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Bondage Number of Interval Graphs

B. Maheswari[1]
Department of Applied Mathematics,
S.P.M.Visvavidyalayam,
Tirupati – 517502, A.P., India

A. Sudhakaraiah[2]
Department of Mathematics,
S.V.University,
Tirupati – 517502, A.P.India

Abstract---

Among the various applications of the theory of domination, the most often discussed is a communication network. This network consists of communication links between a fixed set of sites. The problem is to select a smallest set of sites at which transmitters are placed so that every site in the network that does not have a transmitter, is joined by a direct communication link to the site, which has a transmitter. Then this problem reduces to that of finding a minimum dominating set in the graph corresponding to this network. Suppose communication network does not work due to link failure. Then the problem is, what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites be possible. This leads to the introduction of the concept of bondage number of a graph. In this paper we discuss bondage number of interval graphs.

Keywords-- Interval family, Interval graph, Dominating Set, Bondage Number, Dominating Number

I . INTRODUCTION

The bondage number b(G) of a non – empty graph G is the minimum cardinality among all sets of edges E_1 for which $\gamma(G - E_1) > \gamma(G)$. Here $\gamma(G)$ indicates the domination number of G. Thus, the bondage number of G is the smallest number of edges whose removal will render every minimum dominating set in G a non – dominating set in the resultant spanning subgraph. Since the domination number of every spanning subgraph of a non – empty graph G is at least as great as $\gamma(G)$, the bondage number of a non – empty graph is well defined.

This concept was introduced by Fink et. al [3] and they have studied this parameter for some standard graphs, trees and general bounds are obtained.

2. BONDAGE NUMBER – INTERVAL GRAPHS

Let $I = \{I_1, I_2, \ldots, I_n\}$ be an interval family where each I_i is an interval on the real line and $I_i = [a_i, b_i]$ for i = 1,2,3,……,n. Here a_i is called the left endpoint and b_i the right endpoint of I_i. Without loss of generality, we assume that all endpoints of the intervals in I are distinct numbers between 1 and 2n. Two intervals i and j are said to intersect each other if they have non-empty intersection.

A graph G (V, E) is called an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect.

Let I={1,2,……n} be a given interval family where each I_i is represented by i and the intervals are labelled in the increasing order of their right endpoints. Let G be its corresponding interval graph. We assume that G is a connected graph.

Theorem1: Let G be an interval graph corresponding to the interval family I. Let i, j $\in$ I and suppose j is contained in i, $i \neq 1$ and there is no other interval that intersects j, other than i. Then b(G) = 1.

Proof: Let G be the interval graph corresponding to the given interval family I. Let i, j be any two intervals in I, which satisfy the hypothesis of the theorem. Then clearly $i \in D$, where D is a minimum dominating set of G because there is no other interval in I, other than i, that dominates j. Consider the edge e = (i, j) in G. If we remove this edge from G, then j becomes an isolated vertex in G – e, as there is no other vertex in G, other than i, that is adjacent with j. Hence $D_1 = D \cup \{ j \}$ becomes a dominating set of G – e and since D is a minimum dominating set of G it follows that D_1 is also a minimum dominating set of G – e. Therefore $|D_1| = \gamma(G - e) = |D| + 1 > |D|$. Thus b(G) =1.
As a special case of the above theorem, we have the following two corollaries.

Corollary 1: Let i = 2, j = 1, and j is contained in i. Suppose there is no interval other than i that intersects j. Then b (G) = 1.

Proof: Let i = 2, j = 1 satisfy the hypothesis of the corollary. Let e = (1, 2) and D be a minimum dominating set of G. The vertex 1 becomes an isolated vertex in G – e as there is no vertex other than 2 that is adjacent with 1.Then $D_1 = D \cup \{1\}$ becomes a minimum dominating set of G – e so that $\gamma(G - e) > \gamma(G)$. Thus b (G) = 1.
The proof of the following corollary follows on similar lines to that of corollary 1.

Corollary 2: Let i = n, j = n-1 and j is contained in i. Suppose there is no interval other than i that intersects j. Then b(G) = 1.

Theorem 2: Let the dominating set D of G consists of two vertices only, say u and v. Suppose u dominates the vertex set $S_1 = \{1,\ldots,i\}$ and v dominates the vertex set $S_2 =\{ i+1,\ldots,n \}$.

1. Suppose there is no vertex in S_1 other than u that dominates S_1 and no vertex in S_2 other than v that dominates S_2. Then b(G) = 1.

