Face Recognition Using Eigen Faces and Dimensionality Reduction by PCA

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Abstract— Face recognition is a key biometric technology with a wide range of potential applications. In this paper we present on of the idea of feature extraction using eigen faces. Dimensionality reduction techniques using linear transformations have been very popular in determining the intrinsic dimensionality of the manifold as well as extracting its principal directions (i.e., basis vectors). The most prominent method in this category is Principal Component Analysis PCA. In this we determine the Euclidean distance. Matlab is used for this algorithm as interface.

Keywords— Biometrics, Dimensionality Reduction, Eigen faces, Feature Extraction, Matching, Principal component analysis

I. INTRODUCTION

PCA is a variable reduction procedure and useful when obtained data have some redundancy. This will result into reduction of variables into smaller number of variables which are called Principal Components which will account for the most of the variance in the observed variable. Problems arise when we wish to perform recognition in a high-dimensional space. Goal of PCA is to reduce the dimensionality of the data by retaining as much as variation possible in our original data set. On the other hand dimensionality reduction implies information loss. The best low-dimensional space can be determined by best principal-components. The major advantage of PCA is using it in eigenface approach which helps in reducing the size of the database for recognition of a test images. The images are stored as their feature vectors in the database which are found out projecting each and every trained image to the set of Eigen faces obtained. PCA is applied on Eigen face approach to reduce the dimensionality of a large data set. The algorithm basically involves projecting a face onto a face space, which captures the maximum variation among faces in a mathematical form. During the training phase, each face image is represented as a column vector, with each entry corresponding to an image pixel. These image vectors are then normalized with respect to the average face. Next, the algorithm finds the eigenvectors of the covariance matrix of normalized faces by using a speedup technique that reduces the number of multiplications to be performed. This eigenvector matrix is then multiplied by each of the face vectors to obtain their corresponding face space projections. Lastly, the recognition threshold is computed by using the maximum distance between any two face projections. In the recognition phase, a subject face is normalized with respect to the average face and then projected onto face space using the eigenvector matrix. Next, the Euclidean distance is computed between this projection and all known projections. The minimum value of these comparisons is selected and compared with the threshold calculated during the training phase. Based on this, if the value is greater than the threshold, the face is new. Otherwise, it is a known face.

II. IMPLEMENTATION OF PRINCIPAL COMPONET ANALYSIS

In Principal component analysis (PCA) can generate one of the most valuable results from applied linear algebra. It is adequate and efficient method to be used in face recognition due to its simplicity, speed and learning capability. Eigen faces are a set of Eigen vectors used in the Computer Vision problem of human face recognition. They refer to an appearance based approach to face recognition that seeks to capture the variation in a collection of face images and use this information to encode and compare images of individual faces in a holistic manner..

III. DIMENSIONALITY REDUCTION

The main purpose of PCA is to reduce dimensionality as much as possible. Problem arise when oerforming recognition in higher-dimensional sub-space. Significance improvements can be achieved by first mapping the data in to a lower dimensional sub-space achievements.

$$x = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_N \end{bmatrix} --> reduce\ dimensionality --> y = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_K \end{bmatrix} \quad (K << N)$$

PCA allows us to compute linear transformation that maps data from high-dimensional sub-space to lower dimensional sub-space.

$$b_1 = t_{11}a_1 + t_{12}a_2 + \ldots + t_{1n}a_N$$
$$b_2 = t_{21}a_1 + t_{22}a_2 + \ldots + t_{2n}a_N$$
$$\ldots$$
$$b_K = t_{K1}a_1 + t_{K2}a_2 + \ldots + t_{KN}a_N$$

$$\text{or } y = Tx \text{ where } T = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1N} \\ t_{21} & t_{22} & \cdots & t_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ t_{K1} & t_{K2} & \cdots & t_{KN} \end{bmatrix}$$

a). *Vector representation in higher dimensional sub-space.*

$$x = a_1 v_1 + a_2 v_2 + \cdots + a_N v_N$$

$v_1, v_2, \ldots, v_N$ is a basis of the N-dimensional space

b). *Vector representation in lower dimensional sub-space.*

$$\hat{x} = b_1 u_1 + b_2 u_2 + \cdots + b_K u_K$$

$u_1, u_2, \ldots, u_K$ is a basis of the K-dimensional space

- *Note:* if both bases have the same size ($N = K$), then $x = \hat{x}$)

As shown in an example;-

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$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ (standard basis)}$$

$$x_v = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = 3v_1 + 3v_2 + 3v_3$$

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ (some other basis)}$$

$$x_u = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = 0u_1 + 0u_2 + 3u_3$$

$$\text{thus, } x_v = x_u$$

the best lower dimensional sub-space can be determined by the "best" eigen vectors of covariance matrix of x.

