

# Comparative Analysis of PCA, DCT & DWT based Image Fusion Techniques

Nisha Gawari,  
IV Sem M.Tech Student,  
Dept of ECE, AIET, Gulbarga, India

Dr. Lalitha.Y.S  
Professor, Dept. of ECE,  
AIET, Gulbarga, India

## Abstract-

**Image fusion is the process of combining relevant information from two or more images into a single image. The resulting image will be more informative than any of the input images. The object of image fusion is to retain the most desirable characteristics of each image. This paper discusses about the Formulation, Process Flow Diagrams and algorithms of PCA (principal Component Analysis), DCT (Discrete Cosine Transform) and DWT (Discrete Wavelet Transform) based image fusion techniques. The results are also furnished in picture and table format for comparative analysis of above techniques. This paper presents the three different image fusion techniques and there comparative analysis, as the conventional fusion techniques PCA and DCT has some drawbacks. The comparative study concludes that DWT is the best approach for image fusion. In this paper DWT based two algorithms are proposed, these are maximum pixel replacement and pixel averaging approach.**

**Keywords-image fusion; principal component analysis; discrete cosine transform; discrete wavelet transform.**

## 1. INTRODUCTION

Image fusion is a useful technique for merging single sensor and multi-sensor images to enhance the information. The objective of image fusion is to combine information from multiple images in order to produce an image that deliver only the useful information. Any piece of information makes sense only when it is able to convey the content across. The clarity of information is important. By the process of image fusion the good information from each of the given images is fused together to form a resultant image whose quality is superior to any of the input images This is achieved by applying a sequence of operations applied on the images that would make the good information in each of the image prominent. The fused image is constructed by combining magnified information from the input images

## 2. PRINCIPAL COMPONENT ANALYSIS

It is a mathematical tool from applied linear algebra. It is a simple non-parametric method of extracting relevant information from confusing data sets. PCA is a useful statistical technique that has found application in fields such as face recognition and image compression, and is a common technique for finding patterns in data of high dimension. The origins of PCA lie in multivariate data analysis, it has a wide range of other applications PCA has been called, 'one of the most important results from applied linear algebra and perhaps its most common use is as the first step in trying to analyses large data sets. In general terms, PCA uses a vector space transform to reduce the dimensionality of large data sets. Using mathematical projection, the original data set, which may have involved many variables, can often be interpreted in just a few variables (the principal components).

### 2.1. Formulation

Let us consider  $X$  be a  $d$ -dimensional random vector and assume it to have zero empirical mean. The orthonormal projection matrix  $V$  would be such that  $Y=V^T X$  with the following constraints.

The covariance of  $Y$ , i.e.,  $\text{cov}(Y)$  is a diagonal and inverse of  $V$  is equivalent to its transpose

( $V^{-1}=V^T$ ). Using Matrix Algebra,

$$\text{cov}(Y) = E\{YY^T\} \quad (1)$$

$$\text{cov}(Y) = E\{(XV^T)(V^T X)^T\} \quad (2)$$

$$\text{cov}(Y) = E\{(XV^T)(VX^T)\} \quad (3)$$

$$\text{cov}(Y) = V^T \text{cov}(X) V \quad (4)$$

Multiplying both sides of equation (4) by  $V$ , we get,

$$V \text{cov}(Y) = V V^T \text{cov}(X) V = \text{cov}(X) V \quad (5)$$

Substituting equation (4) into the equation (5) gives,

$$\begin{aligned} & [\lambda_1 V_1, \lambda_2 V_2, \dots, \lambda_d V_d] \\ & = [\text{cov}(X)V_1, \text{Cov}(X)V_2, \text{cov}(X)V_d] \end{aligned} \quad (6)$$

This could be rewritten as

$$\lambda_i V_i = \text{cov}(X) V_i \quad (7)$$

Where,  $i=1, 2, \dots, d$  and  $V_i$  is an eigenvector of  $\text{cov}(X)$ .

## 2.2. PROCES FLOW DIAGRAM OF PCA

The process flow diagram of PCA algorithm is shown in below figure 1. The input images  $I_1(x, y)$  and  $I_2(x, y)$  are arranged in two column vectors and their empirical means are subtracted. The resulting vector has a dimension of  $n \times 2$ , where  $n$  is length of the each image vector.

