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On Basics of Cosmology

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Abstract:

Some discussion of physical and geometrical interpretation of Einstein's theory of gravitation which is on basic of cosmology.

I. INTRODUCTION

The fundamental idea of geometrical theory of gravity start from the fact to assign four coordinates, say, (x,y,z,t) to any event observed in our vicinity. Locally space appears flat but this does not prefridge the global shape of the space.

The surface of a four-dimensional hyper-sphere:

$$(x')^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = a^2 \qquad (1)$$

Where a is the radius of the sphere .The distance between any two nearby point on this surface is

$$dl^2 = (dx')^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2 \qquad (2)$$

Here it should be noted that x^4 has nothing to do with time. It is extra and unphysical as curved three dimensional space is a subspace of a flat Euclidian space .Eq,(1) can be used to eliminate unphysical co-ordinate x^4 in (2):

$$dl^2 = (dx')^2 + (dx^2)^2 + (dx^3)^2 + \frac{(x'dx' + x^2dx^2 + x^3dx^3)^2}{a^2 - (x')^2 - (x^2)^2 - (x^3)^2} \qquad (3)$$

In this expression there appears only the physical co-ordinates x', x2 , x 3 .

II. THE CASE OF POSITIVE CURVATURE

Introduce the polar co-ordinates:

$$x' = r\sin\theta\cos\phi \qquad , x^2 = r\sin\theta\sin\phi \qquad x^3 = r\cos\theta$$

There the space interval equation (3) takes the form

$$dl^2 = \frac{dr^2}{1 - \frac{r^2}{a^2}} + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2 \qquad (4)$$

It is easy to see that rectangular coordinates x^k and the radial coordinate r are actually periodic coordinates.

To have understanding of our space of positive curvature, let us work at the radius and circumference of a wide placed in this space. For convenience, consider a circle defined by r=b= const. around the origin. This circle has as radius the distance between r=0 and r=b

$$\text{l= radius} = \int_0^b \frac{dr}{\sqrt{1 - r^2/a^2}} = a\sin^{-1}\left(\frac{b}{a}\right) \qquad (5)$$

If the circle is in the plane $\theta = \frac{\pi}{2}$

$$\text{Circumference} = \int_0^{2\pi} b\sin\theta\, d\phi = 2\pi b \qquad (6)$$

The ratio of radius to circumference is therefore

Larger than $1/2\pi$ (familiar property of spaces of positive curvature).

The surface area of the sphere r=b=const. surrounding origin is:

$$\text{Area} = \int_0^{2\pi}\int_0^{\pi} b^2 \sin\theta\, d\theta d\phi = 4\pi b^2$$

Hence the ratio of the radius squared to the area is larger than $1/4\pi$. For a radius larger than $\pi a/2$ the area decreases as radius increases. The volume inside the sphere r =b is

$$\text{Volume} = \int_0^{2\pi}\int_0^{\pi}\int_0^{b} \frac{r^2}{\sqrt{1 - \frac{r^2}{a^2}}}\, dr \sin\theta\, d\theta\, d\phi$$

$$=4\pi\left(\frac{a^2}{2}\sin^{-1}\frac{b}{a}-\frac{ba^2}{2}\sqrt{1-b^2/a^2}\right) \qquad (7)$$

To obtain the total volume of the three –sphere we must have to take

$$b=0 \quad , \sin^{-1}\frac{b}{a}=\pi$$

The bottom of sphere. Then , volume is given by $2\pi^2 a^3$. Our 3- sphere is a "closed space"; it has a finite volume though it has no boundaries.

Since r is a periodic coordinate, it is convenient to interoduce new angular coordinate x such that

$$r = a \sin x \qquad , 0< x < \pi$$

x is a single valued, so advantageous, and we get (4):

$$dl^2 = a^2\left[dx^2 + \sin^2 x + (d\theta^2 + \sin^2\theta\, d\phi^2\right] \qquad (8)$$

and

$$l = \text{radius distance} = ax \qquad (9)$$

from (5).

Now we add the time as the fourth coordinate and we have the space time interval for closed isotropic universes as:

$$ds^2 = c^2dt^2 - a^2(t)\left[dx^2 + \sin^2 x(d\theta^2 + \sin^2\theta\, d\phi^2\right]$$

If we replace time t by cosmic time η defined by

$$cdt = a(\eta)d\eta$$

Then

$$ds^2 = a^2(\eta)\left[d\eta^2 - dx^2 - \sin^2 x(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \qquad (10)$$

The Ricci tensor is entirely determined as

$$R_{00} = \frac{3}{a^2}\left(a\ddot{a} - \dot{a}^2\right)$$

$$R_{kn} = \frac{1}{a^4}\left(2a^2 + \dot{a}^2 + a\ddot{a}\right)g_{kn} \qquad \left(\frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta} = \frac{\dot{a}}{a}\right)$$

$$R = R_0^0 + R_k^k = \frac{6}{a^3}(a + \ddot{a})$$

The Einstein equation

$$R_i^k - \frac{1}{2}\delta_i^k R = -8\pi\, GT_i^k$$

00-component of the equation reduces to

$$-\frac{3}{a^4}(a^2 + \dot{a}^2) = -8\pi G T_0^0 \qquad (11)$$

Note that in this equation a is cancelled. This equation determines a(t) if T_{00} is given. The other components of Einstein equations can be regarded as ignored because they tell us nothing different from eq(11).