IV. EIGEN VECTORS FROM COVARIANCE MATRICES.

This solution is based on an important property of eigenvector decomposition. The data set is X which is an N x M matrices where N is the number of measurement types and M is the number of samples.Suppose there are m number of total images i.e $x_1, x_2, \ldots, x_M$ are N x 1 vectors. We have to find the mean image in order to find the normalize face vector:-

Step 1: $\bar{x} = \frac{1}{M} \sum_{i=1}^{M} x_i$

Step 2: subtract the mean: $\Phi_i = x_i - \bar{x}$

Step 3: form the matrix $A = [\Phi_1 \; \Phi_2 \cdots \Phi_M]$ (NxM matrix), then compute:

$$C = \frac{1}{M} \sum_{n=1}^{M} \Phi_n \Phi_n^T = AA^T$$

(sample **covariance** matrix, NxN, characterizes the *scatter* of the data)

Step 4: compute the eigenvalues of C: $\lambda_1 > \lambda_2 > \cdots > \lambda_N$

Step 5: compute the eigenvectors of C: $u_1, u_2, \ldots, u_N$

- Since C is symmetric, $u_1, u_2, \ldots, u_N$ form a basis, (i.e., any vector x or actually $(x - \bar{x})$, can be written as a linear combination of the eigenvectors):

$$x - \bar{x} = b_1 u_1 + b_2 u_2 + \cdots + b_N u_N = \sum_{i=1}^{N} b_i u_i$$

Step 6: (**dimensionality reduction step**) keep only the terms corresponding to the K largest eigenvalues:

$$\hat{x} - \bar{x} = \sum_{i=1}^{K} b_i u_i \text{ where } K << N$$

- The representation of $\hat{x} - \bar{x}$ into the basis $u_1, u_2, \ldots, u_K$ is thus

$$\begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_K \end{bmatrix}$$

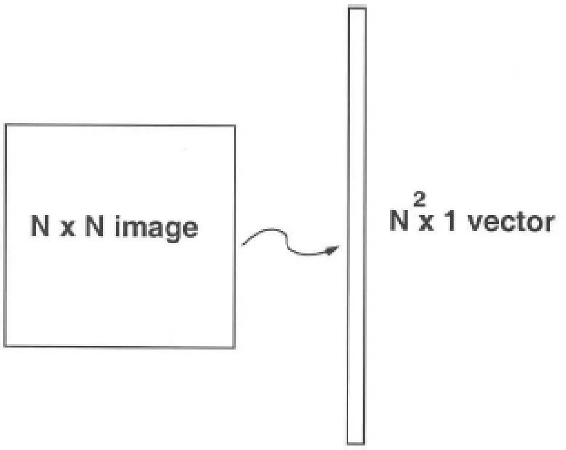
Thus, the linear transformation $R^N \to R^K$ that performs the dimensionality reduction is:

$$\begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_K \end{bmatrix} = \begin{bmatrix} u_1^T \\ u_2^T \\ \ldots \\ u_K^T \end{bmatrix} (x - \bar{x}) = U^T (x - \bar{x})$$

PCA works on the decomposition of covariance matrix

V. FACE RECOGNITION

In face recognition we have to match the faces. Problem arise when performing recognition in higher dimensional space. Now we have to find lower dimensional sub-space.



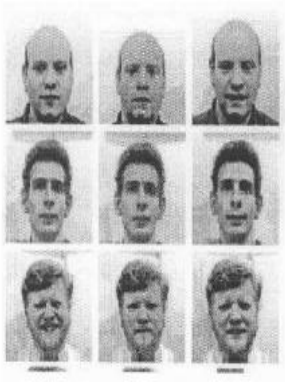
- Suppose Γ is an N^2x1 vector, corresponding to an NxN face image I.