Compute the eigenvector and eigenvalues for this resulting vector are computed and the eigenvectors corresponding to the larger eigenvalue obtained.

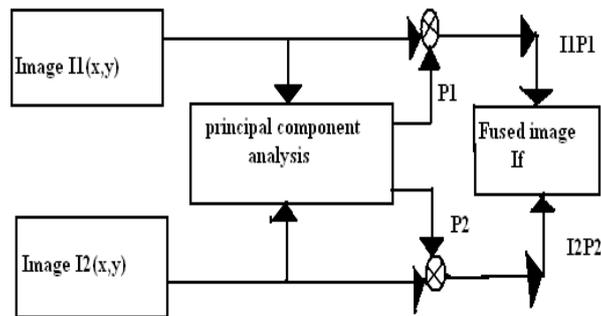


Figure 1. Image Fusion Process using PCA

The normalized components  $P_1$  and  $P_2$  are computed from the obtained eigenvector.

The fused image is given by equation,

$$I_f(x,y) = P_1 I_1(x,y) + P_2 I_2(x,y) \quad (8)$$

## 2.3. PCA ALGORITHM

Let the source images be arranged in two-column vectors. The steps followed to project this data into 2-D subspaces are:

1. From the input images matrices arrange the data into column vectors. The resulting matrix  $Z$  is of dimension  $2 \times n$ .
2. Then Compute the empirical mean along each column. The empirical mean vector  $M_e$  has a dimension of  $1 \times 2$ .
3. Subtracting the empirical mean vector  $M_e$  from each column of the data matrix  $S$ . The resulting matrix  $X$  is of dimension  $2 \times n$ .
4. Find the covariance matrix  $C$  of  $X$  i.e.  $C = XX^T$  mean of expectation =  $cov(X)$
5. Compute the eigenvectors  $V$  and eigenvalue  $D$  of  $C$  and sort them by decreasing eigenvalue. Both  $V$  and  $D$  are of dimension  $2 \times 2$ .
6. Finally consider the first column of  $V$  which corresponds to larger eigenvalue to compute  $P_1$  and  $P_2$  as,

$$P_1 = \frac{v(1)}{\sum v} \text{ and } P_2 = \frac{v(2)}{\sum v}$$

## 3. DISCRETE COSINE TRANSFORM

The digital images are displaying on a screen immediately after they are captured. There are two represent types for digital image that is spatial domain or frequency domain. Spatial domain image can be realizes through our human eyes, but frequency domain use to analysis of spatial domain A Discrete Cosine Transform (DCT) is an important transform in image processing. It is always used to express a sequence of finite data points in terms of a sum of cosine functions oscillating at different frequencies. Large DCT coefficients are concentrated in the low frequency region; hence, it is known to have excellent energy compactness properties. Discrete Cosine Transformation (DCT) are important to numerous applications in science, engineering and in images compress, like MPGE, JVT, etc

### 3.1. Formulation of DCT

The 2D DCT is nothing but a direct extension of 1D DCT. The 2D discrete cosine transform of an image or 2D signal  $x(n_1, n_2)$  of size  $N_1 \times N_2$  is defined as,

$$x(k_1, k_2) = \alpha(k_1) \alpha(k_2) \quad (9)$$

$$\sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2) \cos\left(\frac{\pi(2n_1+1)k_1}{2N_1}\right) \quad (10)$$

$$\cos\left(\frac{\pi(2n_2+1)k_2}{2N_2}\right) \quad \text{for } 0 \leq k_1 \leq N_1-1 \text{ and } 0 \leq k_2 \leq N_2-1$$

Where,

$$\alpha(k_1) = \begin{cases} \frac{1}{\sqrt{N_1}} & , \quad k_1 = 0 \\ \sqrt{\frac{2}{N_1}} & , \quad 1 \leq k_1 \leq N_1 - 1 \end{cases} \quad (11)$$

$$\alpha(k_2) = \begin{cases} \frac{1}{\sqrt{N_2}}, & K_2 = 0 \\ \sqrt{\frac{2}{N_2}}, & 1 \leq k_2 \leq N_2 - 1 \end{cases} \quad (12)$$

Where  $K_1$  &  $K_2$  are discrete frequency variables.

