 addition, and b_w represents the phase factor bit of the w^{th} block, i.e.,

$$b_w = \begin{cases} 0, & p_w = 1, \\ 1, & p_w = -1, \end{cases} \quad \text{for } w = 1, 2, \ldots, W.$$

We call A_{PTS} as the equivalent transmitted codeword in this project. In this project, the concatenated LDPC-PTS codeword is defined as the codeword consisting of the equivalent transmitted codeword and phase factor bits, although the phase factor bits are not transmitted by the system.

The concatenated LDPC-PTS code for the previously described OFDM system is a block code with the following codeword, which is represented by a vector of bits:

$A_{CLP} = [A_0, (A_{CLP})_1, \ldots, (A_{CLP})_W]$

where $(A_{CLP})_w = [(A_{PTS})_w, b_w]$, for $w = 1, 2, \ldots, W$.

Apparently, the equivalent transmitted codeword A_{PTS} is a punctured version of the concatenated LDPC-PTS codeword A_{CLP}. In the rest of this section, we define the extended Tanner graph for the OFDM system and discuss its relationship with the concatenated LDPC-PTS code.

B. Belief Propagation Algorithm

In this paper we propose use of belief propagation (BP) algorithm for decoding the transmitted concatenated LDPC-PTS codeword A_{CLP}.

While decoding LDPC codeword BP algorithm uses Tanner Graph model to derive logical conclusions about the value of bits which were transmitted. A tanner graph is a graphical representation of LDPC code consisting of variable nodes V, check nodes C and E the set of edges, where $E \subseteq V \times C$..

In this paper we extend the tanner graph (V_{EX}, C, E_{EX}) for the previously described OFDM system from (V,C,E) with the following rule: the extended variable-node set V_{EX} consists of subsets V_0, $(V_{EX})_1$, . . . , $(V_{EX})_W$, where $(V_{EX})_w$ is the extension of the subset V_w by including phase factor node b_w, i.e., $(V_{EX})_w = V_w \cup \{b_w\}$, for $w = 1, 2, \ldots, W$. In addition, the extended edge set E_{EX} consists of E and new edges that are connected to phase factor nodes $b_1, b_2, \ldots, b_W$. The new edges are generated with the following rule: For any check node $c \in C$ and phase factor node b_w, $w = 1, 2, \ldots, W$, an edge is added between c and phase factor node b_w if the total number of edges connecting c to all variable nodes in V_w is odd. (Without sacrificing clarity, we abuse the notation b_w for the phase factor node and phase factor bit in this project.)

An example of the extended Tanner graph is presented in Fig. 4, in which phase factor nodes b_1 and b_2 are inserted into the Tanner graph for variable-node subsets V_1 and V_2, respectively. In addition, four edges (between c_3 and b_1, c_5 and b_1,

c_4 and b_2, and c_5 and b_2) are added in the extended Tanner graph so that the total number of edges connecting each check node to all variable nodes in $(V_{EX})_w$ ($w = 1, 2$) is even.
As a Tanner graph defines a parity-check matrix, we can obtain the extended parity-check matrix H_{EX} of dimension M × (N +W) from (V_{EX},C,E_{EX}). H_{EX} consists of W + 1 sub matrices as
$H_{EX} = [H_0, (H_{EX})_1, \ldots, (H_{EX})_W]$
where $(H_{EX})_W$ ($1 \leq w \leq W$) is the submatrix of H_{EX}, whose columns correspond to the extended variable-node subset $(V_{EX})_W$, and H_0 is the submatrix corresponding to V_0.