- The idea is to represent Γ (Φ=Γ - mean face) into a low-dimensional space:

$$\hat{\Phi} - mean = w_1 u_1 + w_2 u_2 + \cdots w_K u_K \ (K << N^2)$$

Step 1: obtain face images $I_1, I_2, ..., I_M$ (training faces)

(**very important:** the face images must be *centered* and of the same *size*)



Step 2: represent every image I_i as a vector Γ_i

Step 3: compute the average face vector Ψ:

$$\Psi = \frac{1}{M} \sum_{i=1}^{M} \Gamma_i$$

Step 4: subtract the mean face:

$$\Phi_i = \Gamma_i - \Psi$$

Step 5: compute the covariance matrix C:

$$C = \frac{1}{M} \sum_{n=1}^{M} \Phi_n \Phi_n^T = AA^T \quad (N^2 \mathrm{x} N^2 \text{ matrix})$$

$$\text{where } A = [\Phi_1 \; \Phi_2 \cdots \Phi_M] \quad (N^2 \mathrm{x} M \text{ matrix})$$

Step 6: compute the eigenvectors u_i of AA^T

The matrix AA^T is very large --> not practical !!

Step 6.1: consider the matrix $A^T A$ (MxM matrix)

Step 6.2: compute the eigenvectors v_i of $A^T A$

$$A^T A v_i = \mu_i v_i$$

What is the relationship between us_i and v_i?

$A^T A v_i = \mu_i v_i$ => $AA^T A v_i = \mu_i A v_i$ =>

$CAv_i = \mu_i A v_i$ or $Cu_i = \mu_i u_i$ where $u_i = Av_i$

Thus, AA^T and $A^T A$ have the same eigenvalues and their eigenvectors are related as follows: $u_i = Av_i$!!

Note 1: AA^T can have up to N^2 eigenvalues and eigenvectors.

Note 2: $A^T A$ can have up to M eigenvalues and eigenvectors.

Note 3: The M eigenvalues of $A^T A$ (along with their corresponding eigenvectors) correspond to the M *largest* eigenvalues of AA^T (along with their corresponding eigenvectors).

Step 6.3: compute the M best eigenvectors of AA^T: $u_i = Av_i$

(**important:** normalize u_i such that $\|u_i\| = 1$)

Step 7: keep only K eigenvectors (corresponding to the K largest eigenvalues)

- Each normalized training face Φ_i is represented in this basis by a vector:

$$\Omega_i = \begin{bmatrix} w_1^i \\ w_2^i \\ \dots \\ w_K^i \end{bmatrix}, \quad i = 1, 2, \dots, M$$

VI. RECOGNITION USING EIGEN FACES

- Given an unknown face image Γ (centered and of the same size like the training faces) follow these steps:

Step 1: normalize Γ: $\Phi = \Gamma - \Psi$

Step 2: project on the eigenspace

$$\hat{\Phi} = \sum_{i=1}^{K} w_i u_i \quad (w_i = u_i^T \Phi)$$

Step 3: represent Φ as: $\Omega = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_K \end{bmatrix}$

Step 4: find $e_r = \min_l \|\Omega - \Omega^l\|$

Step 5: if $e_r < T_r$, then Γ is recognized as face l from the training set.

we can use the common Euclidean distance to compute e_r:-

$$\|\Omega - \Omega^k\| = \sum_{i=1}^{K} \frac{1}{\lambda_i} (w_i - w_i^k)^2$$

(variations along all axes are treated as equally significant)

VII. Procedures And Working

We have to create a function using matlab i.e
Function T = Create Database (TrainDatabasePath)
This creates a set of face images and reshapes all 2D images of the training database into 1D column vectors. Then, it (As from table I) puts these 1D column vectors in a row to construct 2D matrix.
Now, We use Principle Component Analysis to determine the most discriminating features between images of faces. This function *(function[m,A,Eigenfaces] = EigenfaceCore(T)* gets a 2D matrix, containing all training image vectors and returns three outputs which are extracted from the training database. In the arg., T is a 2D matrix, containing all 1D image vectors.