Similarly, the 2D inverse discrete cosine transform is defined as

$$X(n_1, n_2) = \alpha(k_1) \alpha(k_2) X(k_1, k_2) \quad (13)$$

$$\sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \alpha(k_1) \alpha(k_2) X(k_1, k_2) \cos\left(\frac{\pi(2n_1+1)k_1}{2N_1}\right) \quad (14)$$

$$\cos\left(\frac{\pi(2n_2+1)k_2}{2N_2}\right) \quad \text{For } 0 \leq k_1 \leq N_1-1 \quad \text{and } 0 \leq k_2 \leq N_2-1$$

Where  $\alpha(k_1)$  &  $\alpha(k_2)$  are same as equation (11) & (12).

### 3.2. Process flow graph of DCT

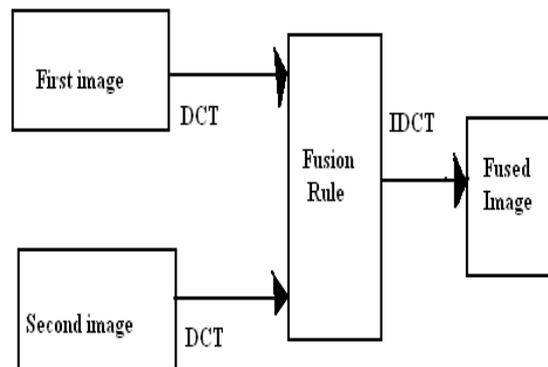


Figure 2. process flow graph of DCT

In DCT the Images to be fused are divided into non-overlapping blocks of size  $N \times N$  as shown in above Fig-2.

For each block DCT coefficients are computed and fusion rules are applied to get fused DCT coefficients. Then apply the IDCT on the fused coefficients to produce the fused image/block.

The following only two fusion rule is used for image fusion process. They are the simple averaging method and by using equation method as in eq (15).

Let the  $X_1$  be the DCT coefficients of image block from image 1 and similarly let  $X_2$  be the DCT coefficients of image block from image 2. Assume the image block is of size  $N \times N$  and  $X$  be the fused DCT coefficients. Here, all DCT coefficients from both image blocks are averaged to get fused DCT coefficients. It is very simple and basic image fusion technique in DCT domain

$$X_f(k_1, k_2) = 0.5 X_1(k_1, k_2) + X_2(k_1, k_2) \quad (15)$$

Where  $k_1, k_2 = 0, 1, 2, \dots, N-1$

### 4. DISCRETE WAVELET TRANSFORM

The Discrete Wavelet Transform (DWT) also converts the image from the spatial domain to frequency domain. The image is divided by vertical and horizontal lines and represents the first-order of DWT, and the image can be separated with four parts those are LL1, LH1, HL1 and HH1. In additional, those four parts are represented four frequency areas in the image. For the low-frequency domain LL1 is sensitively with human eyes. In the frequency domains LH1, HL1 and HH1 have more detail

Information more than frequency domain LL1.

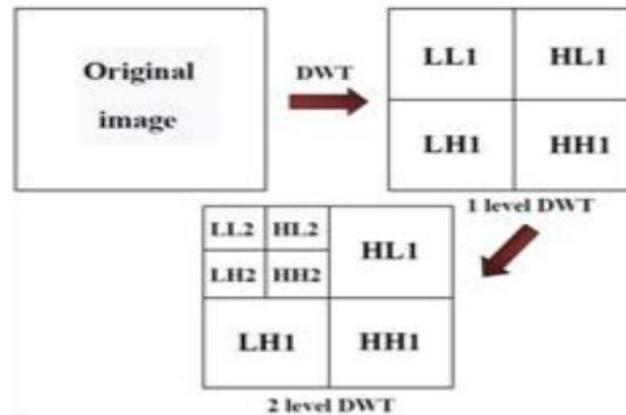


Figure 3. Frequency distribution of DWT

#### 4.1. Process Flow Diagram of DWT

Wavelet transform is first performed on each source images to generate a fusion decision map based on a set of fusion rules. The fused wavelet coefficient map can be constructed from the wavelet coefficients of the source images according to the fusion decision map. Finally the fused image is obtained by performing the inverse wavelet transform