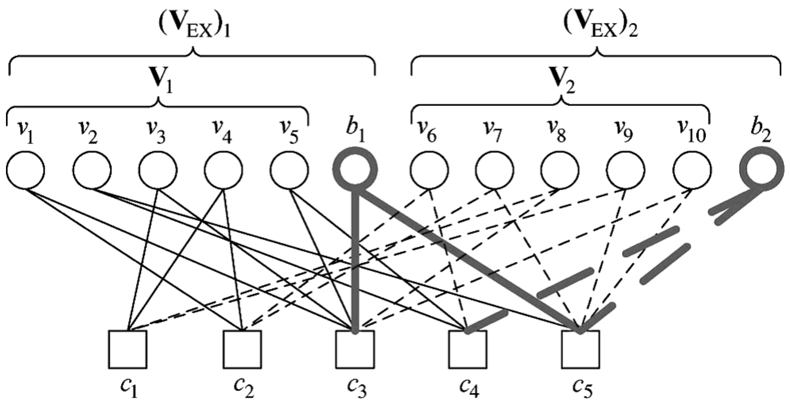

Figure 4: Example of the extended Tanner graph.

C. Parity-Check Matrix and Decoding of the Concatenated LDPC-PTS Code
The concatenated LDPC-PTS code for the previously defined OFDM system is a linear block code with parity-check matrix H_{EX} and Tanner graph (V_{EX},C,E_{EX}).
It can be written over GF(2) for $1 \leq w \leq W$

$$(H_{EX})_w [(A_{CLP})_w]^T$$
$$= (H_{EX})_w [(A_{PTS})_w, b_w]^T$$
$$= (H_{EX})_w [a_w(1) \oplus b_w, \ldots, a_w(N_w) \oplus b_w, b_w]^T$$
$$= (H_{EX})_w [a_w(1), \ldots, a_w(N_w), 0]^T + (H_{EX})_w [\underbrace{b_w, \ldots, b_w}_{N_w}, b_w]^T$$

$$= H_w[A_w]T + (H_{EX})_w [b_w, \ldots, b_w, b_w]^T$$

where H_w is the submatrix of H, whose columns correspond to the variable-node subset V_w, and $a_w(k)$ is the k^{th} element of A_w. Since the total number of edges connecting each check node to all variable nodes in $(V_{EX})w$ is even, each row of the associated submatrix $(H_{EX})_w$ has even Hamming weight (even number of 1s) ($1 \leq w \leq W$). Therefore

$$(H_{EX})_w [\underbrace{b_w, .., b_w}_{N_w}, b_w]^T = 0, \text{ for } 1 \leq w \leq W.$$

Then, we have

$$(H_{EX})_w [(A_{CLP})_w]^T = H_w[A_w]^T, \text{ for } 1 \leq w \leq W.$$

Finally, we have

$$\mathbf{H}_{\mathrm{EX}}[\mathbf{A}_{\mathrm{CLP}}]^T = \mathbf{H}_0[\mathbf{A}_0]^T + \sum_{w=1}^{W} (\mathbf{H}_{\mathrm{EX}})_w [(\mathbf{A}_{\mathrm{CLP}})_w]^T$$
$$= \sum_{w=0}^{W} \mathbf{H}_w[\mathbf{A}_w]^T = \mathbf{H}[\mathbf{A}]^T = \mathbf{0}.$$

Therefore, the concatenated LDPC-PTS codeword A_{CLP} is a valid codeword of the linear block code with parity-check matrix H_{EX} and Tanner graph (V_{EX},C,E_{EX}).
Fig. 3 also shows the receiver diagram of the proposed combined decoding scheme. Since A_{PTS} is the puncture version of the concatenated LDPC-PTS codeword A_{CLP}, A_{CLP} can be decoded with H_{EX} and the received A_{PTS}. Note that the decoded A_{CLP} consists of the decoded A_{PTS} and phase factor bits. Assuming correct decoding, phase factor bits are extracted, and then, the original LDPC codeword **A** can be found by phase compensation as the following:

$$\mathbf{A}_w = \begin{cases} (\mathbf{A}_{\mathrm{PTS}})_w \oplus [\underbrace{b_w, \ldots, b_w}_{N_w}], & w = 1, 2, \ldots, W \\ \mathbf{A}_0, & w = 0. \end{cases}$$

The complexity of the proposed combined decoding is higher, but not very significant, compared with that of decoding of the original LDPC code, since the dimension of the parity-check matrix of the concatenated LDPC-PTS code (H_{EX}) is only greater than that of the original LDPC code (H) for W columns, and W<< N.