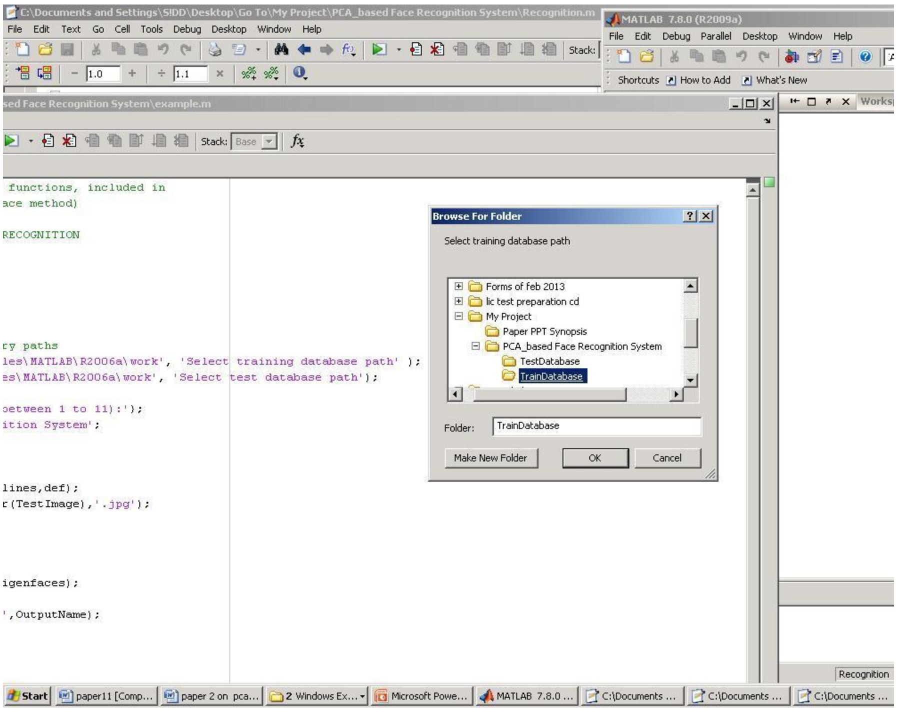


Fig. 1.Selection of Train Database Path

Let all the images in train database are of same size MxN

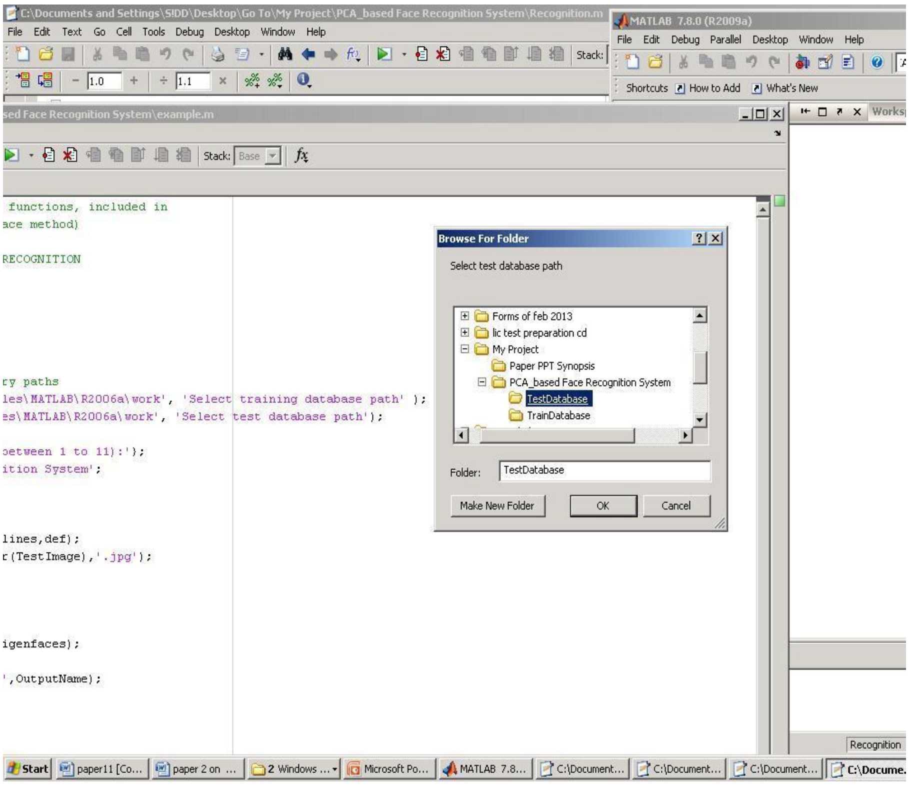


Fig. 2. Selection of Test Database Path

In Fig.2 we have to give the path of test database which contain all the images to be tested using principal component analysis after that we have to enter the number of images to be tested and verified from the training database whether it is a known person or unknown image.this whole idea totally works on the concept of PCA by using best K selected eigen faces from the higher dimensional sub-space. Here, dimensionality reduction plays a vital role in reducing the computational and time consuming mathematical process. This is how the face rocgnition through eigen faces is done.