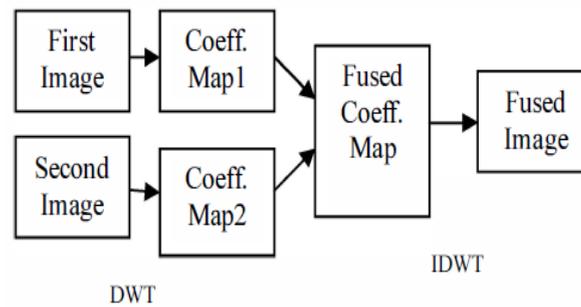


Figure 4. Process of image fusion using DWT

The fusion rules play a very important role during the fusion process.

1. Implement the DWT on both the input images to create lower decomposition wavelets.
2. By using different fusion rules fuse each decomposition levels.
3. Apply IDWT on fused decomposition levels, to reconstruct the original image that is fused image.

#### 4.2. DWT Decomposition

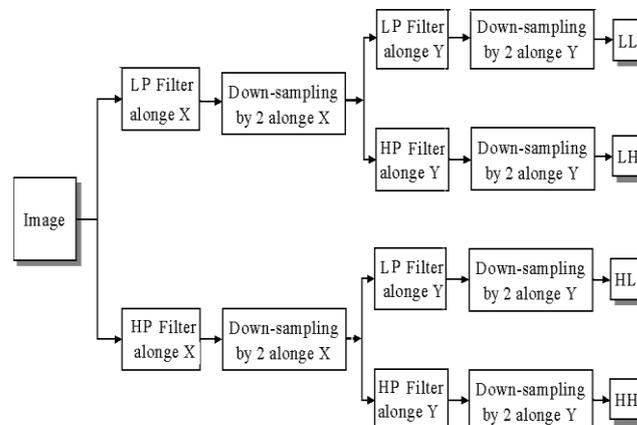


Figure 5. The block diagram of 2D- DWT

In discrete wavelet transform (DWT) decomposition, the filters are specially designed so that successive layers of the pyramid only include details which are not already available at the preceding levels. The DWT decomposition uses a

cascade of special low pass and high-pass filters and a sub-sampling operation. The outputs from 2D-DWT are four images having size equal to half the size of the original image. So from first input image we will get HHa, HLa, LHa, LLa images and from second input image we will get HHb, HLb, LHb, LLb images. LH means that low-pass filter is applied along x and followed by high pass filter along y. The LL image contains the approximation coefficients. LH image contains the horizontal detail coefficients. HL image contains the vertical detail coefficients, HH contains the diagonal detail coefficients. The wavelet transform can be performed for multiple levels. The next level of decomposition is performed using only the LL image. The result is four sub-images each of size equal to half the LL image size.

### 4.3. Algorithms

The algorithm of image fusion using DWT has following common steps applicable to proposed methods of fusion.

- a) Accept the two input images.
- b) Resize both the images to 256 x 256.
- c) Convert to Gray scale image.
- d) Convert to double precision format.
- e) Take Discrete Wavelet Transform of both the images.
- f) Let for first image OUT bands be HHa, HLa, LHa, LLa and for second image be HHb, HLb, LHb, LLb.

#### 4.3.1. Image fusion using Maximum Pixel replacement

- 1) Take the pixel having the maximum value of the two bands i.e. HHa and HHb, and put in HHn.
- 2) Take the pixel having the maximum value of the two bands i.e. HLa and HLb, and put in HLn.
- 3) Take the pixel having the maximum value of the two bands i.e. LHa and LHb, and put in LHn.
- 4) Take the pixel having the maximum value of the two bands i.e. LLa and LLb, and put in LLn.
- 5) Thus we will get HHn,HLn,LHn and LLn as new coefficients.
- 6) Take Inverse Discrete Wavelet Transform.
- 7) Obtain the fused Image and Display.

#### 4.3.2. Image Fusion using Pixels Averaging

- 8) Take the average of pixels of the two bands i.e. HHa and HHb, and put in HHn .
- 9) Take the average of pixels of the two bands i.e. HLa and HLb, and put in HLn.
- 10) Take the average of pixels of the two bands i.e. LHa and LHb, and put in LHn.
- 11) Take the average of pixels of the two bands i.e. LLa and LLb, and put in LLn.
- 12) Thus we will get HHn,HLn,LHn and LLn as new coefficients.
- 13) Take Inverse Discrete Wavelet Transform.
- 14) Obtain the fused Image and Display.