The main contribution towards energy density would be due to "particles" of one kind another (galaxies, hydrogen gas, etc.). We will neglect the pressure that these particles might exist. Hence

$$T_i^k = \rho u_i u^k$$

Where ρ is the proper mass density. In our co-moving coordinates, the matter is at rest and hence

$$T_0^0 = \rho$$

Again we have shown

$$\rho(t) = \frac{M}{2\pi^2 a^3}$$

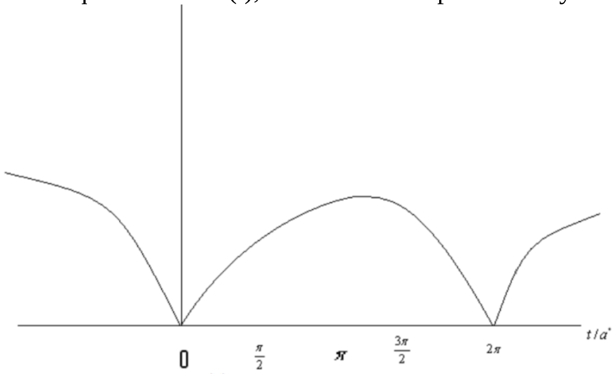
Where M is a constant as it is the "total mass of the universe". It is meaningless to talk of the mass of the universe from an operational point of view, M is really the sum of all the proper masses of the particles in the universe. But, such a sum will fail to account kinetic and binding energies to give the total mass .Hence we get

$$\frac{3}{a^4}\left(\dot{a}^2 + a^2\right) = \frac{4GM}{\pi a^3} \qquad (12)$$

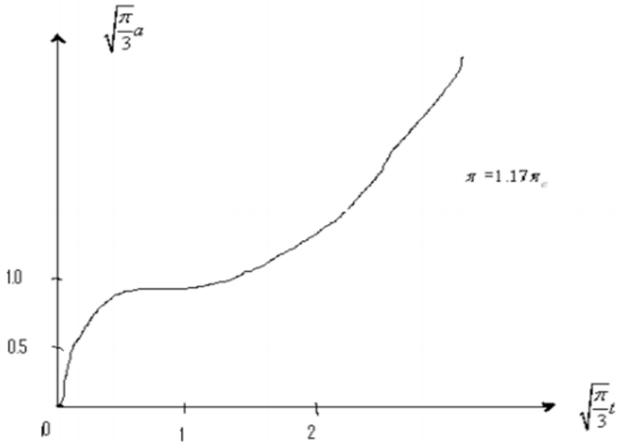
This is the differential equation that describes the closed Freidmaun model.
Its solution is,

$$a(\eta) = a^*(1-\cos\eta)$$
$$a^* = \text{—} \qquad (13)$$
$$t = a^*(\eta - \sin\eta)$$

So, this can be regarded as parametric equations for a (t), and the curve represented by these is a Cycloid:



at $t = 0, \pm 2\pi a^*, \pm 4\pi a^*, \ldots\ldots\ldots\ldots, a(t)$ vanishes , i.e. , the universe contracts to a point. Since the density becomes very large when this is about to happen, Our approximate expression for energy- momentum tensor will fail.



One can therefore conclude that the general Lemaitre model with a non zero mass density and $\Lambda < 0$ must necessarily be of oscillating type. If this universe was monotonic, it would gradually approach a monotonic empty model, which, as we have seen, does not exist.
So we have the basis of three observational facts:
The universe is in a state of uniform expansion.
The universe is filled with photons that come from black body background radiation.
The universe is isotropic on large scales beyond 1000 Mpc to construct or to accept, from GR, a generic cosmological modal.
We need a equation of state for the context of the universe to specify R(t) completely. We often find two cases where $p=0$ (dust) and $p= \frac{1}{3}\rho c^2$ (radiation).
Vacuum is a particular medium. Consider a piston with vacuum in it , and also assume simple vacuum is present outside. The energy inside piston is

$$E = \rho_v c^2 v$$

And if volume changes by a small account, net changes in energy are:

$$dE = d\left(\rho_v c^2 v\right) = -p_v dv$$

So, the equation of state is

$$p_v = -\rho_v c^2$$

Hence for $p \geq 0$, $\rho \geq 0$, there is only one solution $p=0$, $\rho=0$. This is the equation of state of vacuum. This can directly be introduced in the equation governing R(t) by introducing a constant

$$\Lambda = 8\pi G\rho_v$$

Such a term is called Cosmological constant and has been historically introduced by listening by Einstein to modify his theory. So we have

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{Rc^2}{3} + \frac{\Lambda}{3}$$

$$\dot{\rho} = -3\left(\frac{p}{c^2} + \rho\right)\frac{\dot{R}}{R}$$

III. CONCLUSION

Obsevation favors acceleration universe indicating existence of "dark energy" which can be identified with vacuum energy ar called "quintessence".

The Habble parameter $H = \frac{\dot{R}}{R}$

The density parameter $\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$

The deceleration parameter $q = -\frac{R\ddot{R}}{\dot{R}^2}$

The (reduced) cosmological constant $\Omega_\lambda = \lambda = \frac{\Lambda}{3H^2}$

The curvature parameter $\alpha = -\Omega_k = \frac{kc^2}{H^2R^2}$

In this light we have

$$\Omega_M + \Omega_k + \Omega_\lambda = 1$$
$$or \quad \alpha = \Omega + \lambda - 1$$

So that "radius of the universe "can" be written as $R = \frac{C}{H}\frac{1}{\sqrt{|\alpha|}}$

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