2. Suppose there is one more vertex $x \in S_1$ (or S_2) that dominates S_1 (or S_2). Then b(G) = 1.

Proof: 1. Let D = { u, v }. Suppose u and v satisfy the hypothesis of the theorem. Since u alone dominates S_1, there is no vertex in $S_3 = \{ 1,\ldots,i \} \setminus \{u\}$ that can dominate S_1. Let j be any vertex in S_3 and e = (u, j). Consider the graph G – e. In this graph, u dominates every vertex in S_1 except j. Now consider a vertex in S_1 which is adjacent with j, say k. Then clearly the set { u, k } dominates the set S_1 in G – e. If there is no vertex in S_1 that is adjacent with j, then clearly the graph G becomes disconnected. So there is at least one vertex in S_1 that is adjacent with j. Let us assume that there is a single vertex say w, $w \neq u$ such that w dominates the set S_1 in G – e. This implies that w also dominates the set S_1 in G, a contradiction, because by hypothesis u is the only vertex that dominates the set S_1 in G. Hence a single vertex cannot dominate S_1 in G – e.

Thus $D_1 = D \cup \{ k \}$ becomes a dominating set of G – e. Since D is minimum in G, D_1 is also minimum in G – e, so that $\gamma(G – e) > \gamma(G)$. Hence b(G) = 1. A similar argument with vertex v also gives b(G) = 1.

2. Let D = { u, v } and u dominates S_1 and v dominates S_2. Let $x \in S_1$ be such that x also dominates S_1. Let e = (u, x). Consider the graph G – e. In this graph the vertices u and x are not adjacent. Hence u alone cannot dominate the set S_1 in G – e. We require at least two vertices in S_1, which dominate S_1 in G – e. Therefore the dominating set of G – e contains more than two vertices. Thus $\gamma(G – e) > \gamma(G)$. Hence b(G) = 1. Similar is the case if $x \in S_2$.

Theorem 3: Let D = {u, v}. Suppose u dominates $S_1 = \{1,\ldots,i\}$ and v dominates $S_2 = \{i+1,\ldots,n\}$. Suppose there are two vertices say $w_1, w_2 \in S_1$ or S_2 such that w_1, w_2 also dominate S_1 or S_2 respectively. Then b(G) = 3.

Proof: Let D = {u, v} and u, v satisfy the hypothesis of the theorem. Suppose $w_1, w_2 \in S_1$ and w_1, w_2 also dominate S_1. Let k be an arbitrary vertex in S_1, $k \neq i, u, w_1, w_2$. Now delete the edges uk, w_1k, w_2k that are incident with k from G. If d(k) = 3, then k becomes an isolated vertex in $G_1 = G – \{uk, w_1k, w_2k\}$. Thus $D_1 = D \cup \{k\}$ becomes a dominating set of G_1 and since D is minimum it follows that D_1 is minimum in G_1. Hence $\gamma(G_1) > \gamma(G)$ and hence b(G) = 3.

Suppose d(k) > 3. Then there is at least one vertex, say j in S_1 such that j is adjacent to k and $j \neq u, w_1, w_2$. Let $G_1 = G – \{uk, w_1k, w_2k\}$. In G_1, k is not dominated by u, w_1, w_2, but is dominated by j. Further every vertex in S_1 other than k is dominated by u or w_1 or w_2 in G_1. Therefore every vertex in S_1 is dominated by {u, j} or $\{w_1, j\}$ or $\{w_2, j\}$ in G_1. Thus $D_1 = D \cup \{j\}$ becomes a dominating set of G_1 and since D is minimum in G it follows that D_1 is also minimum in G_1. Hence $\gamma(G_1) > \gamma(G)$ so that b(G) = 3. Similar is the case if $w_1, w_2 \in S_2$.

Theorem 4: Let D = { u, v, w }. Suppose u dominates $S_1 = \{1,\ldots,i\}$, v dominates $S_2 = \{ i+1,\ldots,j\}$, w dominates $S_3 = \{j+1,\ldots,n\}$.

1. There are no other vertices in S_1 or S_2 or S_3 that dominate the sets respectively. Then b(G) = 1.
2. Suppose there is one more vertex $x \in S_1$ or S_2 or S_3 that dominates S_1 or S_2 or S_3 respectively. Then b(G) = 1.

Proof: 1. The proof is similar to that of case 1 in Theorem 2.
2. The proof is similar to that of case 2 in Theorem 2.

Theorem 5: Let D = { u, v, w }. Suppose u dominates $S_1 = \{1, 2,\ldots,i\}$, v dominates $S_2 = \{ i+1,\ldots,j \}$, and w dominates $S_3 = \{ j+1,\ldots,n \}$. Suppose there are two vertices say $w_1, w_2 \in S_1$ or S_2 or S_3 such that w_1, w_2 also dominate S_1 or S_2 or S_3 respectively. Then b(G) = 3.
Proof: The proof is similar to that of Theorem 3.

II . CONCLUSION

we study the Bondage number of an Interval graph corresponding to an Interval family I. Given an interval model with end points sorted.

III. ACKNOWLEDGEMENT

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