IV. Simulation Results

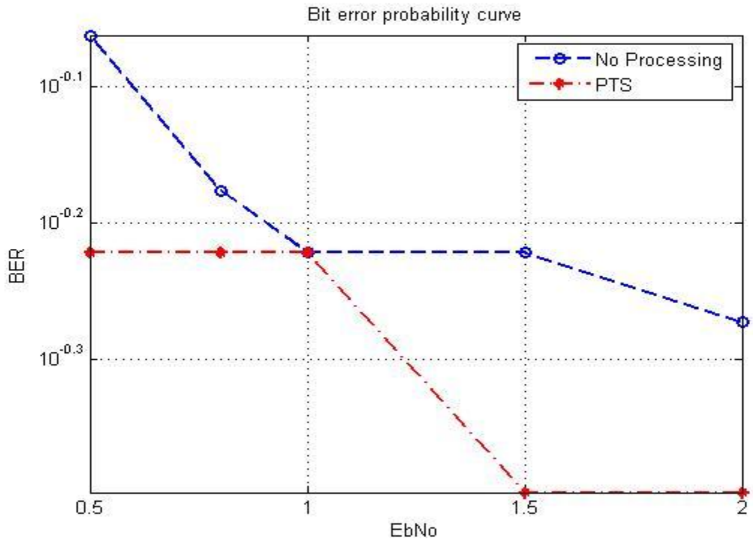

Fig. 5 BER Performance curve for proposed system (MATLAB Result)

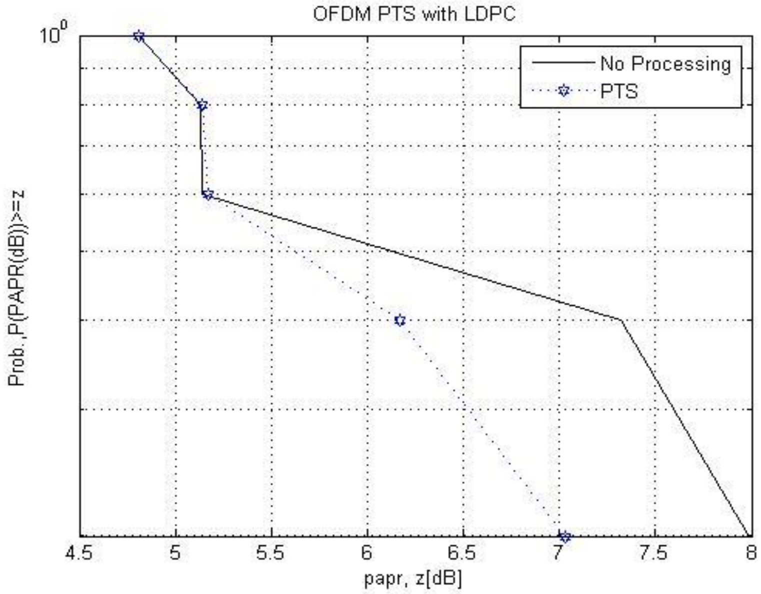

Fig. 6 PAPR performance curve for proposed system(MATLAB Result)

The bit-error-rate (BER) and PAPR performance curves of the proposed combined decoding with QPSK modulation over an AWGN channel are plotted in Fig 6.
The PAPR performance of the simulated system is exactly the same as that of the well-known PTS technique and provides significant reduction in PAPR compared to OFDM system without PTS scheme.
As E_b/N_o increases, the BER of proposed system decreases at larger rate compared to a system without PTS processing as seen from fig.5.

V. Conclusions

These results obtained via MATLAB simulations show that the proposed scheme can provide a significant PAPR reduction, as well as nearly perfect phase factor recovery and LDPC decoding as seen from BER performance curve, with small W.

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