## 5. QUALITY MEASURES

### 5.1 Mean Square Error

$$MSE = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (A_{ij} - B_{ij})^2$$

### 5.2 Peak Signal to Noise Ratio

$$PSNR = 10 \times \log_{10} \left( \frac{peak^2}{MSE} \right)$$

### 5.3 Average Difference

$$AD = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (|A_{ij} - B_{ij}|)$$

### 5.4 Structural Content

$$SC = \frac{\sum_{i=1}^m \sum_{j=1}^n (A_{ij})^2}{\sum_{i=1}^m \sum_{j=1}^n (B_{ij})^2}$$

5.5 Normalized Cross Correlation

$$NCC = \frac{\sum_{i=1}^m \sum_{j=1}^n (A_{ij} * B_{ij})}{\sum_{i=1}^m \sum_{j=1}^n (A_{ij})^2}$$

5.6 Maximum Difference

$$MD = \max(|A_{ij} - B_{ij}|), i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

5.7 Normalized Absolute Error

$$NAE = \frac{\sum_{i=1}^m \sum_{j=1}^n (|A_{ij} - B_{ij}|)}{\sum_{i=1}^m \sum_{j=1}^n (A_{ij})}$$

**6. RESULTS**

Following images are results of fusion process with PCA, DCT & DWT fusion technique.

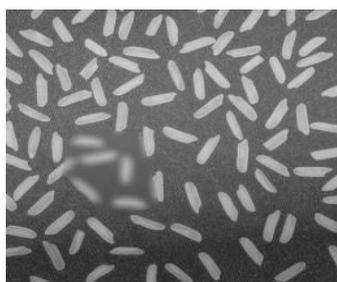


Figure (6a).First input image

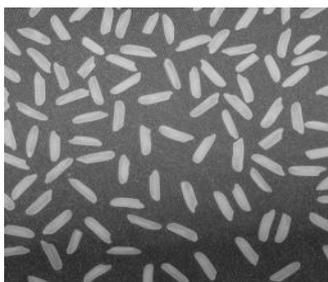


Figure (6b).Second input image

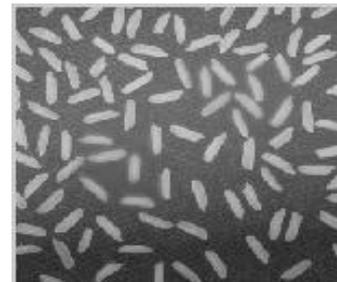


Figure (6c).Fused image using  
PCA

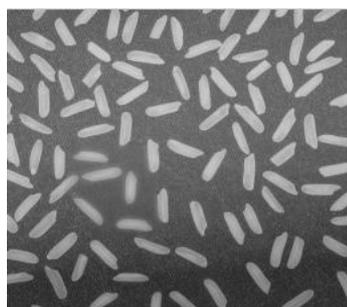


Figure (6d).Fused image using  
DCT (averaging method)

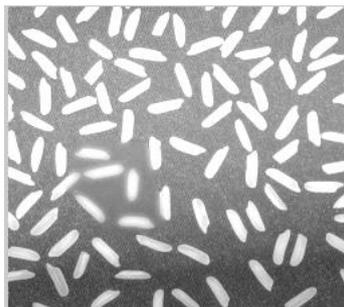


Figure (6e).Fused image using DCT  
(eq 15)

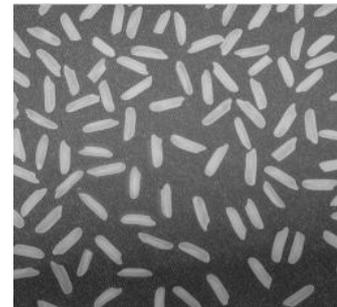


Figure (6f).Fused image using  
maximum pixel replacement

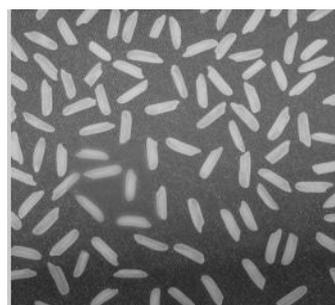


Figure (6g).Fused image using pixel averaging

Following Table demonstrates the various quality measures for different image fusion techniques.