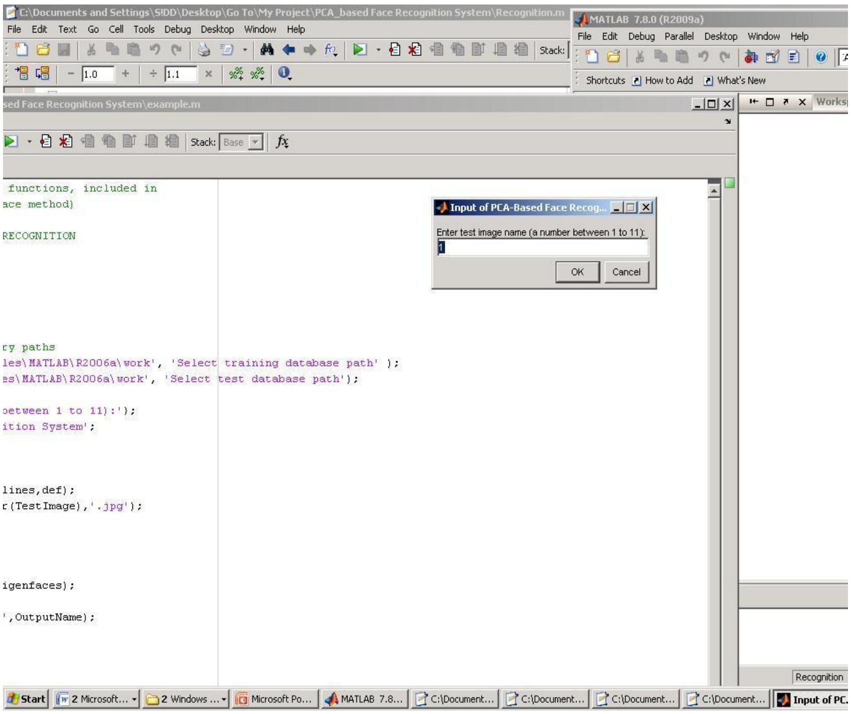


Fig. 3. Assigning the fields

VIII. Results And Conclusion

For the extraction of the PCA features from the test image. We have to calculate the Euclidean distance. This Euclidean distance is find between the input test image and the projection of all centered training images. The main objective of the whole procedure remains to have minimum distance with its corresponding image in the training database. The following figure illustrates the above mentioned step:-

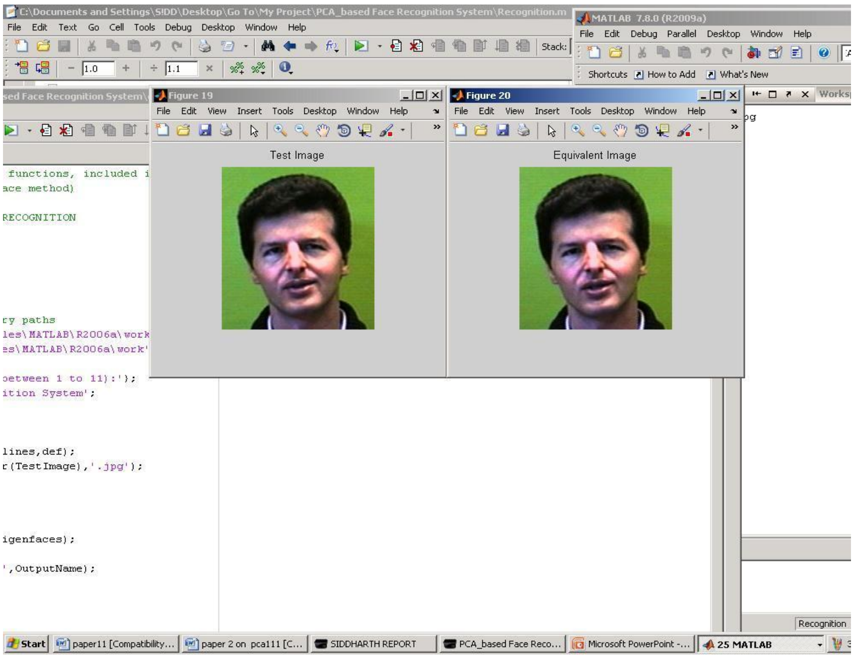


Fig. 4. Resultant output to the assigning field

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