**TABLE I**  
Fusion Techniques & Their Quality Measures

Fusion methods		Quality Measurements						
		Peak signal to noise ratio	Mean square error	Normalized absolute error	Maximum difference	Average difference	Normalized cross correlation	Structural content
PCA		35.8316	10.8669	0.0086	29	0.9573	0.9975	1.0043
DCT	Simple Averaging	39.7047	4.4544	0.0038	27.5000	0.4212	0.9990	1.0016
	By equation(15)	14.2300	3.5355e+03	0.4999	102	55.6173	1.4980	0.4454
DWT	Maximum pixel replacement	46.3594	1.5036	0.0021	21.4172	0.2329	1.0011	0.9977
	Pixel averaging	41.6429	4.4544	0.0038	27.5000	0.4212	0.9990	1.0016

### CONCLUSION

From the above output images and the values of quality measures presented in the table 1, it can be concluded that, PCA & DCT based image fusion technique can be used for applications which does not require high quality & precision. Whereas DWT based fusion techniques provide us good quality fused images than PCA & DCT based techniques.

### REFERENCES

- [1] V.P.S. Naidu and J.R. Raol, "Pixel-level Image Fusion using wavelets and Principal Component Analysis", *Defence Science Journal*, Vol. 58, No. 3, May 2008, pp. 338-352 02008, DESIDOC.
- [2] Nirosha Joshitha J, R. Medona Selin, "Image Fusion using PCA in Multifeature Based Palmprint Recognition", *International Journal of Soft Computing and Engineering (IJSCE)* ISSN: 2231-2307, Volume-2, Issue-2, May 2012.
- [3] VPS Naidu, "Discrete Cosine Transform based Image Fusion techniques," *Journal of Communication, Navigation and Signal Processing* (January 2012) Vol. I, No. I, pp. 35-45.
- [4] Yong Yang, Dong Sun Park, Shuying Huang, and Nini Rao, "Medical Image Fusion via an Effective Wavelet-Based Approach," *Hindawi Publishing Corporation EURASIP Journal on Advances in Signal Processing* Volume 2010, Article ID 579341, 13 pages doi:10.1155/2010/579341.
- [5] M.A.Berbar, S.F.Gahe, N.A.Ismail, "Image Fusion Using Multi Decomposition Levels of Discret Transform, 8 2003 The Institution of Electrical Engineers. Printed and published by IEE, Michael Faraday House, Six Hills Way, Stevenage, SG1 2AY.
- [6] V.P.S. Naidu "Discrete Cosine Transform-based Image Fusion" *Defence Science Journal*, Vol. 60, No. 1, January 2010, pp. 48-54" 2010, DESIDOC
- [7] VPS Naidu, Bindu Elias". A Novel Image Fusion Technique using DCT based Laplacian Pyramid" *International Journal of Inventive Engineering and Sciences (IJIES)* ISSN: 2319-9598, Volume-1, Issue-2

- [8] Ms. V.P.Sawant<sup>1</sup> “Fusion Algorithm for Imagesbased on Discrete Multi wavelet Transform” IOSR Journal of VLSI and Signal Processing (IOSR-JVSP) Volume 2, Issue 3 (May. – Jun. 2013), PP 22-27 e-ISSN: 2319 – 4200, p-ISSN No. : 231– 4197
- [9] Kanagaraj Kannan<sup>1</sup>, Subramonian Arumuga Perumal<sup>2</sup>, Kandasamy Arulmozhi<sup>3</sup>” Optimal Decomposition Level of Discrete, Stationary and Dual Tree Complex Wavelet Transform for Pixel based Fusion of Multi-focused Images” Serbian Journal of Electrical Engineering Vol. 7, No. 1, May 2010, 81-93 UDK: 004.932.4
- [10] V. P. S. Naidu “Novel Image Fusion Techniques using DCT International Journal of Computer Science and BusinessInformatics ISSN: 1694-2108 | Vol. 5, No.1. SEPTEMBER 2013